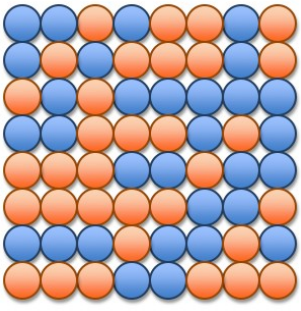


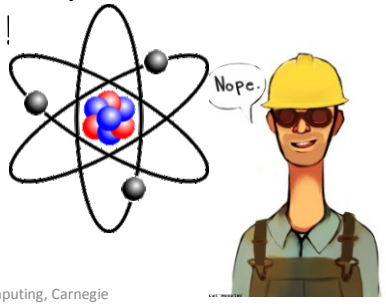
Quantum Computation, and Epilog: The Future of Computing

Promise of Quantum Computation

- Classical computers have their limitations:
  - Factoring large numbers takes exponential time.
    - No faster algorithm is known.
  - Searching an unordered list takes O(n) time.
    - No faster algorithm is possible.
- Quantum computers can solve some problems more efficiently than classical computers.
  - Prime factoring: could break RSA encryption.

What Is Quantum Mechanics?

- Objects at the subatomic level behave in ways that have no analog at the macroscopic level.
- Protons, neutrons, and electrons are not little billiard balls. They are both *particles* and *waves* at the same time!
- Quantum mechanics describes how these objects really behave. It's quite weird.



Examples of Quantum Weirdness

- A particle (or an atom) can:
  - Be in two different states at the same time.
  - Be in several places at the same time.
  - Move from A to B without ever occupying the space between them (tunneling).
  - Communicate information to another distant particle instantly (quantum teleportation).

Quantum Computers

- We can exploit 3 weird quantum phenomena to build a new kind of computer.

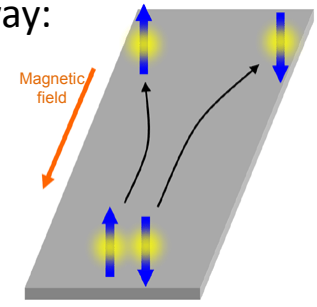
Intrinsic Angular Momentum

- Particles have a property (intrinsic angular momentum) that has two distinct values.
- Call the values “up” and “down”.
- Or +½ and -½.
- Or |1> and |0>.

- Intrinsic angular momentum is called “spin” but that is misleading. Nothing is spinning.

Measurement

- We can measure a particle's state and we will always get one of two results: |0> or |1>.
  - There are no intermediate values. Spin is quantized.
- How do we measure? One way:
  - Pass the particle through a magnetic field.
  - It will go left if its state is |0> and right if |1>.
  - Put a detector on each side.



Q Weirdness 1: Mixtures of States

- Before we measure, a particle's state can be a mixture of "up" and "down".
- Suppose it's ¾ "up" and ¼ "down".
- When we measure the state, we will get:
  - "Up" with probability 0.75
  - "Down" with probability 0.25
- Once we measure, the state is fixed; it's either "up" or "down". No more mixture.

Bits vs. Qubits

- Conventional computers use bits:
  - Value is either 0 or 1. Might be encoded by a voltage, e.g., "0" = 0 volts, "1" = +5 volts.
  - There are no mixture states. A value of +2 volts would indicate a broken computer.
- Quantum computers use qubits instead of bits. Qubits can have mixture states.

Qubits in Mixture States

- Let |0> denote the 100%"down" state and |1> the 100% "up" state. These are *basis* states.
- Any qubit's state can be expressed in terms of the basis states using two coefficients *a* and *b*:

  $a|0> + b|1>$

  where $|a|^2 + |b|^2 = 1$.

Mixture States (cont.)

- Mixture state is: a|0> + b|1>
- So the 100% "down" state is a=1,b=0
  The 100% "up" state is a=0,b=1
- Equal mixture of "up" and "down" would be:

$$a = b = \frac{1}{\sqrt{2}}$$

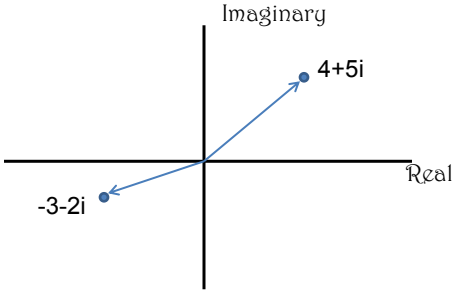
because we must have: $a^2 + b^2 = \frac{1}{2} + \frac{1}{2} = 1$

Q Weird. 2: Complex Amplitudes

- "Normal" mixture coefficients: $0 \leq x \leq 1$.
- Combine by simple addition: $a + b = 1$.
- Negative values would make no sense.
  - Can you have a dog that is 4/3 golden retriever and -1/3 german shepherd? No!
- But in quantum mechanics, the mixture coefficients are *complex numbers*!
- That's why the mixture rule is $|a|^2 + |b|^2 = 1$.

Complex Numbers: Cartesian Form

- Define *i* as $\sqrt{-1}$
- Complex numbers: p = a + b*i*, q = c + d*i*

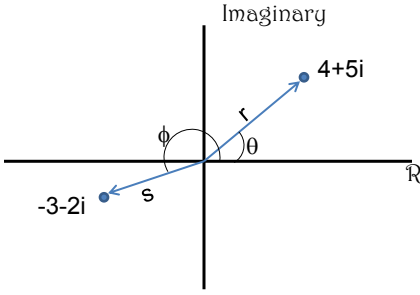


Complex Arithmetic

- Complex numbers: p = a + b*i*, q = c + d*i*
- p+q = (a+b*i*) + (c+d*i*) = (a+c) + (b+d)*i*
- p×q = (a+b*i*) × (c+d*i*)
  = a×c + a×d*i* + b*i*×c + b*i*×d*i*
  = (ac-bd) + (ad+bc)*i*

Complex Numbers: Polar Form

- Defined in terms of a magnitude and phase.
- Complex numbers: p = <r,θ>, q = <s,ϕ>



$$r = \sqrt{4^2 + 5^2}$$

$$\theta = \arctan\frac{5}{4}$$

Complex Arithmetic (Polar)

- Complex numbers: p = <r,θ>, q = <s,ϕ>
- p+q = <r,θ> + <s,ϕ> = *something messy*
- p×q = <r,θ> × <s,ϕ> = <r·s, θ+ϕ>
- Some common constants:

  1 = <1, 0°> -1 = <1, 180°>

  i = <1, 90°> -i = <1, 270°>

  So i × i = <1·1, 90°+90°> = <1, 180°> = -1
- Multiplication is just scaling plus rotation!

Complex Magnitude

- In polar form:
  p = <r,θ> so |p| = r
- In rectangular form:
  p = a + bi, so $|p| = \sqrt{a^2 + b^2}$
- In quantum mechanics, probability is the square of the complex coefficient: $|p|^2$

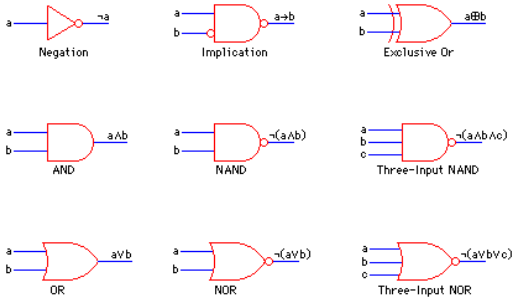
Quantum Weirdness 2a: Phase

- Consider a photon in state a|0> + b|1>.
- The complex coefficients ("amplitudes") *a* and *b* have both magnitude and phase.
- Photons have *polarization* determined by the relative phases of *a* and *b*.
  - Vertically polarized, horizontally polarized, left or right circularly polarized, elliptically polarized, etc.
- Polarized sunglasses filter out photons based on phase to reduce glare.

Logic Gates

Conventional Boolean logic gates:

- 1-input: the NOT gate
- 2-input: AND, OR, NAND, NOR, XOR, EQV, …

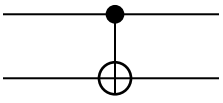


Quantum Gates

- 1-input quantum gates change the magnitudes and/or phases of *a* and *b*. Assume state is: a|0> + b|1>.
- Pauli-X gate: (a,b) → (b,a) quantum NOT
- Pauli-Y gate: (a,b) → (b*i*,-a*i*)
- Pauli-Z gate: (a,b) → (a,-b) phase flip
- Hadamard: (a,b) → (a+b,a-b) /$\sqrt{2}$

Quantum Gates

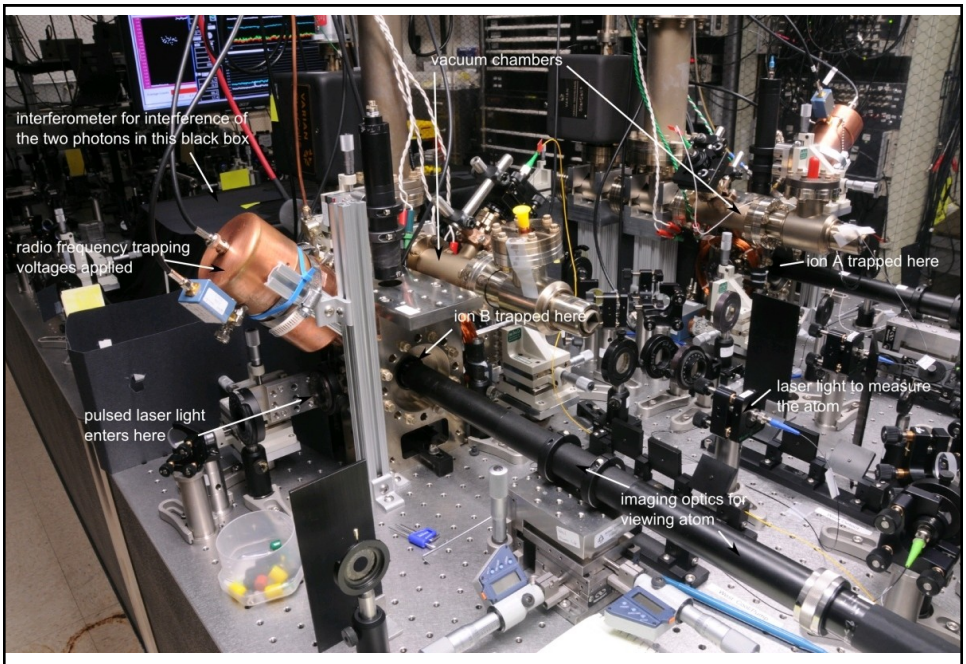
- 2- and 3-input quantum gates perform operations on one qubit based on the values of one or two other qubits.
- Controlled-NOT gate performs NOT on second qubit when first qubit is |1>.



- More gates: Toffoli, Fredkin, etc.

How to Make a Quantum Gate

- Use trapped ions for qubits.
  - Trap them in a vacuum using magnetic fields.
- Zap the ions with:
  - Magnetic fields
  - Lasers
  - Radio waves



QW3: Entanglement (Big Payoff)

- Suppose we have two independent qubits:
  $q_1 = a_1|0> + b_1|1>$
  $q_2 = a_2|0> + b_2|1>$
- If we measure them, we find that:
  q_1 is "down" with probability $|a_1|^2$
  q_2 is "down" with probability $|a_2|^2$
- For n qubits, we have 2n amplitudes.
- But qubits don't have to be independent…

Entanglement

- We can "hook up" two qubits so that their states are bound together, or "entangled".
- Now they have a joint state space:

  $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

  where a, b, c, d can all vary freely, subject to $|a|^2+|b|^2+|c|^2+|d|^2 = 1$.
- If a=d=0 then q_1 and q_2 have opposite states.

Implications of Entanglement

- If we entangle *n* entangled qubits, the resulting system has 2^n independent coefficients.
- You can operate on all 2^n coefficients *in parallel* by applying quantum gates.
- 50 entangled qubits give $2^{50} = 10^{15}$ coefficients: more memory than in any computer!

Quantum Algorithms

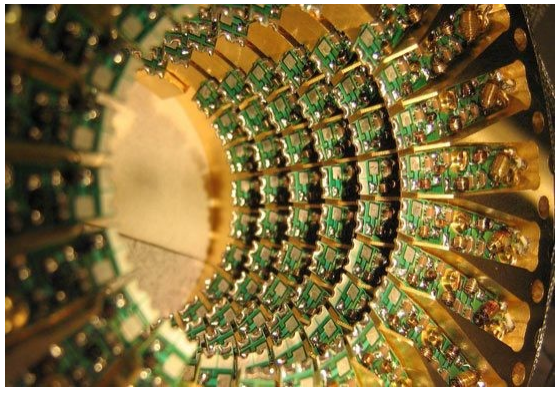
- Shor's algorithm can factor numbers.
  - Runs in time polynomial in # of digits.
  - Exponentially faster than conventional computer.
  - Might break RSA encryption.
- In 2001 IBM demonstrated factorization of 15 into 3 and 5 using a 7-qubit quantum computer.
- Another group has factored 21 into 3 and 7.

Quantum Algorithms

- Grover's algorithm for searching unordered lists (or inverting a function).
  - Runs in time O(sqrt(N)) where N = # of items
  - Conventional computer requires O(N) time.
- Works by exploiting the fact that coefficients have phases that can amplify (if in phase) or attenuate (if out of phase) when added.
- Google wants to use quantum algorithms for fast, sophisticated searching.

Obstacles to Quantum Computers

- Qubits don't last very long (decoherence).
  - Must keep them isolated to preserve their states.
  - Atoms cooled to almost absolute zero.
  - Any collision is a "measurement" that will "collapse the wave function": no more mixture.
- Entanglement is tricky to achieve.
  - Gets harder as the number of qubits goes up.



D-Wave Systems "demonstrated" a 28-qubit quantum computer in November 2007 at a SC07 (a supercomputing conference).

What's Next?

- Will we eventually prove that P = NP or P ≠ NP?
- Will the computers for the next generation be made up of quantum particles rather than silicon?
  - Star Trek computers already use qubits!
- Will humans become more and more robotic as they evolve?
  - Smartphones today; Google glasses tomorrow; cyborgs in 50 years?
- Will robots eventually replace humans as the dominant race due to their superior intelligence?