Strengthening Zero-Knowledge Protocols using Signatures

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Zero Knowledge Proof Protocols

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[hundreds of papers here...]
many many applications in cryptography

- identification protocols
- two-party/multi-party computation
- ...

A Quick Review of ZK Proofs

A protocol between Prover and Verifier.

- Completeness — if $x \in L$ then Verifier always accepts
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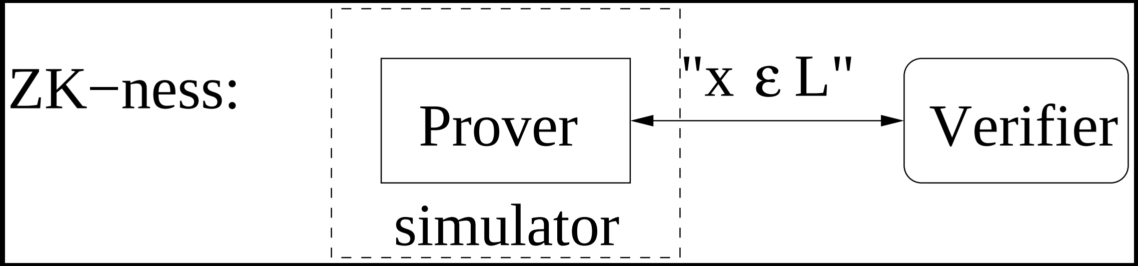

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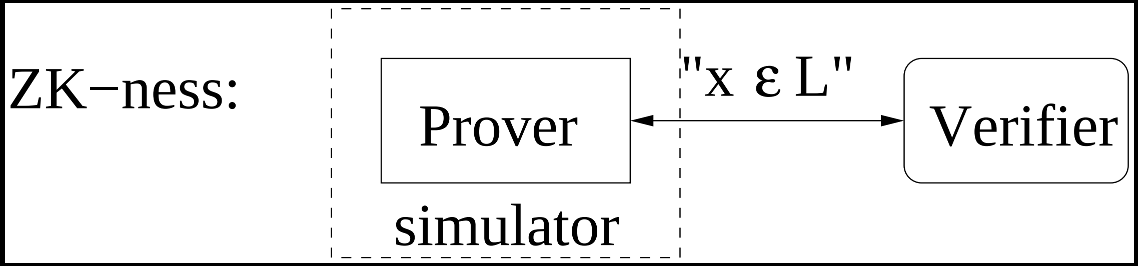

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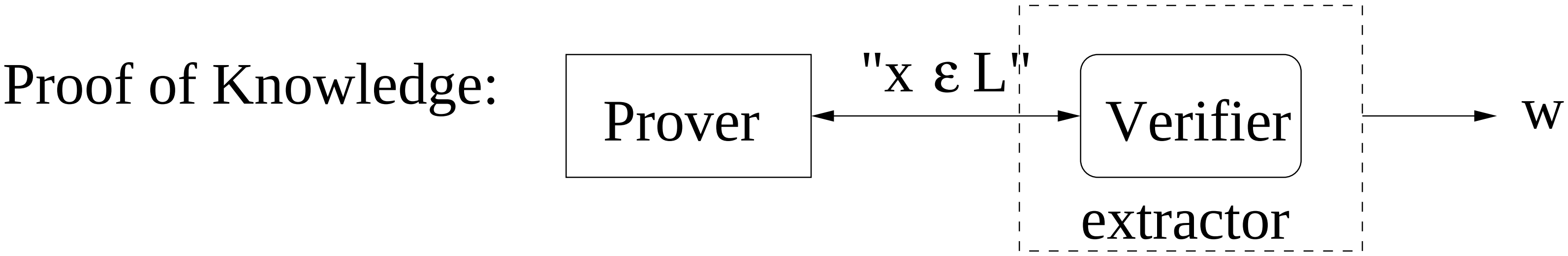

- POK — an extractor produces a witness w from interaction

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- Universal Composability [Canetti 00]
  secure when arbitrarily composed
  (desirable for modularity)

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- Easy in the common reference string (CRS) model
  [Damgård 00] constant round ZK (simulator generates CRS)

We work in the CRS model.

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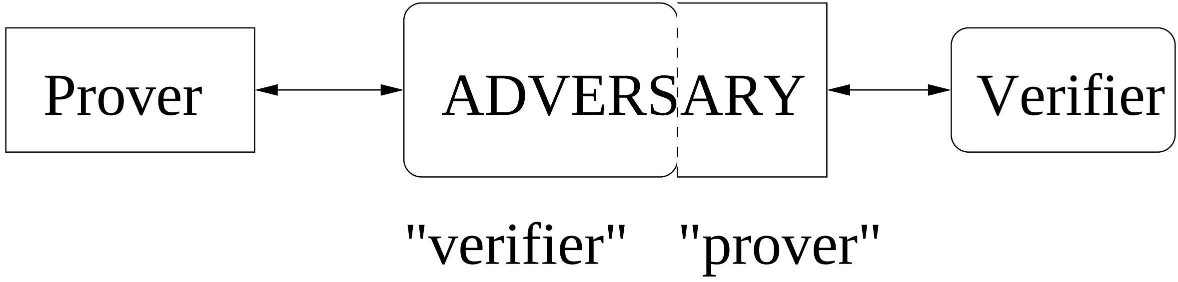

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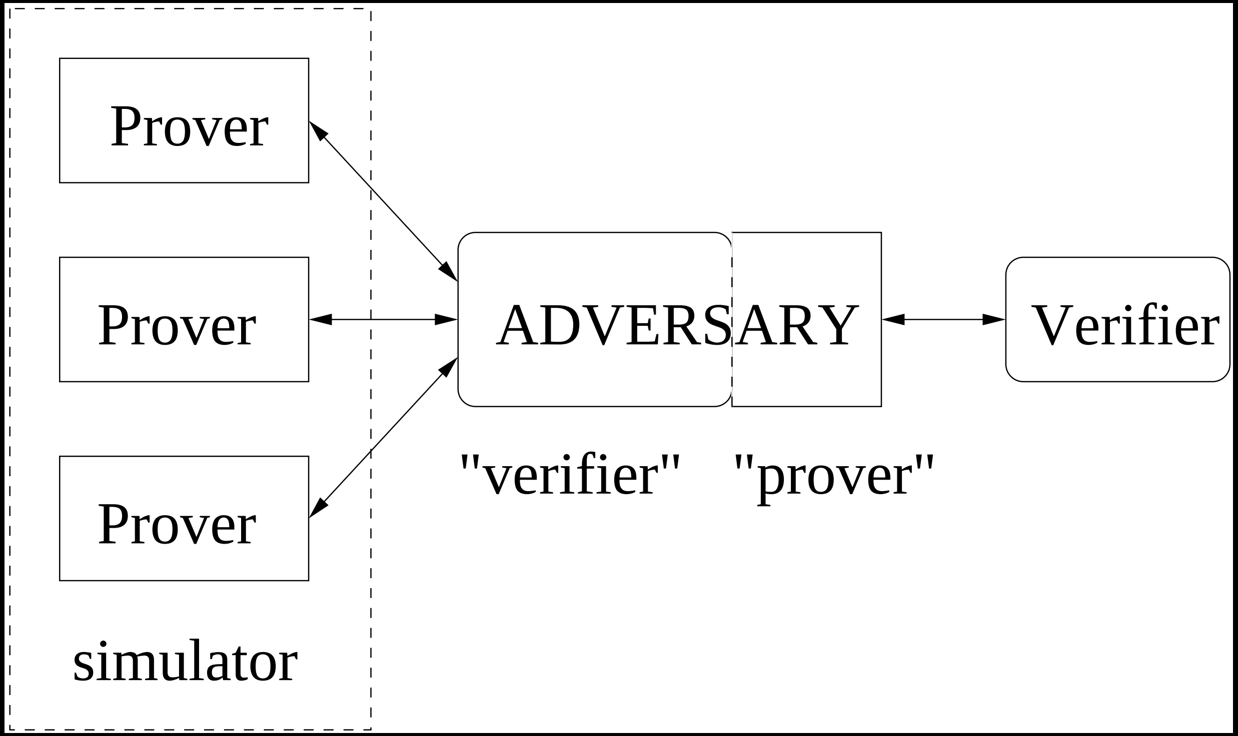

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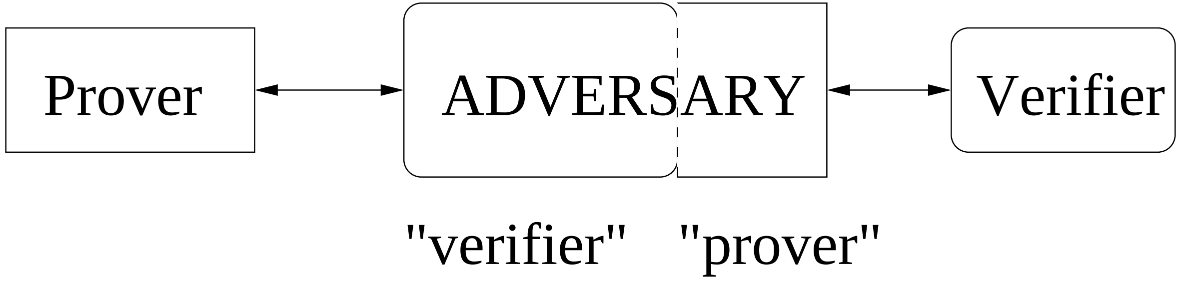

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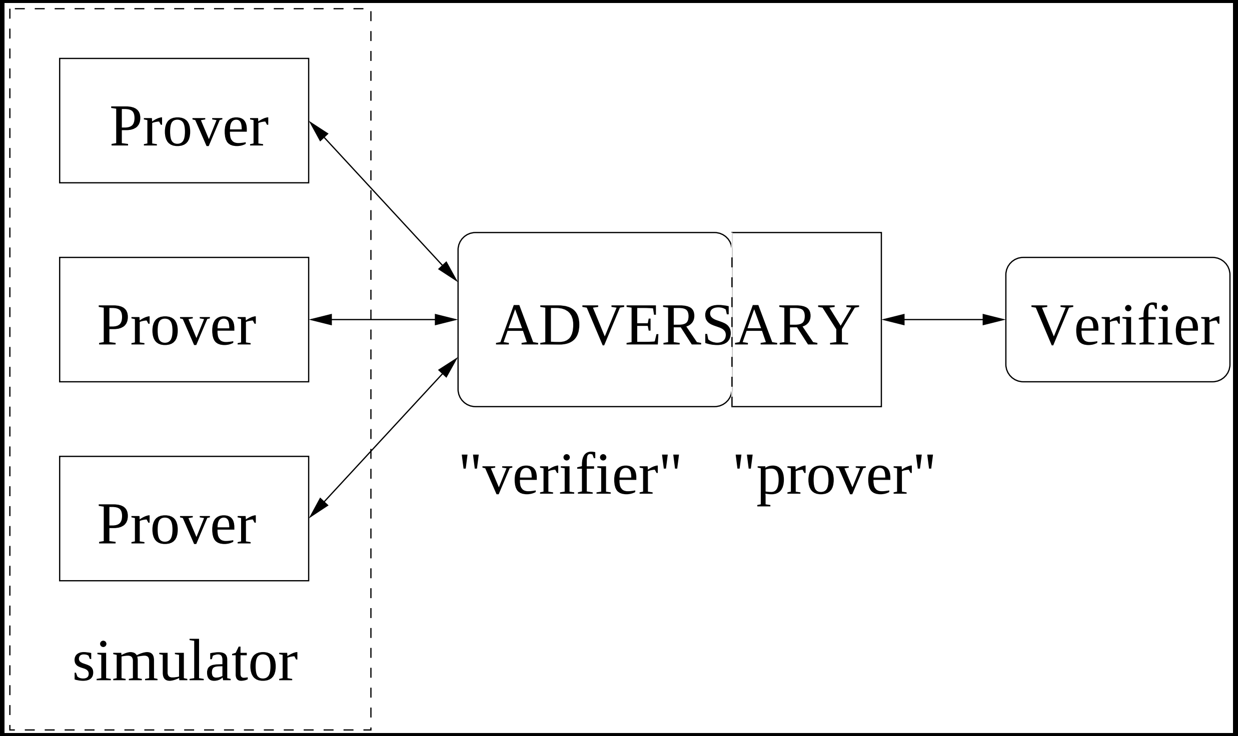
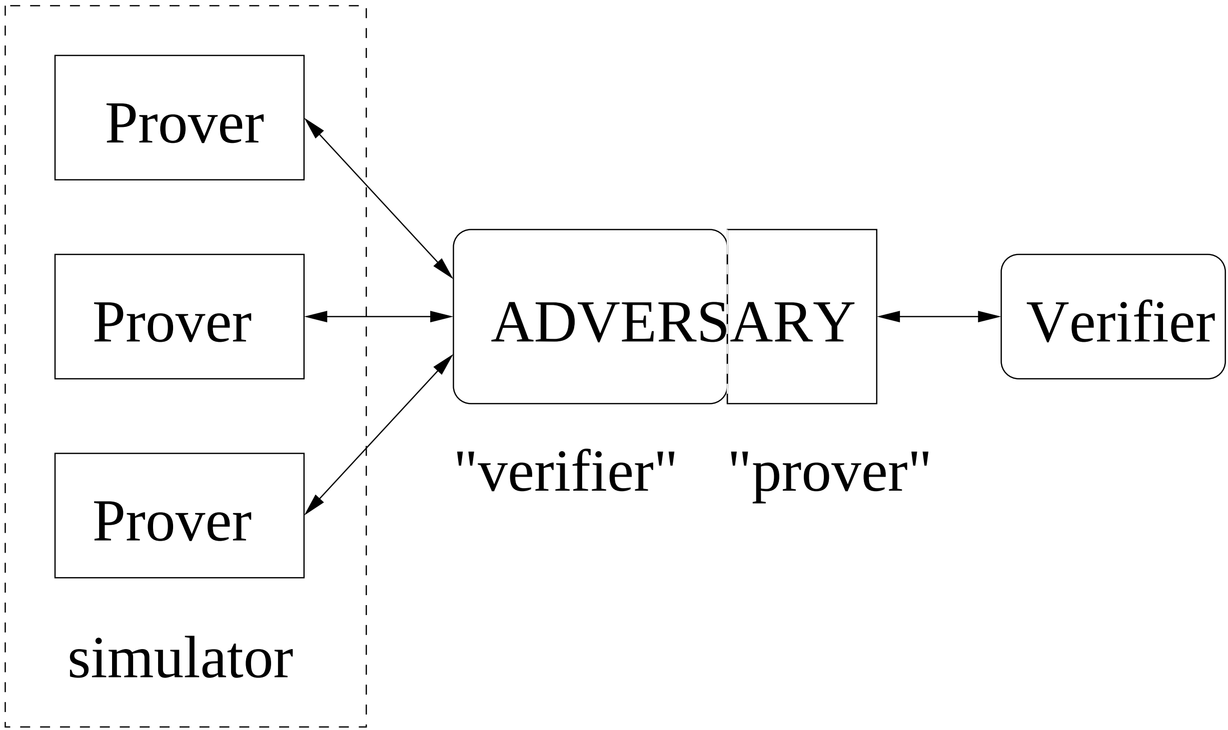


Interactive Simulation Sound ZK



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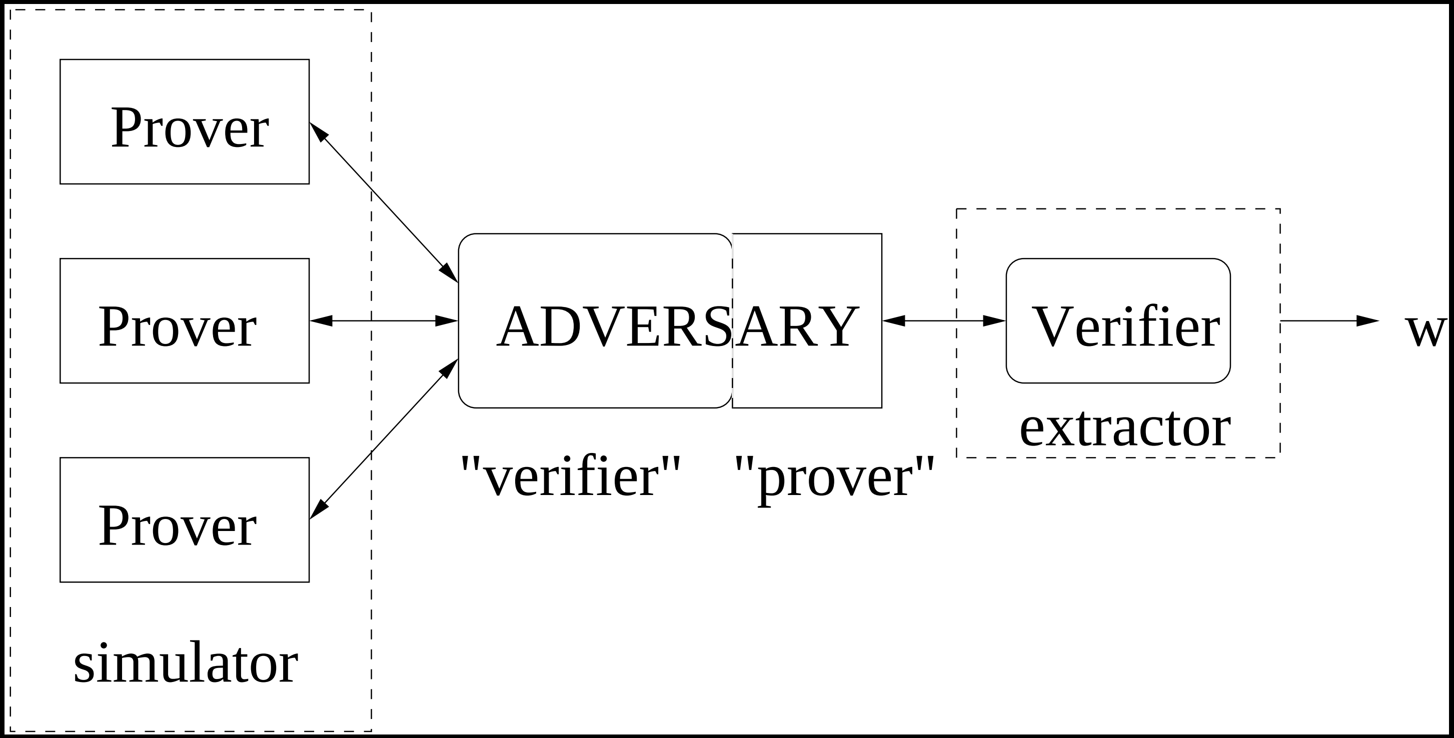
Interactive Non-malleable ZK



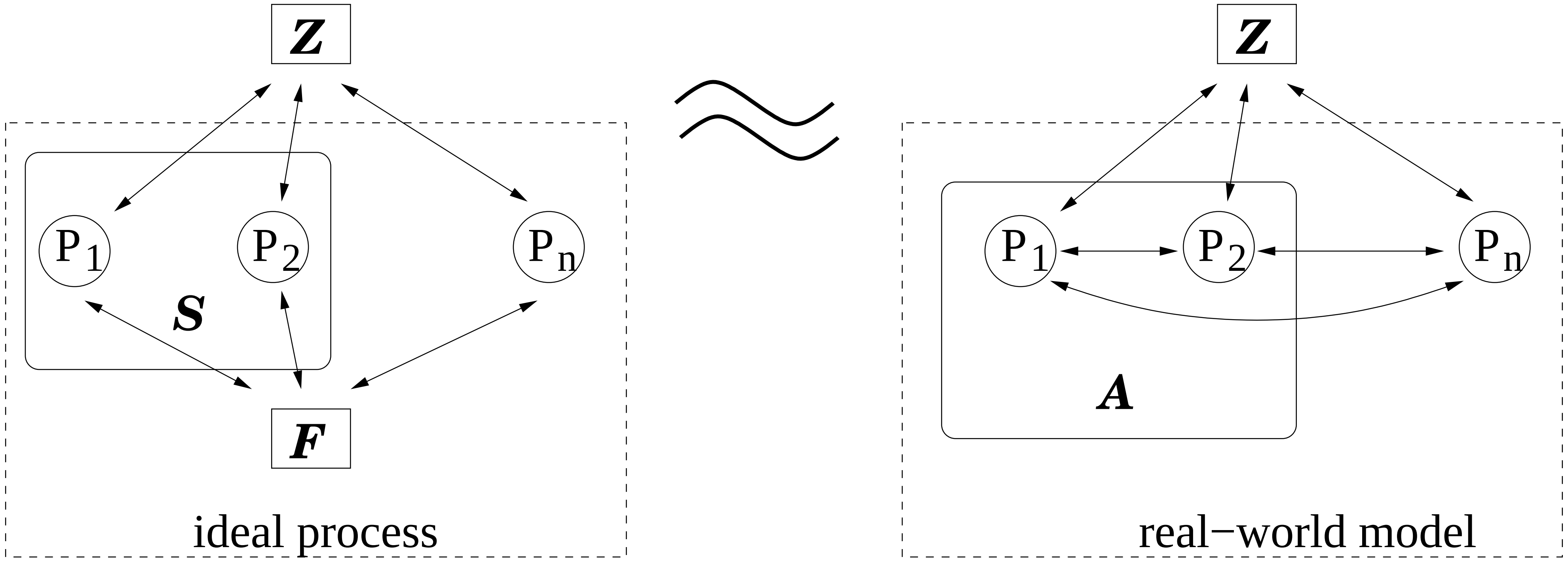
- We allow $\mathcal{A}$ to concurrently interact with many simulated provers.
- Anything $\mathcal{A}$ proves, a witness can be extracted.
- Roughly speaking,
  Non-malleable ZK = Simulation Sound ZK + non-rewinding POK.

Non-malleable/Simulation Sound ZK: Known Constructions

- [Dolev Dwork Naor 91]
  one-time non-malleable ZK, polylogarithmic rounds, plain model
- [Barak 02]
  one-time non-malleable ZK, constant rounds, plain model
- [Katz 03]
  one-time non-malleable ZK, three rounds, CRS model
- [Sahai 00]
  unbounded simulation-sound NIZK
- [De Santis, Di Crescenzo, Ostrovsky, Persiano, Sahai 01]
  unbounded non-malleable NIZK

Universally Composable ZK

- [Canetti 00]
  Universal Composability: a framework for defining (very strong) security that allows arbitrary composition



UCZK: Known Results

- Roughly speaking

  UCZK $\sim$ unbounded non-malleable ZK
- [Canetti 00]

  UCZK impossible in plain model
- [Canetti Fischlin 01]

  three round UCZK in CRS model, adaptive corruption
- [Canetti Lindell Ostrovsky Sahai 02]

  The DDOPS01 construction is NIUCZK, non-adaptive corruption

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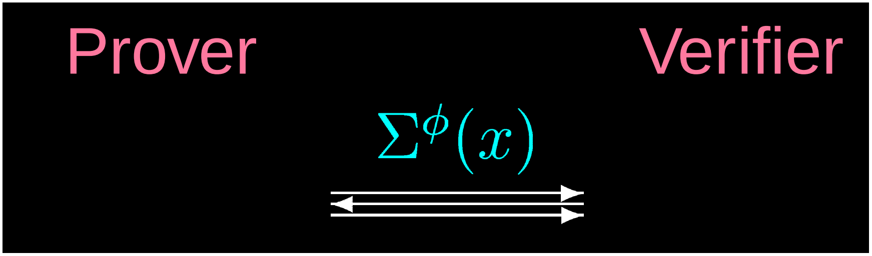
What's special about our technique?

- conceptually simple
- efficient
  - three rounds, small additive overhead (const. pub. key op's)
  - completely avoid the Cook-Levin Theorem
    (c.f. Micciancio and Petrank,
    "Simulatable Commitments and Efficient Concurrent Zero-Knowledge,"
    an hour ago.)

Ideas of the Conversion

Start with a Σ-protocol

Π= “ϕ is true.”



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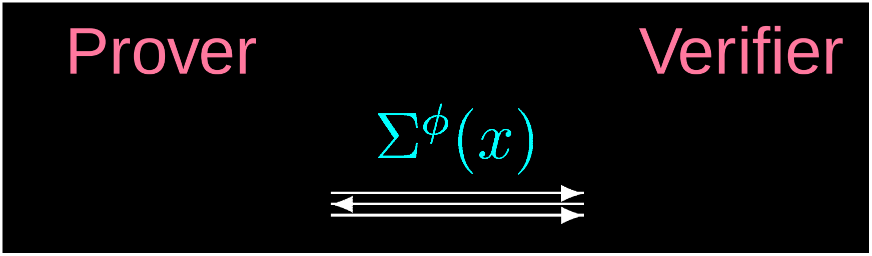

Prover

Verifier

$\Sigma^{\phi}(x)$

Convert to

Π' = “Either ϕ is true, or I know a signature for message m w.r.t. vk.”

Protocol in More Details

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- vk is from a digital signature scheme SIG $=$ (Gen, Sign, Verify) existential unforgeable against chosen message attack.
- vk is in the common reference string (sk unknown).
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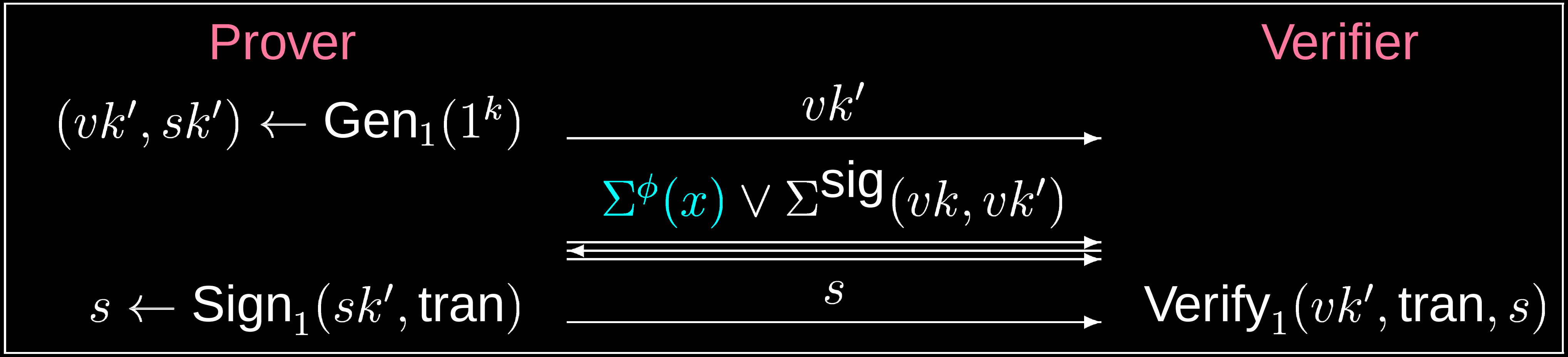

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Verifier

$(vk', sk') \leftarrow \text{Gen}_1(1^k)$ $\xrightarrow{vk'}$

$\Sigma^{\phi}(x) \vee \Sigma^{\text{sig}}(vk, vk')$

$s \leftarrow \text{Sign}_1(sk', \text{tran})$ $\xrightarrow{s}$ $\text{Verify}_1(vk', \text{tran}, s)$

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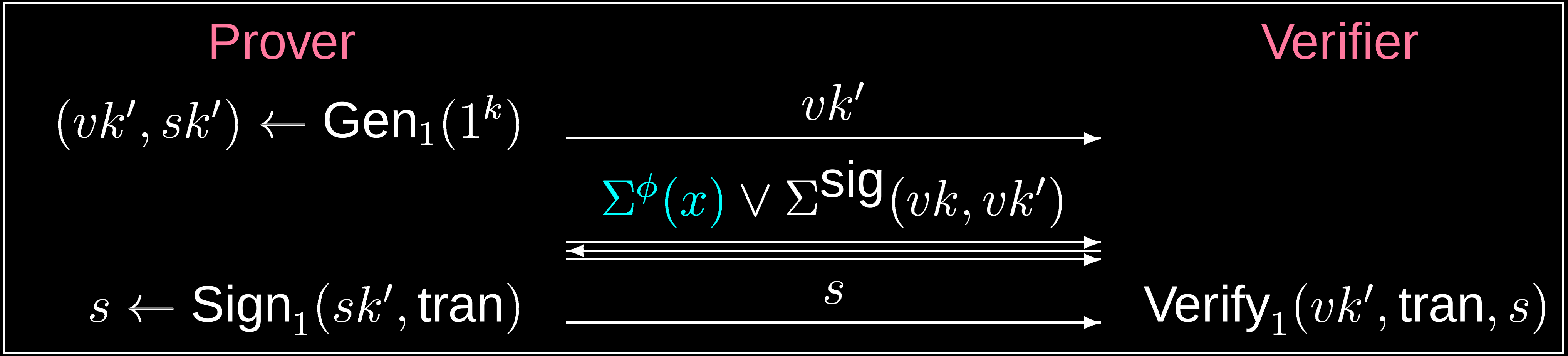

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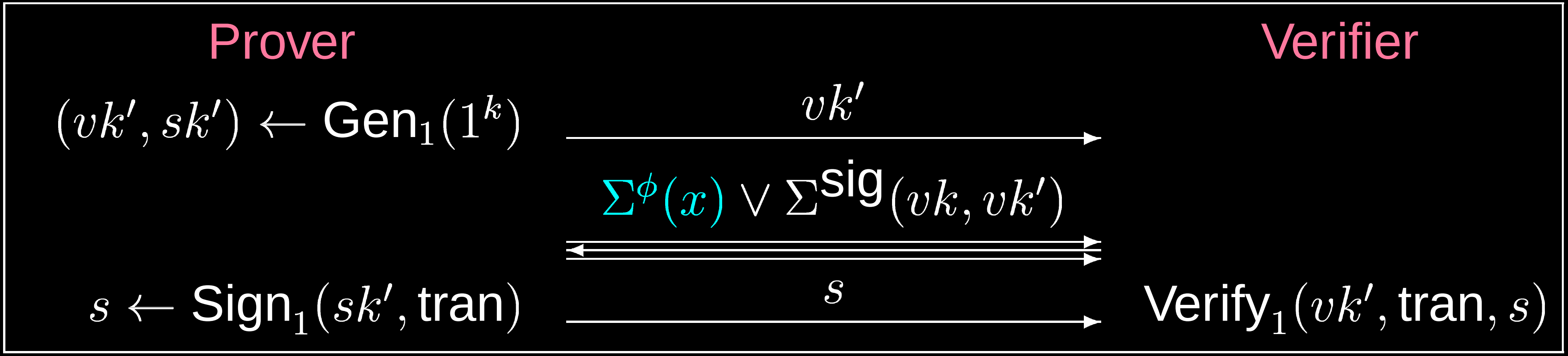

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- Completeness — straightforward

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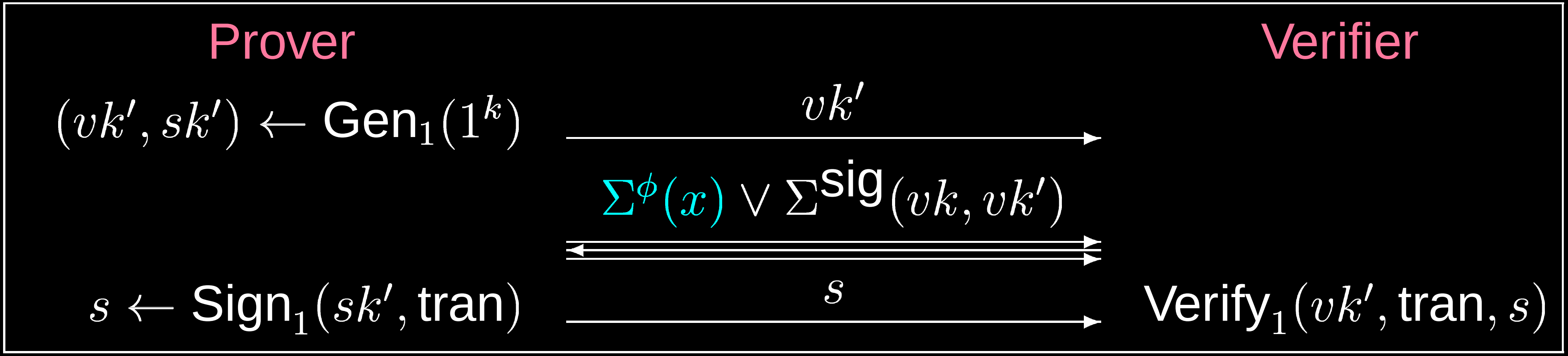

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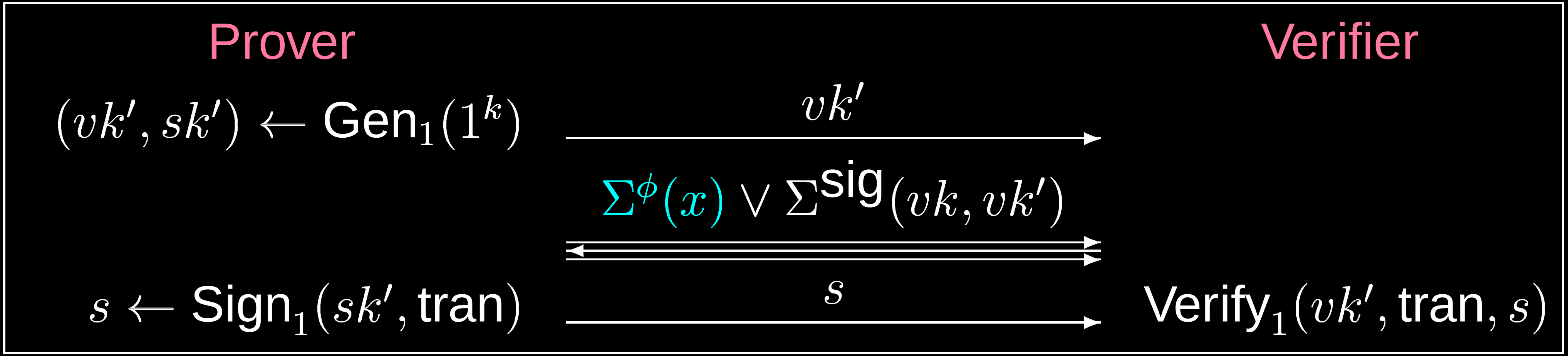

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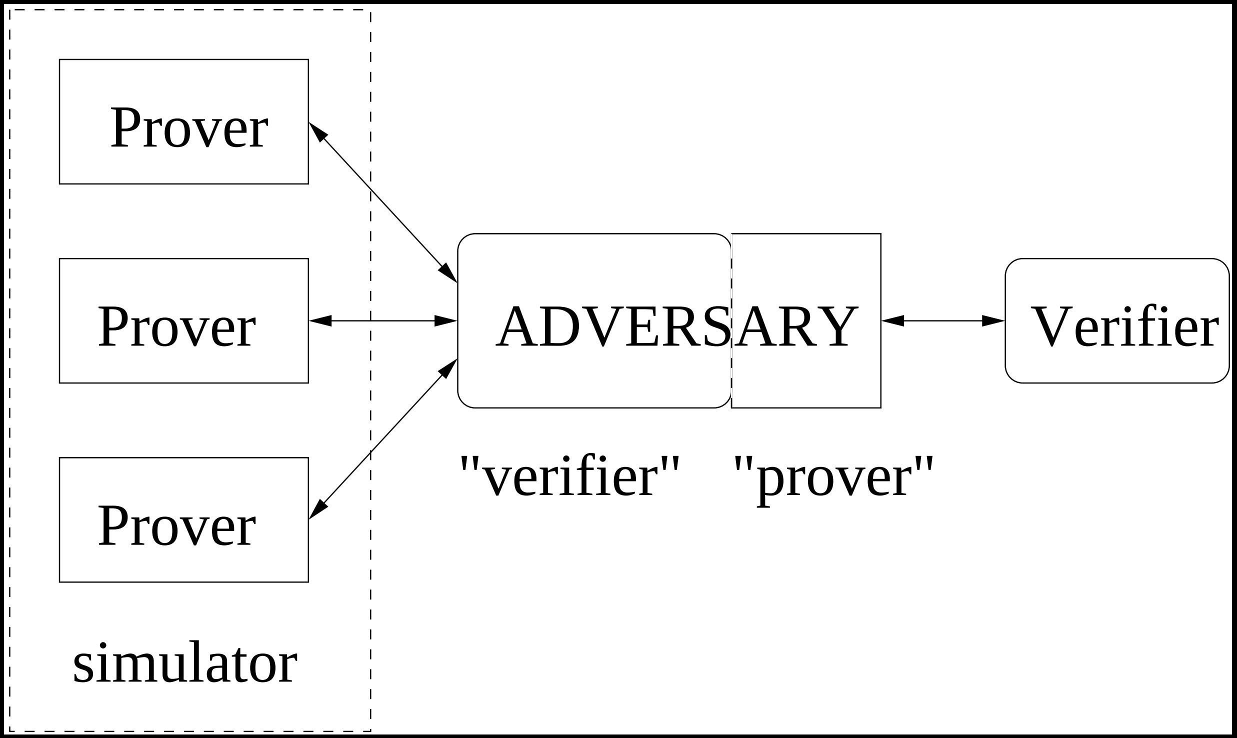
- Completeness — straightforward
- Soundness — since sk unknown, infeasible to fake a signature
- ZK-ness — S generates (vk, sk) and can produce signatures (non-rewinding simulation means concurrency)

How does it Work — Unbounded Simulation Soundness



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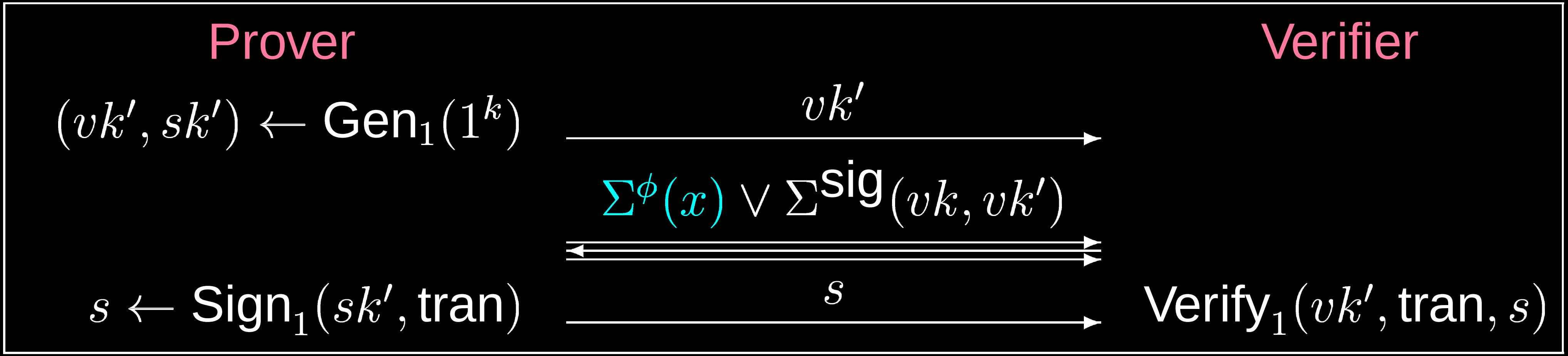

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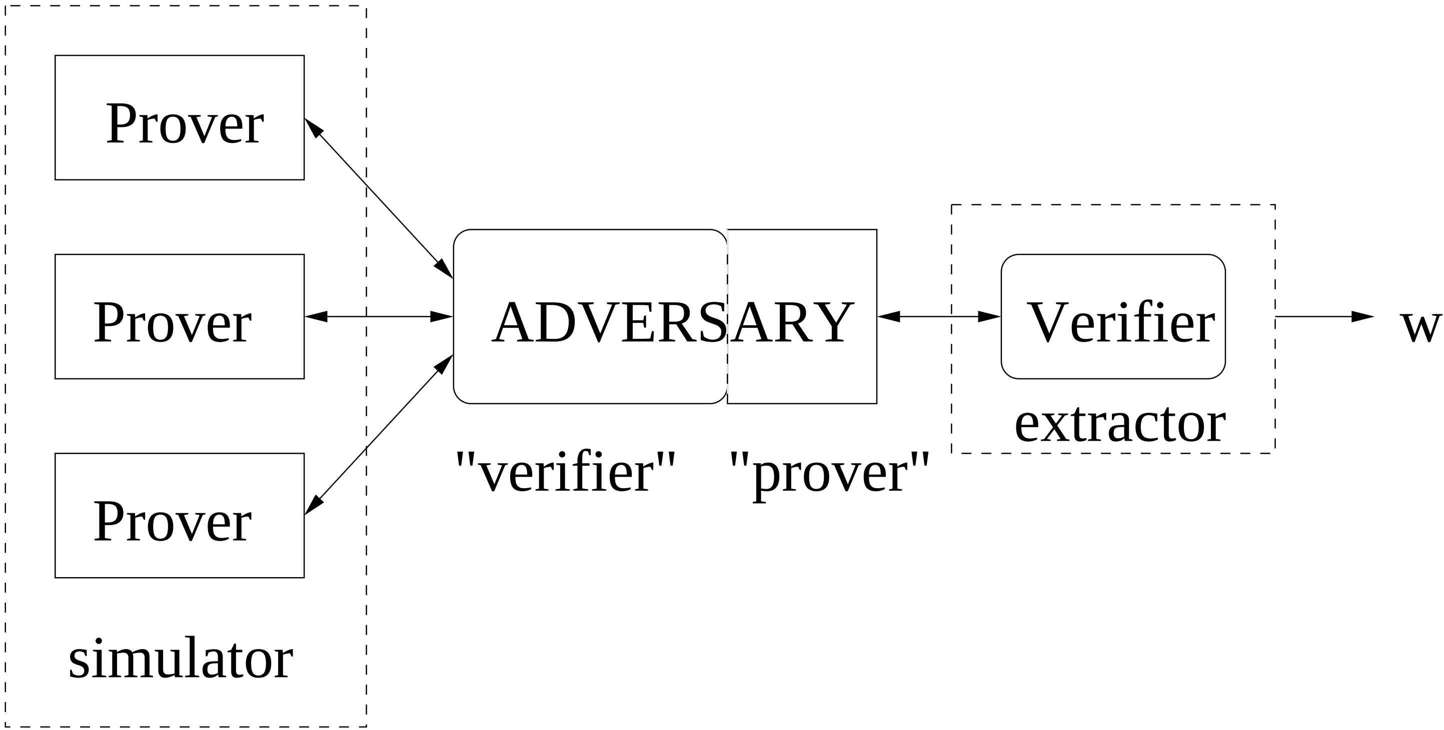
- “producing a false proof” = “faking a signature for vk'”
- $\mathcal{A}$ does not know sk' $\Rightarrow$ cannot reuse vk'
- $\mathcal{A}$ fakes a signature for a fresh vk' $\Rightarrow$ $\mathcal{A}$ breaks SIG

How about Unbounded Non-malleability?



Non-malleable ZK = Simulation Sound ZK + non-rewinding POK

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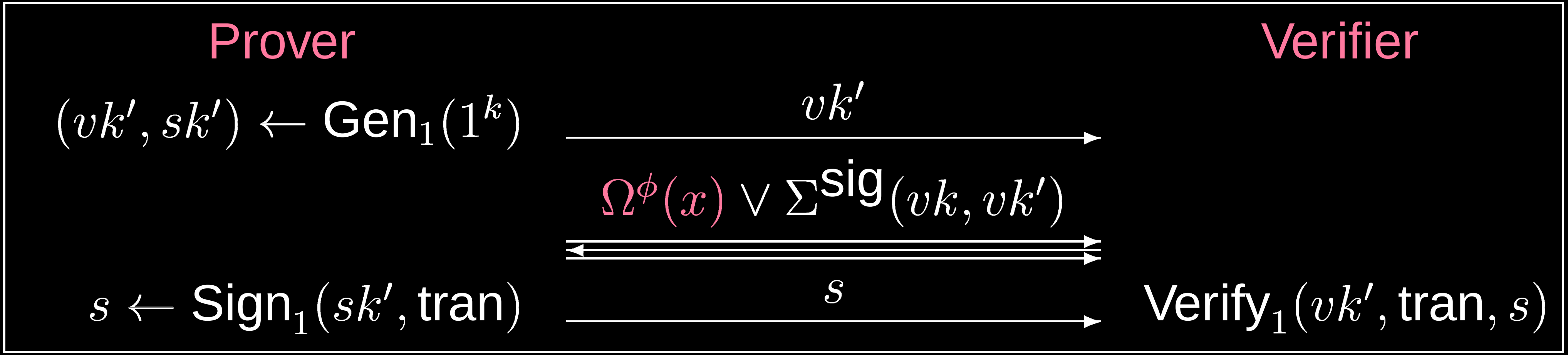
- Anything $\mathcal{A}$ proves, a witness can be extracted.

From Ω-protocols to Unbounded Non-malleability

same construction, let Π be an Ω-protocol



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- Ω-protocol = Σ-protocol + non-rewinding POK

  "failing to extract" = "faking a signature for vk'"
- $\mathcal{A}$ does not know sk' $\Rightarrow$ cannot reuse vk'
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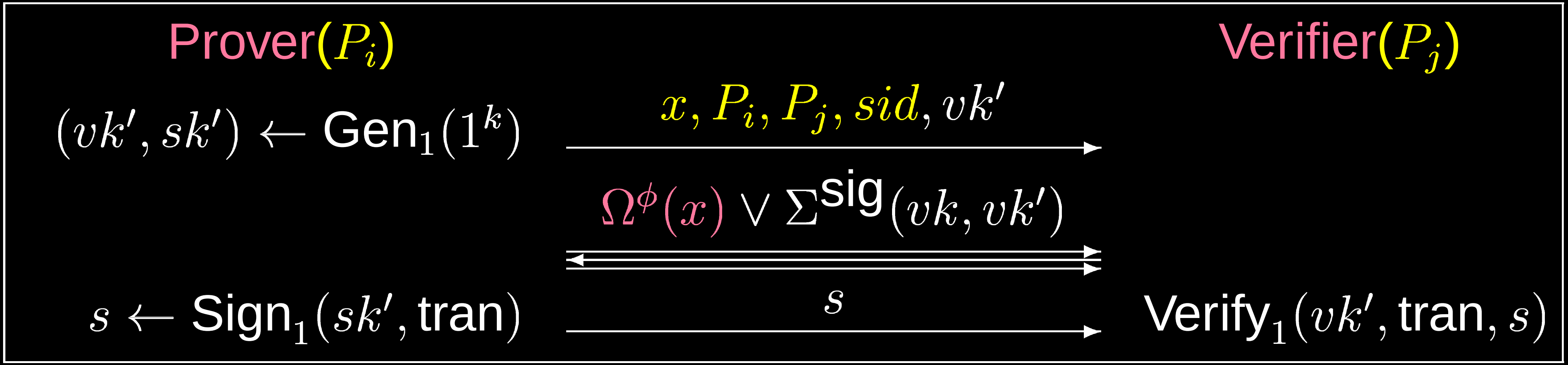
From Unbounded Non-malleability to Universal Composability



Prover(P_i) Verifier(P_j)

$(vk', sk') \leftarrow$ Gen$_1(1^k)$ $\xrightarrow{x, P_i, P_j, sid, vk'}$

$\Omega^{\phi}(x) \vee \Sigma^{\text{sig}}(vk, vk')$

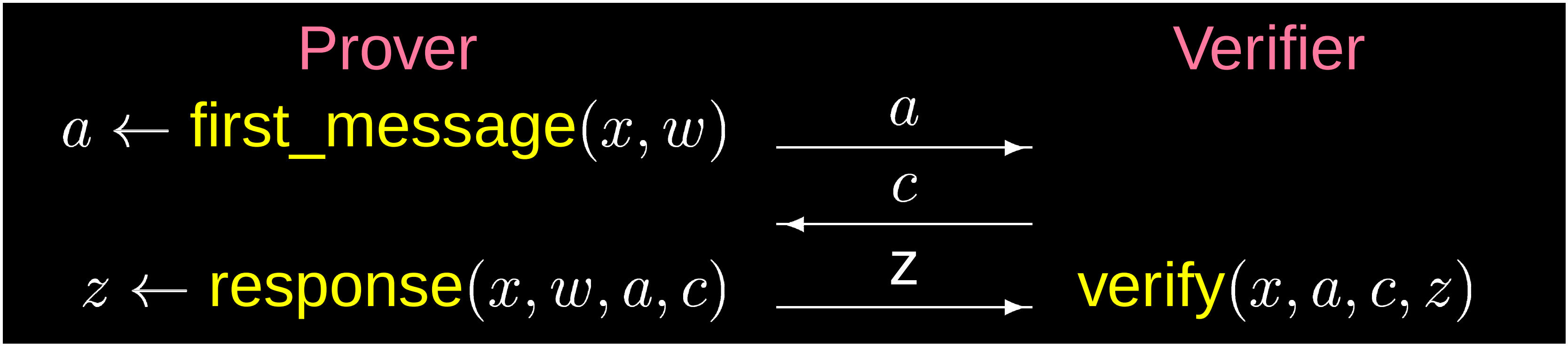
$s \leftarrow$ Sign$_1(sk', \text{tran})$ $\xrightarrow{s}$ Verify$_1(vk', \text{tran}, s)$

- roughly speaking

  UCZK $\sim$ unbounded non-malleable ZK
- easily augmentable to UCZK for non-adaptive corruption (add common input, ProverID, VerifierID, SessionID)

UCZK: Adaptive Corruption (With Erasure)

- start with the UCZK non-adaptive construction
- technique from [Damgård 00, Jarecki Lysyanskaya 00]



Prover | Verifier

$a \leftarrow$ first_message(x, w) $\xrightarrow{a}$

$\xleftarrow{c}$

$z \leftarrow$ response(x, w, a, c) $\xrightarrow{\text{z}}$ verify(x, a, c, z)

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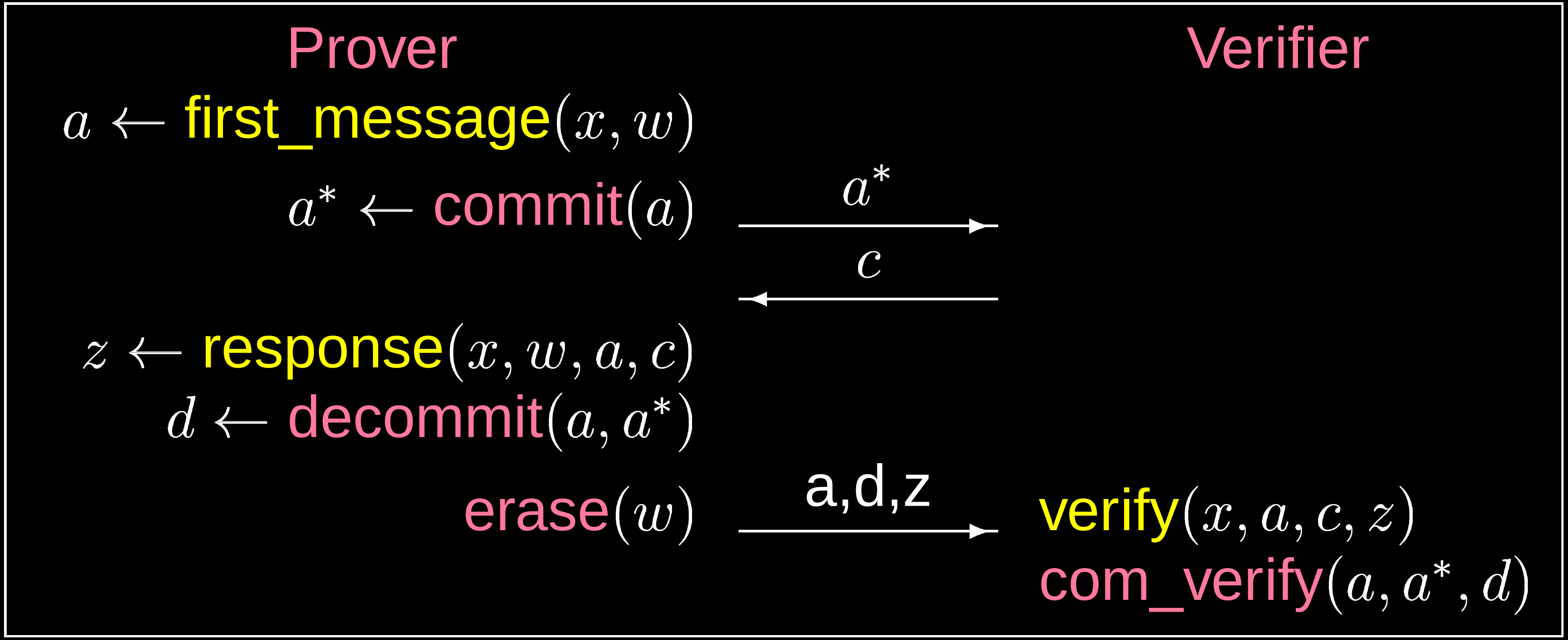

Prover

Verifier

$a \leftarrow$ first_message(x, w)

$a^* \leftarrow$ commit(a) $\xrightarrow{a^*}$

$\xleftarrow{c}$

$z \leftarrow$ response(x, w, a, c)

$d \leftarrow$ decommit(a, a^*)

erase(w) $\xrightarrow{\text{a,d,z}}$ verify(x, a, c, z)

com_verify(a, a^*, d)

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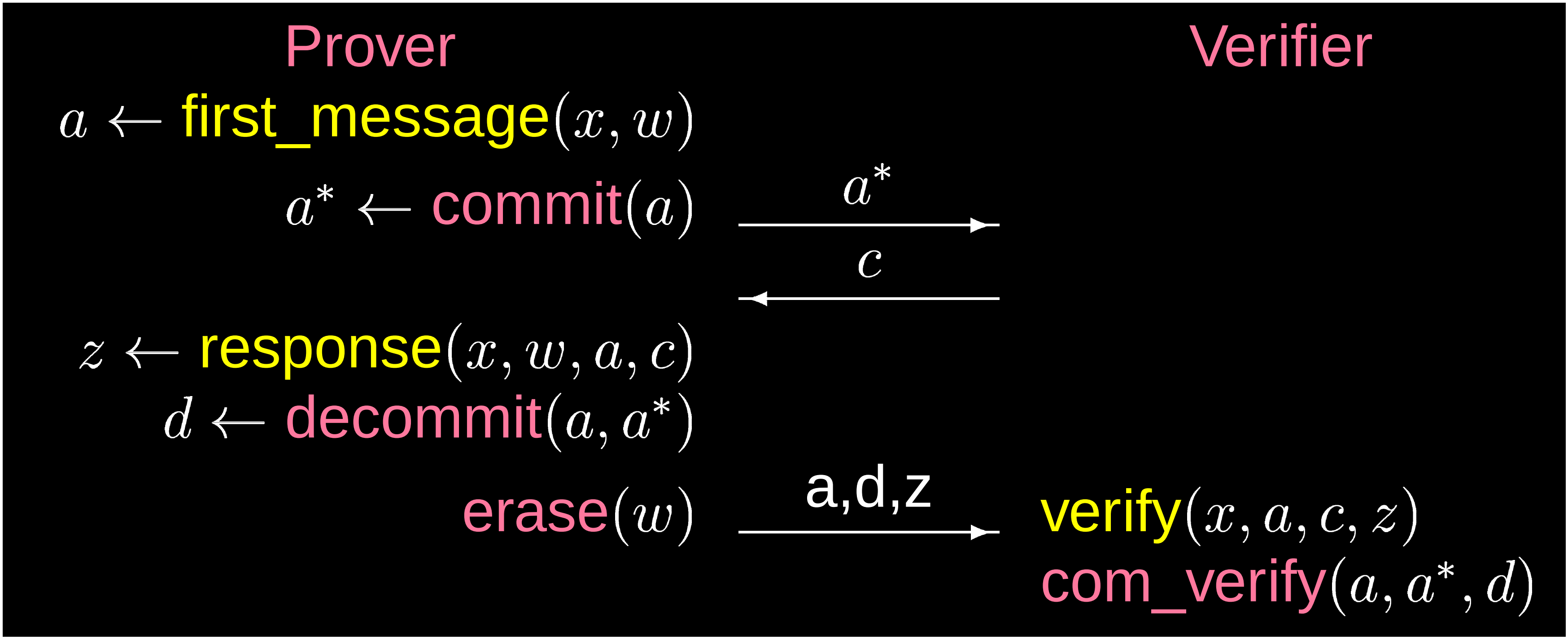

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Verifier

$a \leftarrow$ first_message(x, w)

$a^* \leftarrow$ commit(a) $\xrightarrow{a^*}$

$\xleftarrow{c}$

$z \leftarrow$ response(x, w, a, c)

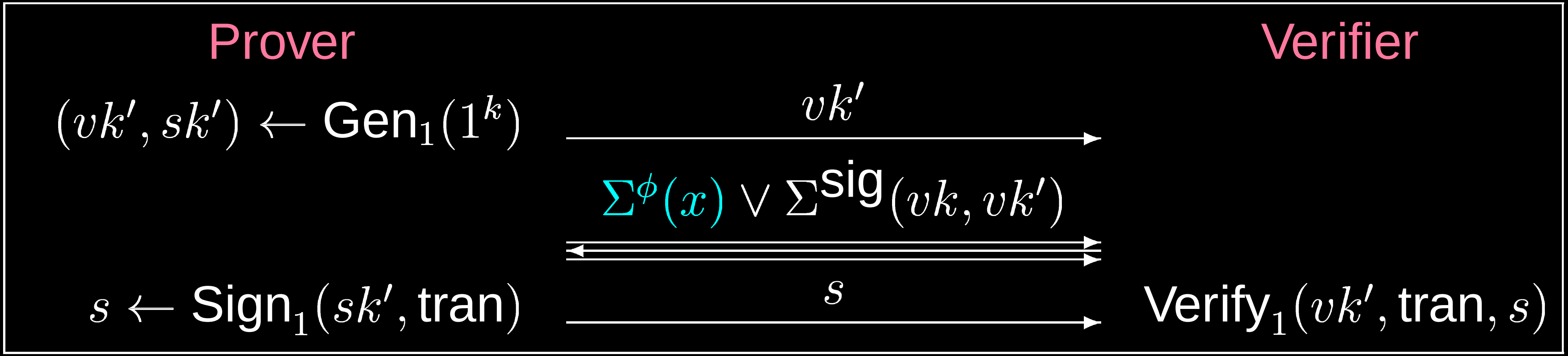
$d \leftarrow$ decommit(a, a^*)

erase(w) $\xrightarrow{\text{a,d,z}}$ verify(x, a, c, z)

com_verify(a, a^*, d)

- "Simulation Sound Trapdoor Commitment": $\mathcal{A}$ cannot fake a decommitment even after seeing a simulator faking

What About Efficiency?



Prover **Verifier**

$(vk', sk') \leftarrow \mathrm{Gen}_1(1^k) \quad \xrightarrow{\quad vk' \quad}$

$\Sigma^{\phi}(x) \vee \Sigma^{\mathrm{sig}}(vk, vk')$

$\xrightarrow{\qquad\qquad} \quad \xleftarrow{\qquad\qquad} \quad \xrightarrow{\qquad\qquad}$

$s \leftarrow \mathrm{Sign}_1(sk', \mathrm{tran}) \quad \xrightarrow{\quad s \quad} \quad \mathrm{Verify}_1(vk', \mathrm{tran}, s)$

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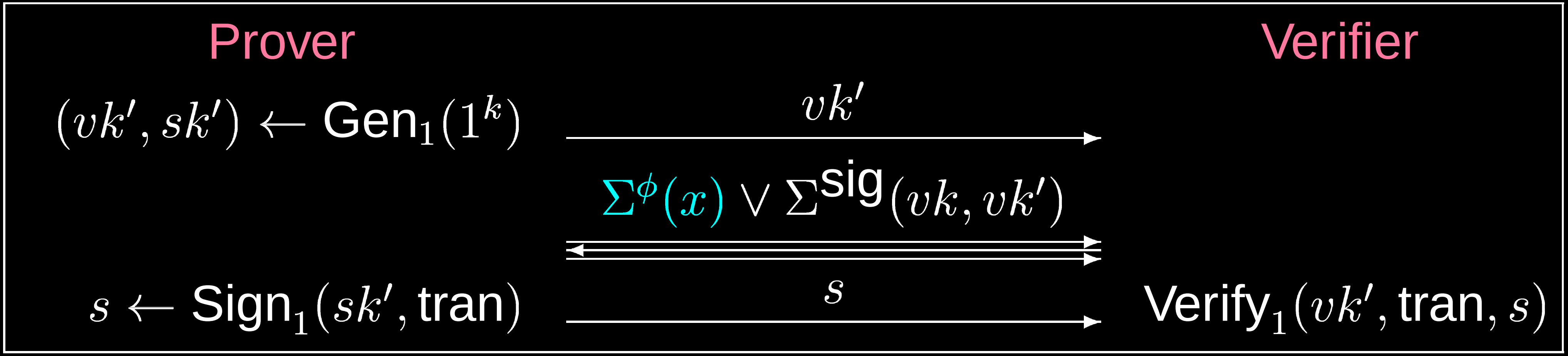

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- Building Π' by adding POK of signature to Π

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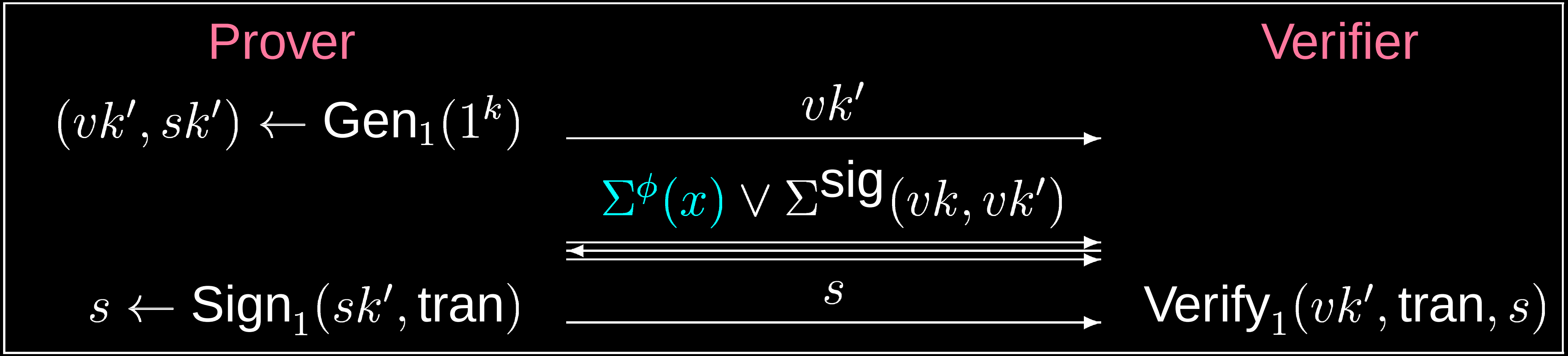

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  - avoids the Cook-Levin Theorem
  - efficient POK of signatures exists (Cramer-Shoup, DSA)
  - Σ-protocols $\Rightarrow$ efficient composition of “OR”

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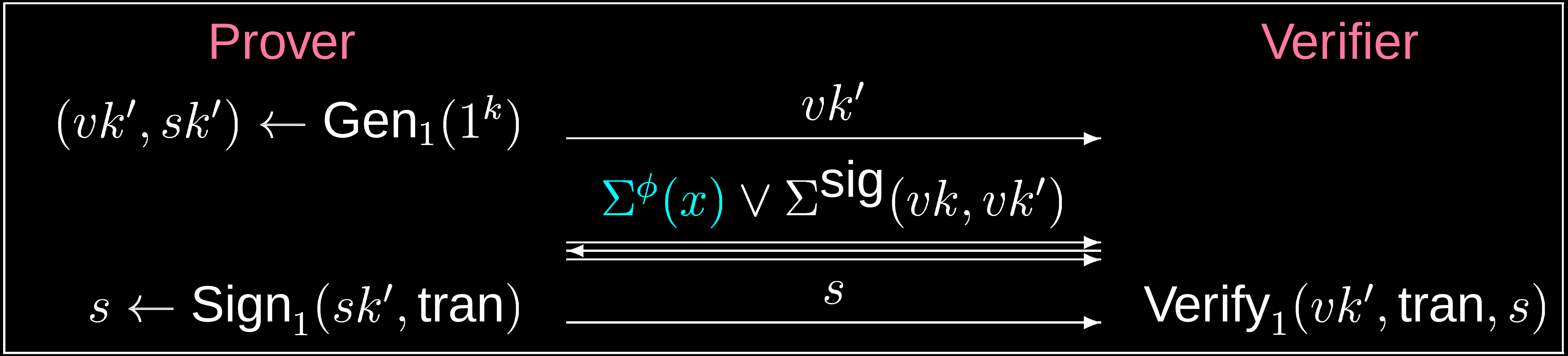

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$\longleftarrow$
$\longrightarrow$

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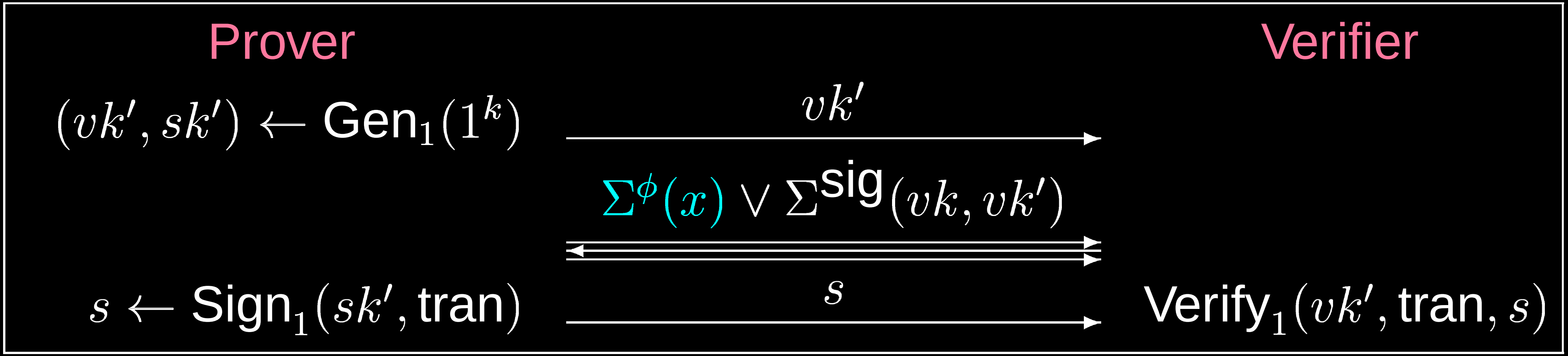

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(honest-verifier ZK) + (additive const. pub. key operations) $\Rightarrow$

(concurrent, non-malleable, and/or universally composable ZK)