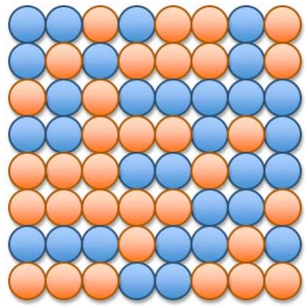


UNIT 6A
Organizing Data: Lists

Last Two Weeks

- Algorithms: Searching and sorting
- Problem solving technique: Recursion
- Asymptotic worst-case analysis using the big O notation

This Week

- Observe: The choice of how to organize your data determines how efficiently you can do certain types of computation
- Data structures
  - Arrays, linked lists, stacks, queues, hash tables, trees, graphs …
- Representation of dynamic sets and supporting operations such as search, insert, delete …

Data Structure

- The organization of data is a very important issue for computation.
- A **data structure** is a way of storing data in a computer so that it can be used efficiently.
  - Choosing the right data structure will allow us to develop certain algorithms for that data that are more efficient.
  - An **array** (or list) is a very simple data structure for holding a sequence of data.

Arrays in Memory

- Typically, array elements are stored in adjacent memory cells. The subscript (or index) is used to calculate an offset to find the desired element.
- Example: data = [50, 42, 85, 71, 99] Assume integers are stored using 4 bytes (32 bits).
- If we want data[3], the computer takes the address of the start of the array and adds the offset * the size of an array element to find the element we want.

Address	Contents
100	50
104	42
108	85
112	71
116	99

Location of data[3] is 100 + 3*4 = 112

- Do you see why the first index of an array is 0 now?

Arrays: Pros and Cons

- Pros:
  - Access to an array element is fast since we can compute its location quickly.
- Cons:
  - If we want to insert or delete an element, we have to shift subsequent elements which slows our computation down.
  - We need a large enough block of memory to hold our array.

Linked Lists

- Another data structure that stores a sequence of data values is the **linked list**.
- Data values in a linked list do not have to be stored in adjacent memory cells.
- To accommodate this feature, each data value has an additional “pointer” that indicates where the next data value is in computer memory.
- In order to use the linked list, we only need to know where the first data value is stored.

Linked List Example

- Linked list to store the sequence: 50, 42, 85, 71, 99

Assume each integer and pointer requires 4 bytes.

Starting Location of List (head)
124

address	data	next
100	42	148
108	99	0 (null)
116		
124	50	100
132	71	108
140		
148	85	132
156		

Linked List Example

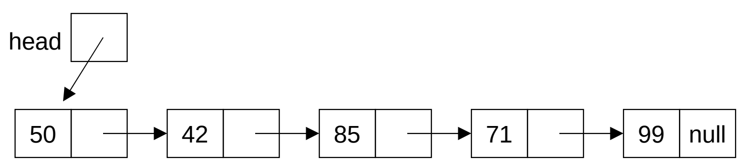
- To insert a new element, we only need to change a few pointers.
- Example: Insert 20 after 42.

Starting Location of List (head)
124

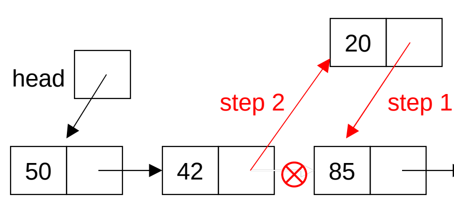
address	data	next
100	42	**156**
108	99	0 (null)
116		
124	50	100
132	71	108
140		
148	85	132
156	**20**	**148**

Drawing Linked Lists Abstractly

- L = [50, 42, 85, 71, 99]



- Inserting 20 after 42:



We link the new node to the list before breaking the existing link.

Linked Lists: Pros and Cons

- Pros:
  - Inserting and deleting data does not require us to move/shift subsequent data elements.
- Cons:
  - If we want to access a specific element, we need to traverse the list from the head of the list to find it which can take longer than an array access.
  - Linked lists require more memory. (Why?)

Two-dimensional arrays

- Some data can be organized efficiently in a **table** (also called a **matrix** or **2-dimensional array**)
- Each cell is denoted with two subscripts, a row and column indicator

B	0	1	2	3	4
0	3	18	43	49	65
1	14	30	32	53	75
2	9	28	38	50	73
3	10	24	37	58	62
4	7	19	40	46	66

B[2][3] = 50

2D Arrays in Ruby

```
data = [ [ 1, 2, 3, 4],
         [5, 6, 7, 8],
         [9, 10, 11, 12]
     ]
```

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12

```
data[0]    =>   [1, 2, 3, 4]
data[1][2]  =>  7
data[2][5]  =>  nil
data[4][2]  =>  undefined method '[]' for nil
```


2D Array Example in Ruby

- Find the sum of all elements in a 2D array

```
def sumMatrix(table)
  sum = 0
  for row in 0..table.length-1 do
     for col in 0..table[row].length-1 do
          sum = sum + table[row][col]
     end
  end
  return sum
end
```

number of rows in the table

number of columns in the given row of the table

Tracing the Nested Loop

```
for row in 0..table.length-1 do
  for col in 0..table[row].length-1 do
    sum = sum + table[row][col]
  end
end
```

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12

```
table.length = 3
table[row].length = 4 for every row
```

row	col	sum
0	0	1
0	1	3
0	2	6
0	3	10
1	0	15
1	1	21
1	2	28
1	3	36
2	0	45
2	1	55
2	2	66
2	3	78

Stacks

- A **stack** is a data structure that works on the principle of Last In First Out (LIFO).
  - LIFO: The last item put on the stack is the first item that can be taken off.
- Common stack operations:
  - Push – put a new element on to the top of the stack
  - Pop – remove the top element from the top of the stack
- Applications: calculators, compilers, programming

RPN

- Some modern calculators use Reverse Polish Notation (RPN)
  - Developed in 1920 by Jan Lukasiewicz
  - Computation of mathematical formulas can be done without using any parentheses
  - Example:
    (3 + 4) * 5 =
    becomes in RPN:
    3 4 + 5 *

RPN Example

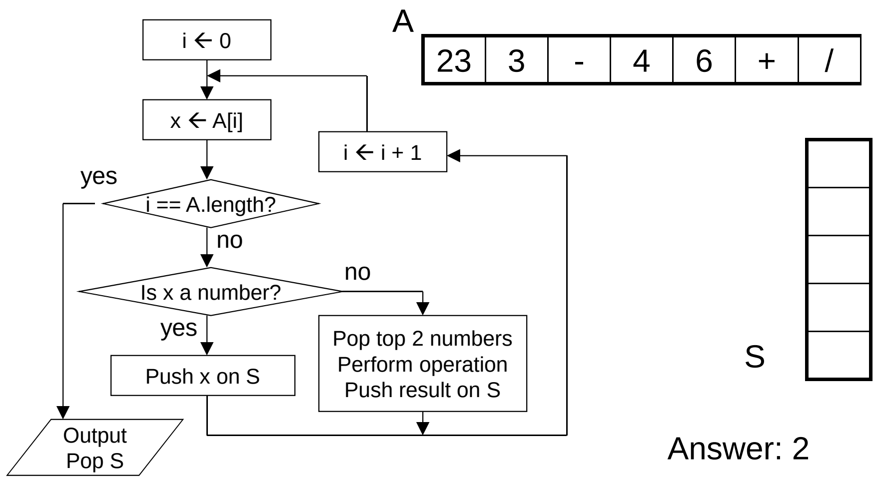
Convert the following standard mathematical expression into RPN:

```
(23 – 3)      /      (4 + 6)

23    3    –          4     6    +
operand1 operand2 operator    operand1 operand2 operator

23    3    –    4    6    +        /
   operand1        operand2      operator
```

Evaluating RPN with a Stack

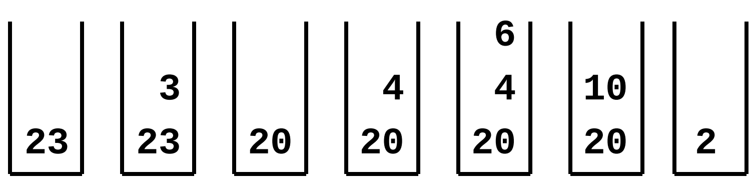


Answer: 2

Example Step by Step

- RPN: **23 3 - 4 6 + /**

- Stack Trace:



Stacks in Ruby

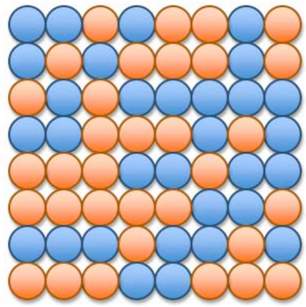
- You can treat arrays (lists) as stacks in Ruby.

	stack	x
`stack = []`	[]	
`stack.push(1)`	[1]	
`stack.push(2)`	[1,2]	
`stack.push(3)`	[1,2,3]	
`x = stack.pop()`	[1,2]	3
`x = stack.pop()`	[1]	2
`x = stack.pop()`	[]	1
`x = stack.pop()`	nil	nil

Queues

- A **queue** is a data structure that works on the principle of First In First Out (FIFO).
  - FIFO: The first item stored in the queue is the first item that can be taken out.

- Common queue operations:
  - Enqueue – put a new element in to the rear of the queue
  - Dequeue – remove the first element from the front of the queue
- Applications: printers, simulations, networks



UNIT 6B
Organizing Data: Hash Tables

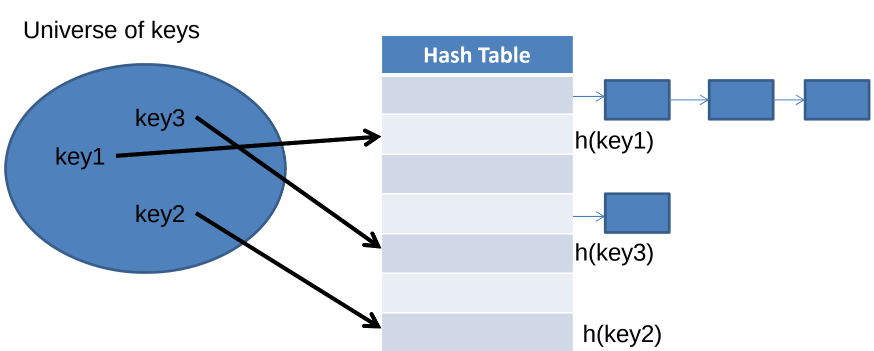
Comparing Algorithms

- You are a professor and you want to find an exam in a large pile of n exams.
- Search the pile using linear search.
  - Per student: O(n)
  - Total for n students: $O(n^2)$
- Have an assistant sort the exams first by last name.
  - Assistant's work: O(n log n) using merge sort
  - Professor:
    - Search for one student: O(log n) using binary search
    - Total for n students: O(n log n)

Another way

- Set up a large number of "buckets".
- Place each exam into a bucket based on some function.
  - Example: 100 buckets, each labeled with a value from 00 to 99. Use the student's last two digits of their student ID number to choose the bucket.
- Ideally, if the exams get distributed evenly, there will be only a few exams per bucket.
  - Assistant: O(n) putting n exams into the buckets
  - Professor: O(1) search for an exam by going directly to the relevant bucket and searching through a few exams.

Hashing



A hash function h is used to map keys to hash-table slots

Strings and ASCII codes

```
s = "hello"
for i in 0..s.length-1 do
  print s[i], "\n"
end
```

```
104
101
108
108
111
```

You can treat a string like an array in Ruby.
If you access the ith character, you get the ASCII code for that character.

Hash table

- Let's assume that we are going to store only lower case strings into an array (**hash table**).

```
table1 = Array.new(26)
=> [nil, nil, nil, nil, nil, nil, nil, nil,
  nil, nil, nil, nil, nil, nil, nil, nil,
  nil, nil, nil, nil, nil, nil, nil, nil,
  nil, nil]
```

Hash table

- We could pick the array position where each string is stored based on the first letter of the string using this hash function:

```
def h(string)
  return string[0] - 97
end
```

The ASCII values of lowercase letters are:

“a” -> 97, “b” -> 98, “c” -> 99, “d” -> 100, etc.

Inserting into Hash Table

- To insert into the hash table, we simply use the hash function h to determine which index (“bucket”) to store the element.

```
def insert(table, name)
  table[h(name)] = name
end

insert(table1, “aardvark”)
insert(table1, “beaver”)  ...
```

Hash function (cont'd)

- Using the hash function h:
  - "aardvark" would be stored in an array at index 0
  - "beaver" would be stored in an array at index 1
  - "kangaroo" would be stored in an array at index 10
  - "whale" would be stored in an array at index 22

```
table1
=> ["aardvark", "beaver", nil, nil, nil,
  nil, nil, nil, nil, nil, "kangaroo", nil,
  nil, nil, nil, nil, nil, nil, nil, nil,
  nil, nil, "whale", nil, nil, nil]
```


Hash function (cont'd)

- But if we try to insert "bunny" and "bear" into the hash table, each word overwrites the previous word since they all hash to index 1:

```
>> insert(table1,"bunny")
>> insert(table1,"bear")
>> table1
=> ["aardvark", "bear", nil, nil, nil, nil,
  nil, nil, nil, nil, "kangaroo", nil, nil,
  nil, nil, nil, nil, nil, nil, nil, nil,
  nil, "whale", nil, nil, nil]
```

Revised Hash table

- Let's make our hash table an array of arrays (an array of "buckets")
- Each bucket can hold more than one string.

```
table2 = Array.new(26)
for i in 0..25 do
   table2[i] = []
end
=> [[], [], [], [], [], [], [], [], [], [],
  [], [], [], [], [], [], [], [], [], [],
  [], [], [], [], [], []]
```


Revised insert function

```
def insert(table, key)
  # find the bucket (array) in the table
  # array using the hash function h
  bucket = table[h(key)]
  # append the key string to the bucket
  # array
  bucket << key
end
```

Inserting into new hash table

```
insert(table2, "aardvark")
>> insert(table2, "beaver")
>> insert(table2, "kangaroo")
>> insert(table2, "whale")
>> insert(table2, "bunny")
>> insert(table2, "bear")
>> table2
=> [["aardvark"], ["beaver", "bunny",
  "bear"], [], [], [], [], [], [], [], [],
  ["kangaroo"], [], [], [], [], [], [], [],
  [], [], [], [], ["whale"], [], [], []]
```


Collisions

- “beaver”, “bunny” and “bear” all end up in the same bucket.
- These are collisions in a hash table.
- The more collisions you have in a bucket, the more you have to search in the bucket to find the desired element.
- We want to try to minimize the collisions by creating a hash function that distribute the keys (strings) into different buckets as evenly as possible.

First Try

```
def h(string)
  k = 0
  for i in 0..string.length-1 do
    k = string[i] + k
  end
  return k
end
h("hello") => 532
h("olleh") => 532
```

Permutations still give same index (collision) and numbers are high.

Second Try

```
def h(string)
  k = 0
  for i in 0..string.length-1 do
    k = string[i] + k*256
  end
  return k
end
h("hello") => 448378203247
h("olleh") => 478560413032
```

Better, but numbers are still high. We probably don't want to (or can't) create arrays that have indices this large.

Third Try

```
def h(string, tablesize)
  k = 0
  for i in 0..string.length-1 do
    k = string[i] + k*256
  end
  return k % tablesize
end
```

We can use the modulo operator to take the large values and map them to indices for a smaller array.

Revised insert function

```
def insert(table, key)
  # find the bucket (array) in the table
  # array using the hash function h
  bucket = table[h(key, table.length)]
  # append the key string to the bucket
  # array
  bucket << key
end
```

Final results

```
table3 = Array.new(13)
for i in 0..12 do table3[i] = [] end
=> [[], [], [], [], [], [], [], [], [], [], [], [],
  []]
>> insert(table3,"aardvark")
>> insert(table3,"bear")
>> insert(table3,"bunny")
>> insert(table3,"beaver")
>> insert(table3,"dog")
>> table3
=> [[], [], [], [], [], [], [], [], [], ["bunny"],
  ["aardvark", "bear"], ["dog"], ["beaver"]]
```

Still have one collision, but b-words are distributed better.

Searching in a hash table

To search for a key, use the hash function to find out which bucket it should be in, if it is in the table at all.

```
def contains?(table, key)
  bucket = table[h(key,table.length)]
  for entry in bucket do
    return true if entry == key
  end
  return false
end
```

Efficiency

- If the keys (strings) are distributed well throughout the table, then each bucket will only have a few keys and the search should take O(1) time.
- Example:
  If we have a table of size 1000 and we hash 4000 keys into the table and each bucket has approximately the same number of keys (approx. 4), then a search will only require us to look at approx. 4 keys => O(1)
  - But, the distribution of keys is dependent on the keys and the hash function we use!