Blocked versus Interleaved Practice with Multiple Representations in an Intelligent Tutoring System for Fractions

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Abstract. Previous research demonstrates that multiple representations can enhance students' learning. However, learning with multiple representations is hard. Students need to acquire representational fluency with each of the representations and they need to be able to make connections between the representations. It is yet unclear how to balance these two aspects of learning with multiple representations. In the present study, we focus on a key aspect of this question, namely the temporal sequencing of representations when students work with multiple representations one-at-a-time. Specifically, we investigated the effects of blocking versus interleaving multiple representations of fractions in an intelligent tutoring system. We conducted an *in vivo* experiment with 296 5th- and 6th-grade students. The results show an advantage for blocking representations and for moving from a blocked to an interleaved sequence. This effect is especially pronounced for students with low prior knowledge.

Keywords: Multiple representations, fractions, intelligent tutoring system, blocked vs. interleaved practice, classroom evaluation.

1 Introduction

The ultimate goal of the work presented in this paper is to generate a set of research-validated principles about how multiple representations best support robust learning in a real-world domain, and to build an intelligent tutoring system (ITS) that reflects these principles and helps students overcome their difficulties with fractions. We report on a study that is a first step in that direction.

We focus on a domain in which multiple graphical representations are often used: fractions [1]. Understanding fractions is foundational for learning algebra and more advanced mathematics [2], yet fractions pose a significant challenge for students in the elementary and middle grades, and even for college students and pre-service teachers [3]. In a recent study, we provide experimental evidence that students working with multiple graphical representations of fractions learn better than students who work with only a single graphical representation, although only when prompted to explain how the graphical representations (e.g., half a pie) of fractions relate to the symbolic representation (e.g., 1/2) [4]. The current study builds on this work; we now

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turn to the question of *how best to temporally sequence* multiple representations. For now, we focus on instruction that presents students with only a single graphical representation for each problem. (In future research we will include problems that involve multiple graphical representations at a time.)

Several studies in cognitive and educational psychology have demonstrated that the use of multiple representations can lead to more robust learning in complex domains [5,6]. However, providing students with multiple representations is not always beneficial [5], which has been attributed to the fact that learning with multiple representations requires several interrelated cognitive competencies [7]. Perhaps most obviously, students need to understand the format and properties of the particular representations and they need to be able to use them appropriately; that is, they need to acquire *representational fluency* with each representation [8]. In addition, students can only benefit from learning with multiple representations if they are able to make comparisons across representations and translate between them; in other words, they need to acquire *representational flexibility* [9,10].

At this point, it is an open question what the best balance is between supporting the acquisition of representational fluency and representational flexibility, and to what degree each facilitates the other. It stands to reason that representational fluency and connection making mutually influence the acquisition of one another. Fluency may facilitate the acquisition of representational flexibility. When learning to work with a new representation, it may help to connect it to one with which one is already fluent. But representational flexibility may also facilitate the development of fluency; a burgeoning understanding of one representation may help make sense of (and develop fluency with) a second, new, representation, even when that first representation is not yet fully understood. Little is known at this point about the relative strength of these mutual influences, which makes it harder to design effective instruction for learning with multiple representations.

In the study presented in this paper, we consider a key aspect of instruction with multiple representations, namely the *temporal sequencing* of representations. We ask what temporal sequence leads to more robust learning when learners work with multiple representations presented one-at-a-time: should practice with different representations be *blocked* (e.g., Pie-Pie-Pie, NL-NL-NL, Set-Set-Set) or *interleaved* (e.g., Pie-NL-Set, Pie-NL-Set, Pie-NL-Set)?

We assume that students develop fluency with a given representation as they work with that representation (e.g., during problem solving). We also assume that when representations are presented one-at-a-time, students are (somewhat) likely to spontaneously compare representations at the points where they switch from one representation to another. When practice with representations is *blocked*, students have the opportunity to build up fluency with one representation before the next one is introduced. When they (spontaneously) make connections between representations in the blocked condition, it is likely therefore that they are fluent with all of these representations except one (the one they are currently learning). In other words, this condition promotes fluency before connection making. On the other hand, when practice with different representations is *interleaved,* students build up fluency with representations in parallel; they may start making connections between representations even before they are fluent with any of them. The interleaved condition therefore facilitates connection making before fluency.

If cross-representational comparisons between different graphical representations strongly rely on students' fluency with at least one of the representations being compared, then students should learn best when practice with each of the representations is blocked – at least until they acquire sufficient representational fluency. If on the other hand, useful comparisons can be made even between representations students are not yet fluent with, then the interleaved condition should support more robust learning. Indeed, several studies have shown that interleaving practice with tasks that structurally differ from one another leads to better learning outcomes than blocked practice [11,12]. We know of no studies however that compared the blocking and interleaving of different external representations.

We investigated how multiple representations should be sequenced temporally in the context of a proven intelligent tutoring system (ITS) technology, namely, Cognitive Tutors [13]. Specifically, we developed a set of example-tracing tutors for fractions learning. Example-tracing tutors are a type of tutors that are behaviorally similar to Cognitive Tutors, but that rely on examples of correct and incorrect solution paths rather than on a cognitive model of student behavior. We created these tutors with the Cognitive Tutor Authoring Tools (CTAT [14]). We used these tutors in an *in vivo* experiment (i.e., a rigorously controlled experiment in a real educational setting).

Students in all conditions worked on the same problems, but practice with the different representations of fractions was blocked across problems to varying degrees. We used four conditions: blocked, moderate, interleaved, and increased. We hypothesize that spontaneous cross-representational comparison making builds on representational fluency, and thus predict that the increased condition will yield the best learning results. Additionally, we explore whether low and high prior knowledge students differ regarding which condition is most beneficial.

2 Methods

2.1 Material and Fractions Tutors

The tutors used in the study included three different graphical representations of fractions: pie charts (see Figure 1), numberlines (see Figure 2, top), and sets (see Figure 2, bottom). Each graphical representation emphasizes certain aspects of the different interpretations of fractions [15]. The pie chart as a part-whole representation depicts fractions as parts of an area that are partitioned into equally-sized pieces. The numberline is considered a measurement representation and thus emphasizes that fractions can be compared in terms of their magnitude, and that they fall between whole numbers. Finally, the set is a ratio representation and presents fractions in the context of discrete objects that have several features.

We employed different orders of graphical representations in addition to the blocked versus interleaved factor. Students never worked with the set representation first, because the set appears to be the representation students are least familiar with. We randomly assigned students to one of four different orders of representations: Pie chart – numberline – set, pie chart – set – numberline, numberline – pie chart – set, or numberline – set – pie chart.

The fractions tutor covered fraction identification, fractions as division, equivalent fractions, ordering fractions, and fraction addition. One graphical representation was presented at a time, but across the whole sequence of tutor problems, all graphical representations were crossed with all topics, except for obvious mismatches.

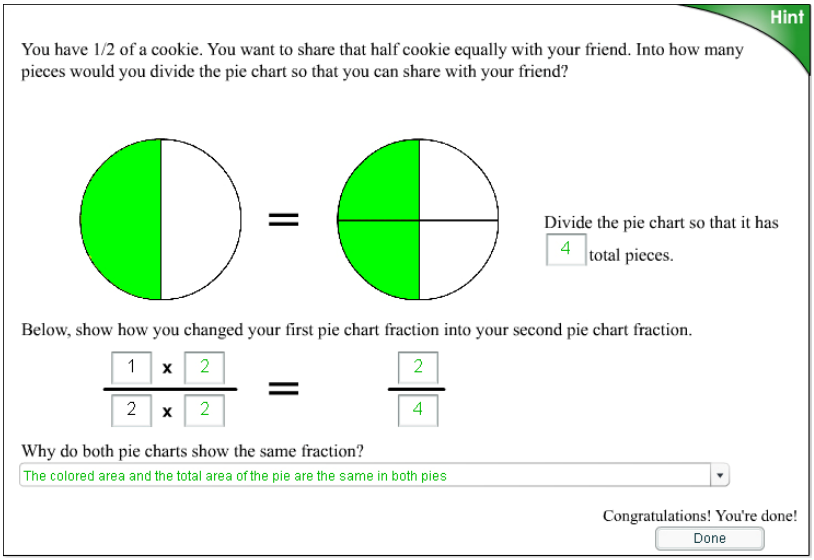

Fig. 1. Example of equivalent fractions problems with the pie chart

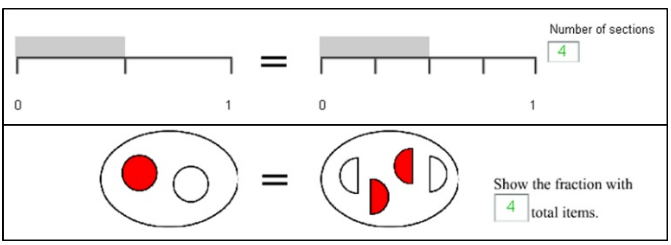

Fig. 2. Example of equivalent fractions problems with the numberline (top) and sets (bottom)

Before solving a problem symbolically, students were asked to perform the same steps by manipulating the graphical representations. For instance, students could partition them into smaller sections (for the numberline), pieces (for the pie chart), or objects (for the set). Figure 1 shows an example of equivalent fractions problems with the pie chart. Figure 2 shows corresponding problems with the numberline (top) and set representation (bottom). The tutors provided students with realistic cover stories for each problem. Students received error feedback and hints on all problem-solving steps. Error feedback messages were designed to make students reconsider their answer by either reminding them of a previously-introduced principle, or by providing them with an explanation for their error. Hint messages usually had three levels. First, students received a clarification of the goal (e.g., "You now added all pieces into the same pie chart. Before you know what fraction of the whole cake you won, you need to divide the pie chart into equally sized pieces."). They were then given conceptually oriented help, by reminding them of a specific concept (e.g., "The pieces are part of the same cake. Therefore, you keep the same denominator in the sum fraction."). Finally, students received explicit instructions regarding the next step (e.g., "Please divide the pie chart into four pieces.").

Students were prompted to self-explain their problem solution. We found this procedure to be effective in an earlier experimental study [4]. Students selected their answer from a drop-down menu, as shown in Figure 1. Previous research shows that asking students to select their answers from a menu rather than to explain in their own words, promotes a self-explanation effect [16].

2.2 Experimental Design and Procedure

A total of 269 5th- and 6th-grade students from three different schools in the United States participated in the study during their regular mathematics instruction. All students worked with a set of ITS for fractions (example-tracing tutors, as mentioned) designed and created specifically for this study. Students were randomly assigned to one of four conditions that varied regarding the degree to which practice with the representations was blocked versus interleaved: blocked, moderate, interleaved, or increased.

Students in the blocked condition encountered the representations in three blocks: They first worked through the whole sequence of topics with the first graphical representation (corresponding to 36 problems), then with a second representation, and finally with the last representation. In the moderate condition, the blocks were much smaller: students switched representations after every third problem. Students in the interleaved condition switched representations after every single problem. And finally, in the increased condition, the length of the blocks was gradually reduced from twelve problems at the beginning to a single problem at the end.

We assessed students' knowledge of fractions three times. On the first day, students completed a 30-minute pre-test. They then worked on the fractions tutor, for a total of 5 hours, spread across five to six (depending on specific school schedules) consecutive days. The day following the tutor sessions, students completed a 30-minute post-test. Seven days later, we gave students an equivalent delayed post-test.

2.3 Test Instruments

Students' understanding of fractions was assessed with respect to *representational knowledge* and *operational knowledge*. By representational knowledge, we mean the ability to interpret representations of fractions and to use them to make sense of fractions. In our specific test, we asked students to identify and order fractions using different graphical representations some of which students were not familiar with (meaning that they did not encounter them in the set of tutor problems) to assess their representational knowledge. Operational knowledge describes the ability to solve fractional tasks procedurally, by applying algorithms. The operational test items in our test assessed students' ability to convert and add fractions with and without the help of graphical representations. Test items were adapted from standardized national tests and from examples from the fractions literature. We randomly assigned students to different versions of the fractions test at the pre-test, the immediate and the delayed post-test. We validated the theoretical structure of the test with a confirmatory factor analysis using data from a large sample in a pilot study of the test instruments. The test's theoretical structure was replicated with data from the presented experiment.

3 Results

Students who stayed in their assigned condition, who were present for all test days, and who did not work on the tutoring system during the weekend were included in the analysis, yielding a total of $N = 215$. Neither the number of excluded students differed between experimental conditions, χ^2 (3, N = 269) = 1.21, $p > .10$, nor did the number of problems completed ($F < 1$), or the time spent on the tutor problems ($F < 1$).

A hierarchical linear model (HLM; [17]) with four nested levels was used to analyze the data. At level 1, we modeled performance for each of the three tests for each student. At level 2, we accounted for differences between students. At level 3, we modeled differences between classes, and at level 4 accounted for differences between schools. In addition, we used post-hoc comparisons to clarify the effect of blocking versus interleaving. The reported p-values are adjusted using the Bonferroni correction. Since there was no effect for order of representation ($F < 1$), only the results for blocked versus interleaved practice with the representations are reported.

Table 1. Rel. means and standard deviations (in brackets) for representational and operational knowledge at pre-test, immediate post-test, delayed post-test by low and high prior knowledge

		Blocked	Moderate	Interleaved	Increased
	low prior knowledge				
pre-test	representational knowledge	.39 (.12)	.38 (.17)	.35 (.15)	.42 (.13)
	operational knowledge	.22 (.15)	.28 (.14)	.26 (.16)	.27 (.14)
immediate post-test	representational knowledge	.50 (.24)	.49 (.20)	.36 (.23)	.55 (.23)
	operational knowledge	.37 (.27)	.33 (.31)	.26 (.26)	.31 (.26)
delayed post-test	representational knowledge	.51 (.25)	.27 (.30)	.29 (.27)	.50 (.26)
	operational knowledge	.29 (.27)	.27 (.28)	.26 (.20)	.36 (.27)
	high prior knowledge				
pre-test	representational knowledge	.78 (.11)	.77 (.14)	.77 (.11)	.70 (.09)
	operational knowledge	.75 (.16)	.81 (.15)	.76 (.16)	.76 (.15)
immediate post-test	representational knowledge	.78 (.21)	.70 (.22)	.72 (.17)	.67 (.16)
	operational knowledge	.84 (.22)	.73 (.33)	.68 (.34)	.75 (.23)
delayed post-test	representational knowledge	.72 (.19)	.58 (.32)	.53 (.34)	.67 (.25)
	operational knowledge	.76 (.28)	.68 (.29)	.67 (.34)	.65 (.36)

3.1 Learning Effects

First, we looked at student learning across conditions across the three test times. The tendency for the overall effect for test was in the opposite than the predicted direction, so that the overall hypothesis of a learning effect was not confirmed. Post-hoc comparisons showed a significant gain from pre-test to immediate post-test on representational knowledge for the blocked condition ($p < .01$).

To clarify this result, we split the data into groups based on median performance in the pre-test. Table 1 shows the means and standard deviations for representational and operational knowledge by test and condition for low and high prior knowledge students. For the low prior knowledge group, the results showed a significant improvement from pre-test to immediate post-test for representational knowledge in the blocked and increased conditions, which for the increased condition was also significant at the delayed post-test (ps < .05). No significant differences were found for operational knowledge.

3.2 Effects of Blocked versus Interleaved Representations

We had predicted an advantage for the increased condition at the immediate and the delayed post-test. The results partly support this hypothesis. We found a significant interaction effect between test time and blocked versus interleaved practice, for representational knowledge, $F(6, 422) = 5.54$, $p < .01$, and operational knowledge, $F(6, 422) = 2.19$, $p < .05$. Post-hoc comparisons showed that regarding representational knowledge, students in the blocked condition significantly outperformed students in the interleaved condition at the immediate post-test ($p < .05$). At the delayed post-test, both the blocked and the increased condition performed significantly better than the interleaved and moderate condition (ps < .01). As for operational knowledge, the post-hoc comparisons did not reveal statistically significant differences.

The analysis of the effects of blocked versus interleaved practice in the low and high prior knowledge groups further clarifies these results. An interaction between condition and the prior knowledge groups was significant for representational knowledge at both the immediate post-test, $F(4, 201) = 17.56$, $p < .01$, and the delayed post-test, $F(4, 202) = 6.08$, $p < .01$, as well as for operational knowledge at the immediate post-test, $F(4, 199) = 21.74$, $p < .01$, and the delayed post-test, $F(4, 198) = 17.90$, $p < .01$. Post-hoc comparisons showed that for representational knowledge at the immediate post-test, the advantage of the blocked condition and the increased condition over the interleaved condition was only significant for the low prior knowledge group (ps < .01), but not for the high prior knowledge group. At the delayed post-test, the advantage of the blocked condition and the increased condition over the interleaved condition, as well as the advantage of the blocked condition over the moderate condition reached the level of significance for both the low and high prior knowledge groups (ps < .01). The advantage of the increased condition over the moderate condition, in contrast, was significant only for the low prior knowledge group ($p < .01$). Post-hoc comparisons on operational knowledge revealed a significant advantage for the increased condition over the blocked condition in the low prior knowledge group for immediate and delayed post-test ($p < .01$). At the delayed post-test, this advantage was also significant when compared to the interleaved and moderate conditions (*p*s < .01). No further differences were found on operational knowledge.

4 Discussion and Conclusion

We found evidence that our tutoring system improves understanding of fractions for low prior knowledge students in the blocked and increased conditions. This finding in part confirms our hypothesis that the increased condition will yield the best learning results. The reason why our data does not provide evidence for learning in the high prior knowledge group may be that the fractions tutor could not add to their understanding because it provided practice on rather basic fractions concepts and procedures. In fact, the test scores in the high prior knowledge group show that students in the high prior knowledge group already had a relatively good understanding of fractions at the pre-test.

The results on blocked versus interleaved practice support our hypothesis that moving from a blocked scheme towards an interleaved scheme for learning with multiple representations yields the best learning results. The blocked and increased conditions showed more robust learning than the interleaved and moderate conditions. At the level of cognitive processes, the study thus provides some support for the notion that representational fluency facilitates the acquisition of representational flexibility more so than the other way around. While it is important to note that we did not directly support connection making between the different representations, this finding may have implications for the design of curricula that make use of multiple representations at a time. It seems reasonable to believe that instruction explicitly supports connection making between different representations will be most beneficial after students have acquired a good understanding of each individual representation's format.

The advantage of blocked and increased representations was significant for representational knowledge, but not for operational knowledge. One possible explanation is that the tutoring system supports learning of representational knowledge better than the learning of operations. Indeed, we do not find a significant learning gain on operational knowledge. We expected that a deep understanding of graphical representations of fractions would be conducive to a better conceptual understanding of operations, but our data does not support this view. Another explanation is that students are able to gain an abstract understanding of fractions operations regardless of whether a blocked or interleaved design is being used. In fact, the symbolic operations presented in the fractions tutoring system do not change depending on which graphical representation is used to illustrate it. We are currently analyzing the tutor logs to clarify whether the graphical representations in the tutoring system helped students understand fractions operations.

The fact that the interleaved condition yields the best learning results supports the interpretation that providing the opportunity for spontaneous comparison-making is beneficial to students who already acquired representational fluency. To the extent that students in the high prior knowledge group have representational fluency, should the interleaved conditions then not be most suitable for their needs? In fact, we classified students as low versus high prior knowledge students based on their performance in the pre-test, which included many representational test items, so that it seems reasonable to assume that they came in with a higher degree of representational fluency. The fact that we did not find an advantage for the interleaved condition for high prior knowledge students may be due to their performance being at ceiling for representational knowledge.

In conclusion, our study provides preliminary evidence that the acquisition of representational fluency should get higher weight in early instruction that makes use of multiple representations, compared to representational flexibility. One caveat is that the results were obtained with instructional material in which students encounter representations one-by-one, and connection making occurs only to the extent that students spontaneously engage in it. One could argue that the most extreme case of presenting multiple representations in temporal proximity is in fact presenting them simultaneously. In future studies, we will investigate whether the results generalize to situations in which students encounter multiple representations side-by-side, and in which the learning environment provides explicit support for connection making.

Our findings stand in contrast to earlier findings from a variety of domains which demonstrate an advantage for interleaved practice over blocked practice [12]. The difference between our studies and prior research is that we are investigating the effects of blocked versus interleaved practice with graphical representations as opposed to blocked versus interleaved practice of different problem types.

Our results may have implications for the design of instruction that directly supports connection making between multiple representations: It is likely that connection-making tasks will be most effective after students have had the opportunity to acquire fluency with the representations. While most studies on learning with multiple representations have emphasized the importance of helping students in making connections between the different representations [9], more attention should be paid to how to best support students' acquisition of representational fluency which appears to be an important foundation for the acquisition of representational fluency. Our findings are in line with Ainsworth's framework on learning with multiple representations [8] who points out that students often have difficulty in understanding the format of a new representation, as well as to understand how to use them appropriately in subsequent learning tasks. And while it seems logical that students have to acquire this understanding before they can relate different representations to one another, we know of no experimental evidence for this assertion, prior to this study.

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