# 15-213

*"The course that gives CMU its Zip!"*

# Cache Memories
# Oct. 10, 2002

**Topics**

- Generic cache memory organization
- Direct mapped caches
- Set associative caches
- Impact of caches on performance



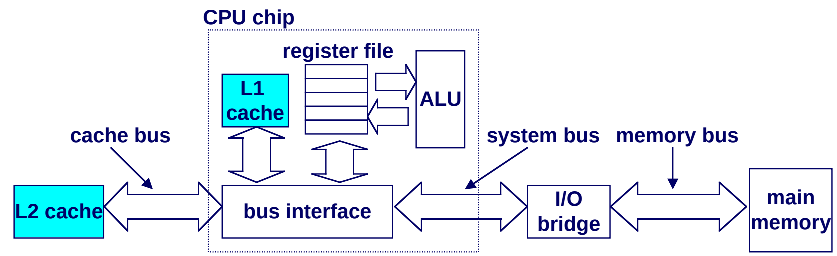
## Cache Memories

**Cache memories are small, fast SRAM-based memories managed automatically in hardware.**

- **Hold frequently accessed blocks of main memory**

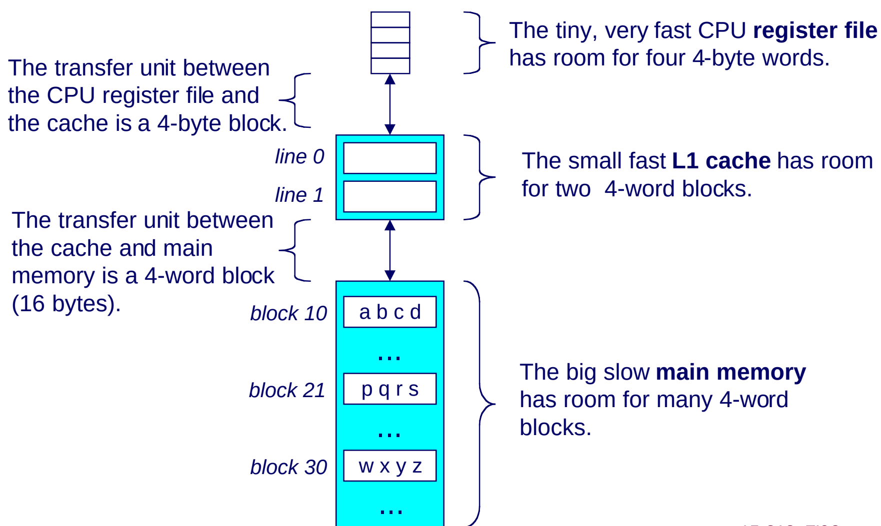
**CPU looks first for data in L1, then in L2, then in main memory.**

**Typical bus structure:**






## Inserting an L1 Cache Between the CPU and Main Memory



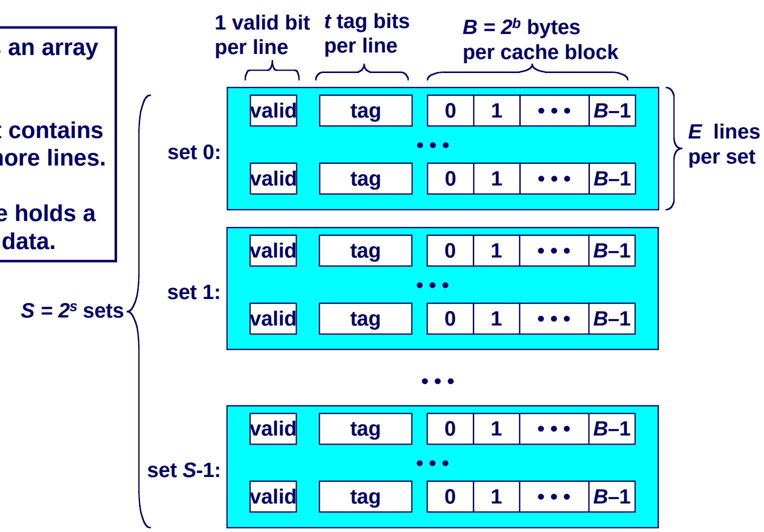



## General Org of a Cache Memory

**Cache is an array of sets.**

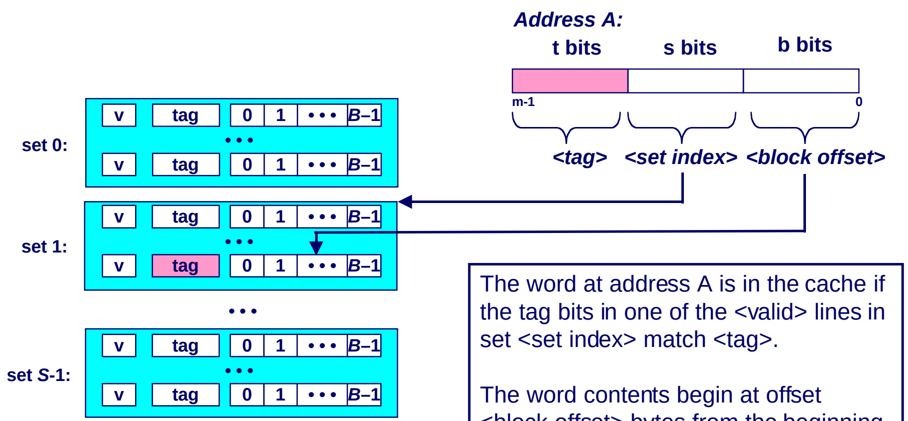
**Each set contains one or more lines.**

**Each line holds a block of data.**




**Cache size: $C = B \times E \times S$ data bytes**

# Addressing Caches




The word at address A is in the cache if the tag bits in one of the <valid> lines in set <set index> match <tag>.

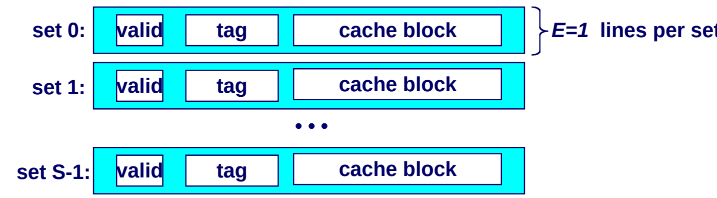
The word contents begin at offset <block offset> bytes from the beginning of the block.



# Direct-Mapped Cache

Simplest kind of cache

Characterized by exactly one line per set.

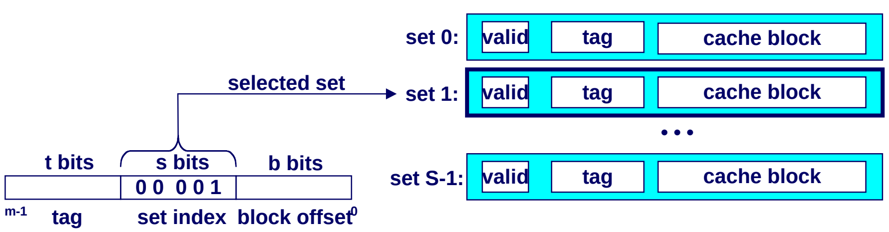





# Accessing Direct-Mapped Caches

## Set selection

- Use the set index bits to determine the set of interest.

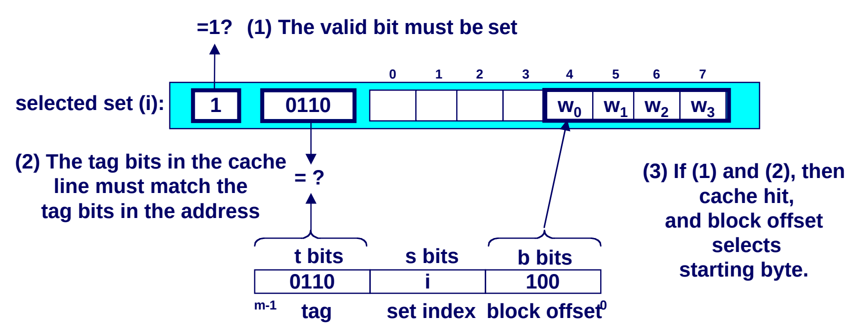





# Accessing Direct-Mapped Caches

## Line matching and word selection

- Line matching: Find a valid line in the selected set with a matching tag
- Word selection: Then extract the word



# Direct-Mapped Cache Simulation

| t=1 | s=2 | b=1 |
|---|---|---|
| x | xx | x |

**M=16 byte addresses, B=2 bytes/block,**
**S=4 sets, E=1 entry/set**

**Address trace (reads):**
**0 [$0000_2$], 1 [$0001_2$], 13 [$1101_2$], 8 [$1000_2$], 0 [$0000_2$]**

(1) 0 [$0000_2$] *(miss)*

| v | tag | data |
|---|---|---|
| 1 | 0 | M[0-1] |
| | | |
| | | |
| | | |

(3) 13 [$1101_2$] *(miss)*

| v | tag | data |
|---|---|---|
| 1 | 0 | M[0-1] |
| | | |
| 1 | 1 | M[12-13] |
| | | |

(4) 8 [$1000_2$] *(miss)*

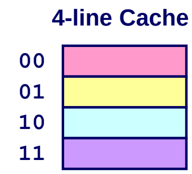
| v | tag | data |
|---|---|---|
| 1 | 1 | M[8-9] |
| | | |
| 1 | 1 | M[12-13] |
| | | |

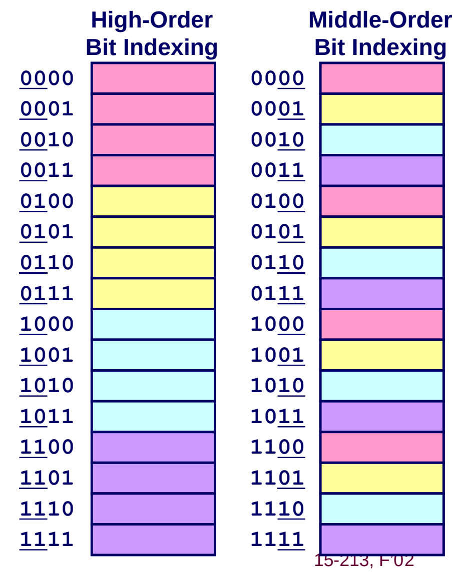
(5) 0 [$0000_2$] *(miss)*

| v | tag | data |
|---|---|---|
| 1 | 0 | M[0-1] |
| | | |
| 1 | 1 | M[12-13] |
| | | |



# Why Use Middle Bits as Index?



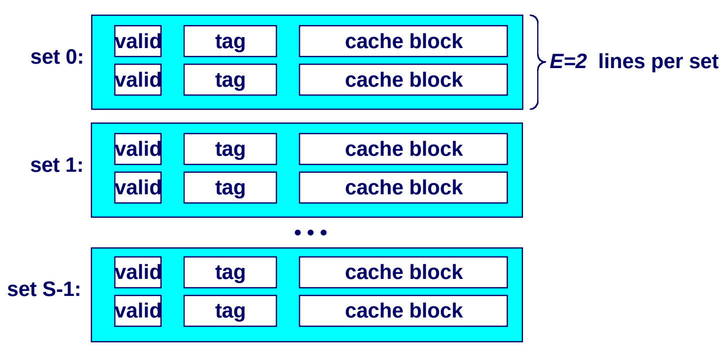

**High-Order Bit Indexing**
- **Adjacent memory lines would map to same cache entry**
- **Poor use of spatial locality**

**Middle-Order Bit Indexing**
- **Consecutive memory lines map to different cache lines**
- **Can hold C-byte region of address space in cache at one time**



# Set Associative Caches

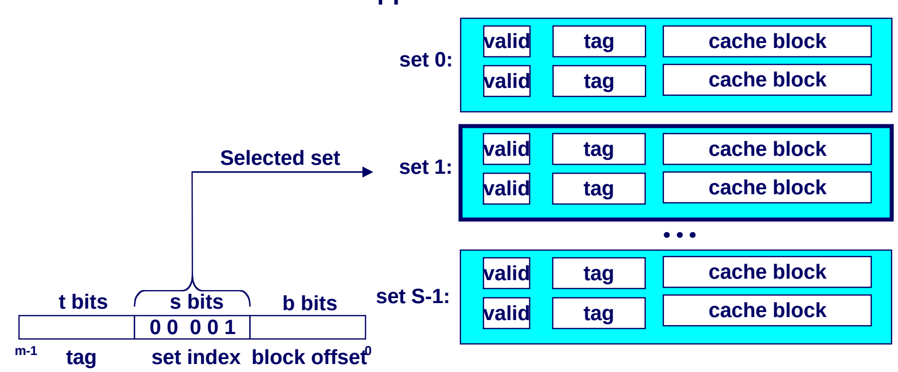
**Characterized by more than one line per set**






# Accessing Set Associative Caches

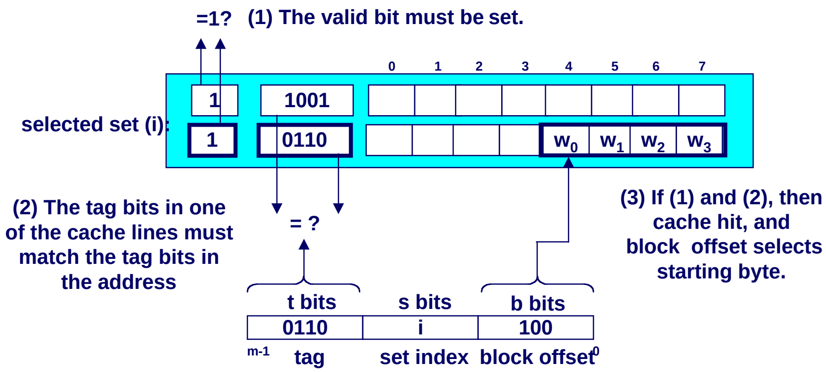
**Set selection**
- **identical to direct-mapped cache**



# Accessing Set Associative Caches

## Line matching and word selection

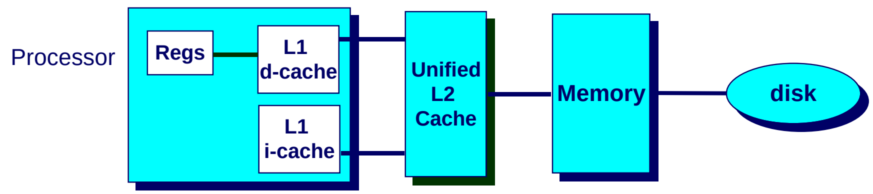
- must compare the tag in each valid line in the selected set.






# Multi-Level Caches

## Options: separate data and instruction caches, or a unified cache

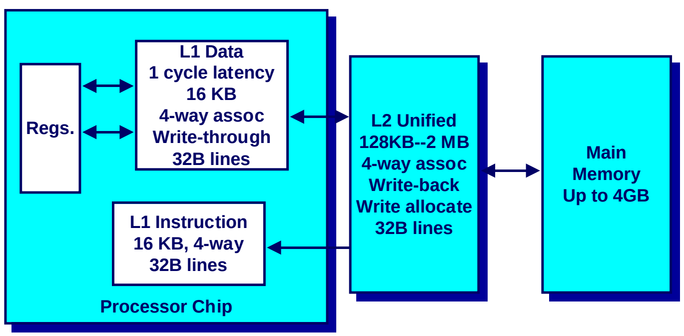



| | | | | | |
|---|---|---|---|---|---|
| size: | 200 B | 8-64 KB | 1-4MB SRAM | 128 MB DRAM | 30 GB |
| speed: | 3 ns | 3 ns | 6 ns | 60 ns | 8 ms |
| $/Mbyte: | | | $100/MB | $1.50/MB | $0.05/MB |
| line size: | 8 B | 32 B | 32 B | 8 KB | |

larger, slower, cheaper



# Intel Pentium Cache Hierarchy






# Cache Performance Metrics

## Miss Rate

- Fraction of memory references not found in cache (misses/references)
- Typical numbers:
  - 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.

## Hit Time

- Time to deliver a line in the cache to the processor (includes time to determine whether the line is in the cache)
- Typical numbers:
  - 1 clock cycle for L1
  - 3-8 clock cycles for L2

## Miss Penalty

- Additional time required because of a miss
  - Typically 25-100 cycles for main memory

# Writing Cache Friendly Code

**Repeated references to variables are good (temporal locality)**

**Stride-1 reference patterns are good (spatial locality)**

**Examples:**

- cold cache, 4-byte words, 4-word cache blocks

```
int sumarrayrows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = 1/4 = 25%

```
int sumarraycols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```

Miss rate = 100%



# The Memory Mountain

**Read throughput (read bandwidth)**

- Number of bytes read from memory per second (MB/s)

**Memory mountain**

- Measured read throughput as a function of spatial and temporal locality.
- Compact way to characterize memory system performance.



# Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride);                     /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0);  /* call test(elems,stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```



# Memory Mountain Main Routine

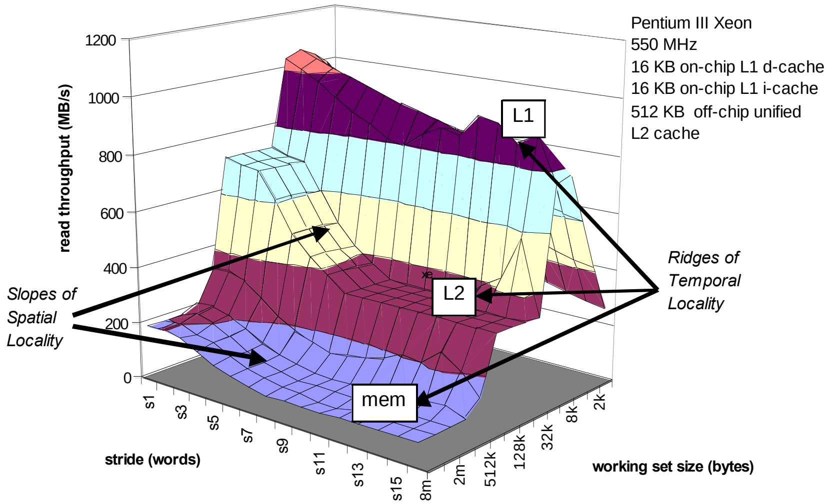
```
/* mountain.c - Generate the memory mountain. */
#define MINBYTES (1 << 10)  /* Working set size ranges from 1 KB */
#define MAXBYTES (1 << 23)  /* ... up to 8 MB */
#define MAXSTRIDE 16        /* Strides range from 1 to 16 */
#define MAXELEMS MAXBYTES/sizeof(int)

int data[MAXELEMS];         /* The array we'll be traversing */

int main()
{
    int size;        /* Working set size (in bytes) */
    int stride;      /* Stride (in array elements) */
    double Mhz;      /* Clock frequency */

    init_data(data, MAXELEMS); /* Initialize each element in data to 1 */
    Mhz = mhz(0);              /* Estimate the clock frequency */
    for (size = MAXBYTES; size >= MINBYTES; size >>= 1) {
        for (stride = 1; stride <= MAXSTRIDE; stride++)
            printf("%.1f\t", run(size, stride, Mhz));
        printf("\n");
    }
    exit(0);
}
```

# The Memory Mountain

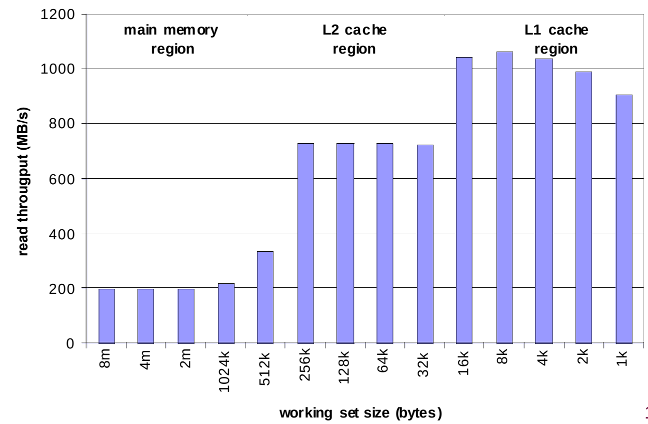





# Ridges of Temporal Locality

## Slice through the memory mountain with stride=1

- illuminates read throughputs of different caches and memory

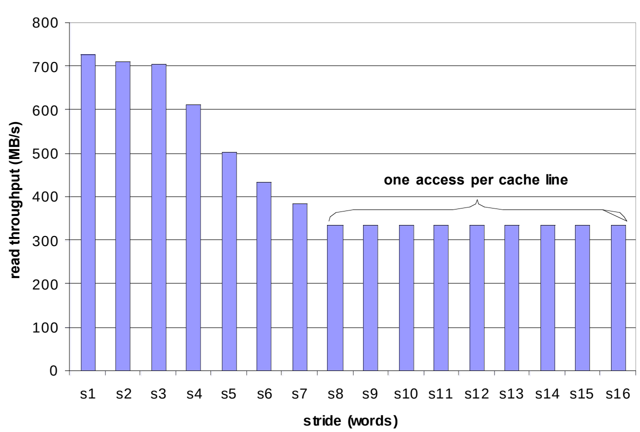





# A Slope of Spatial Locality

## Slice through memory mountain with size=256KB

- shows cache block size.






# Matrix Multiplication Example

## Major Cache Effects to Consider

- Total cache size
  - Exploit temporal locality and keep the working set small (e.g., by using blocking)
- Block size
  - Exploit spatial locality

## Description:

- Multiply N x N matrices
- O(N3) total operations
- Accesses
  - N reads per source element
  - N values summed per destination
    - » but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

*Variable **sum** held in register*

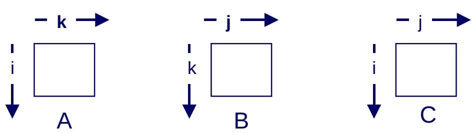
# Miss Rate Analysis for Matrix Multiply

## Assume:

- Line size = 32B (big enough for 4 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

## Analysis Method:

- Look at access pattern of inner loop






# Layout of C Arrays in Memory (review)

## C arrays allocated in row-major order

- each row in contiguous memory locations

## Stepping through columns in one row:

- ```
  for (i = 0; i < N; i++)
    sum += a[0][i];
  ```
- accesses successive elements
- if block size (B) > 4 bytes, exploit spatial locality
  - compulsory miss rate = 4 bytes / B

## Stepping through rows in one column:

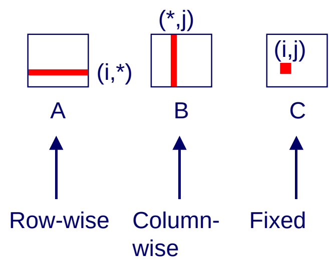
- ```
  for (i = 0; i < n; i++)
    sum += a[i][0];
  ```
- accesses distant elements
- no spatial locality!
  - compulsory miss rate = 1 (i.e. 100%)



# Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:




## Misses per Inner Loop Iteration:

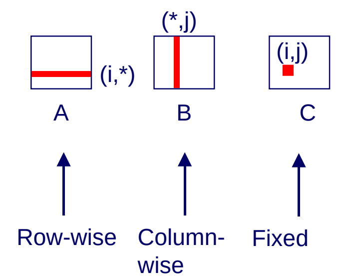
| A | B | C |
|---|---|---|
| 0.25 | 1.0 | 0.0 |



# Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:

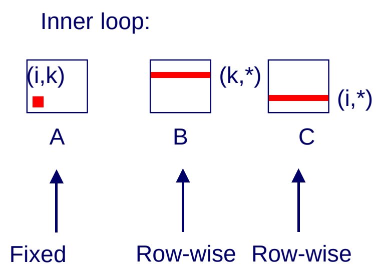



## Misses per Inner Loop Iteration:

| A | B | C |
|---|---|---|
| 0.25 | 1.0 | 0.0 |

# Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```



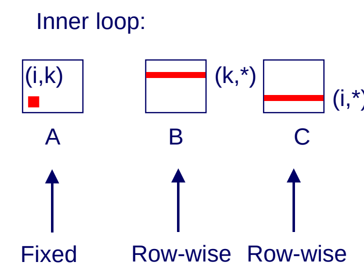

## Misses per Inner Loop Iteration:

| A | B | C |
|---|---|---|
| 0.0 | 0.25 | 0.25 |



# Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```



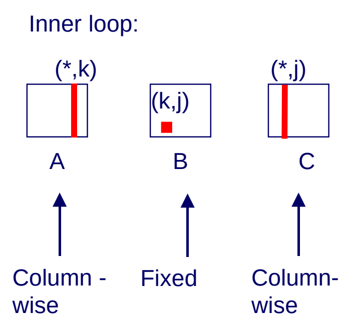

## Misses per Inner Loop Iteration:

| A | B | C |
|---|---|---|
| 0.0 | 0.25 | 0.25 |



# Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



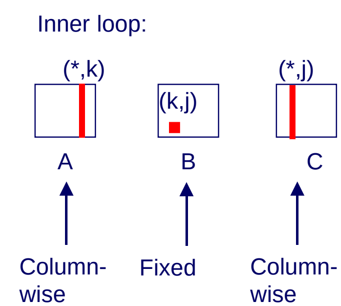

## Misses per Inner Loop Iteration:

| A | B | C |
|---|---|---|
| 1.0 | 0.0 | 1.0 |



# Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```




## Misses per Inner Loop Iteration:

| A | B | C |
|---|---|---|
| 1.0 | 0.0 | 1.0 |

# Summary of Matrix Multiplication

**ijk (& jik):**
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

**kij (& ikj):**
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

**jki (& kji):**
- 2 loads, 1 store
- misses/iter = **2.0**

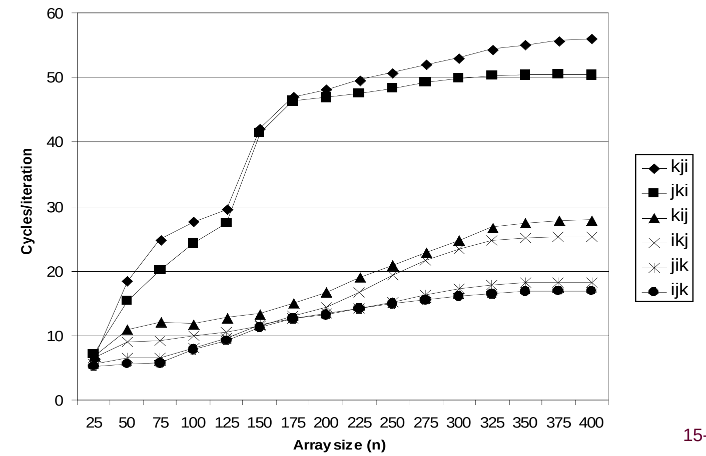
```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



# Pentium Matrix Multiply Performance

## Miss rates are helpful but not perfect predictors.

- Code scheduling matters, too.






# Improving Temporal Locality by Blocking

## Example: Blocked matrix multiplication

- "block" (in this context) does not mean "cache block".
- Instead, it mean a sub-block within the matrix.
- Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., $\mathbf{A}_{xy}$) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$



# Blocked Matrix Multiply (bijk)

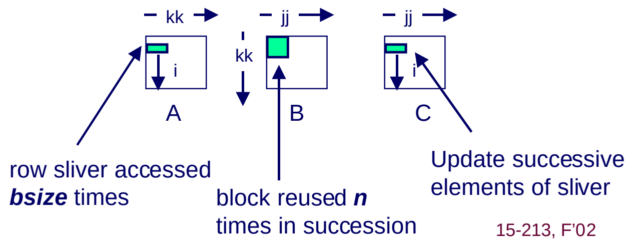
```
for (jj=0; jj<n; jj+=bsize) {
  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;
  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

# Blocked Matrix Multiply Analysis

- Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C*
- Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B*

```
for (i=0; i<n; i++) {
   for (j=jj; j < min(jj+bsize,n); j++) {
     sum = 0.0
     for (k=kk; k < min(kk+bsize,n); k++) {
       sum += a[i][k] * b[k][j];
     }
     c[i][j] += sum;
   }
```

Innermost Loop Pair

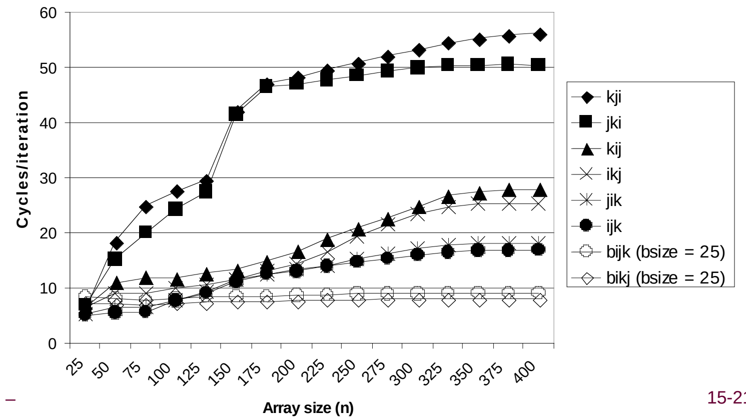





# Pentium Blocked Matrix Multiply Performance

## Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)

- relatively insensitive to array size.






# Concluding Observations

## Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
  - Nested loop structure
  - Blocking is a general technique

## All systems favor "cache friendly code"

- Getting absolute optimum performance is very platform specific
  - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)