15-850(C) Machine Learning Theory

Assignment 2 | Due: October 5, 1995

Decision Trees and Decision Lists

1. The *rank* of a decision tree is defined as follows. If the tree is a single leaf then the rank is 0. Otherwise, let r_L and r_R be the ranks of the left and right subtrees of the root, respectively. If $r_L = r_R$ then the rank of the tree is $r_L + 1$. Otherwise, the rank is the maximum of r_L and r_R. For example, the decision tree in Figure 2.2 in the book has rank 3.

   Prove that a decision tree with ℓ leaves has rank at most $\log_2(\ell)$.

2. Show that the class of rank-r decision trees is a subclass of r-decision lists. (r-decision lists are decision lists in which the antecedent of each rule is a conjunction of up to r literals)

   Hint: Use induction on the depth of the tree. (Actually, there are several different ways of proving this.)

Thus, we conclude that we can learn constant rank decision trees in polynomial time in the PAC model, and we can learn arbitrary decision trees of size s in time and number of examples $O(n^{\log s})$. (So this is "almost" a PAC-learning algorithm for decision trees.)