16-299 Spring 2009: Introduction to Control Systems

Problem Set 1

due 27 January 2009

This problem set is due at the beginning of class on the listed due date. **No late problem sets will be accepted!**

Please show all work and write clearly—all intermediate steps must be shown and legible.

You are encouraged to work with other students to find the answers to these problems, however each student must submit his/her own *unique* set solutions. If you do work with other students, please give them credit by listing their names at the end of your solutions.

1. (10 points) Let $x = 3$, $y = 5$, $z = 0.3$, $s_1 = 7 + 4i$, $s_2 = -3 + i$. Write a single MATLAB script file to evaluate the following expressions. Please submit both a hardcopy of the MATLAB code and the answers it generates.

   (a) $x + y$
   (b) $\frac{5y^z}{x+xy}$
   (c) $3y\pi$
   (d) $\frac{6\left(\frac{y+1}{z}+1\right)}{z}$
   (e) Re(s_1)
   (f) Im(s_2)
   (g) $|s_1 + s_2|$ (the magnitude of $s_1 + s_2$)
   (h) $\angle\frac{s_1}{s_2}$ (the phase of $\frac{s_1}{s_2}$)
   (i) $\frac{3}{2i}s_2$

2. (5 points) Using the values of s_1 and s_2 from above, compute the value of $\angle(s_1 s_2) - \angle(s_1) - \angle(s_2)$. Use Euler's Identity to explain this answer.

3. Consider the complex-valued function $f(t) = Ae^{i\omega t}$, where $A = 2$ and $\omega = 20\pi$. Use MATLAB to generate the following plots:

   (a) (5 points) Im($f(t)$) vs. Re($f(t)$) for t ranging from 0 to 0.1 seconds.
   (b) (5 points) Re($f(t)$) vs. t for t ranging from 0 to 0.5 seconds. What well-known function does this plot look like?
   (c) (5 points) Im($f(t)$) vs. t for t ranging from 0 to 0.5 seconds. What well-known function does this plot look like?

4. (20 points) Consider the savings account system presented in class:

$$-(r+1)y[k-1] + y[k] = (r+1)u[k].$$

   The monthly interest rate $r = 0.005$. Assuming last month's balance was $y[0] = 0$, how much money will you need to put in each month in order to retire 40 years from now with about a million dollars in the bank? Write a MATLAB script to solve the difference equation and then find the answer using trial and error. Submit a hardcopy of your MATLAB code.

5. Find the discrete time approximations for

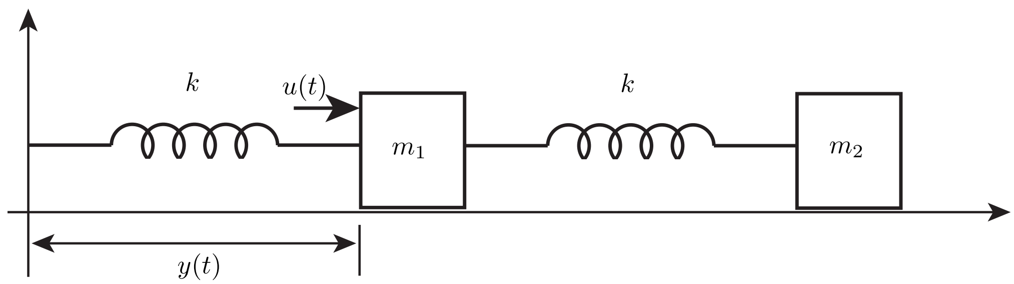
   (a) (5 points) $\frac{d^3}{dt^3}y(t)|_{t=t_0+kT}$

   (b) (5 points) $\frac{d^4}{dt^4}y(t)|_{t=t_0+kT}$

6. (20 points) The following LTI ODE describes the dynamics of the mass-spring system depicted in the figure below. The input $u(t)$ is the force applied to the first mass m_1 and output $y(t)$ is the location of the second mass m_2:

$$m_1 m_2 \frac{d^4}{dt^4}y(t) + k(m_1 + m_2)\frac{d^2}{dt^2}y(t) + k^2 y(t) = m_1 \frac{d^2}{dt^2}u(t) + 2ku(t).$$

   Here, k is the spring constant of both springs. Assume that $m_1 = 1$, $m_2 = 2$, and $k = 3$. Assuming a sampling rate of $T = 0.1$ seconds, find the difference equation that approximates this system.



7. (20 points) Consider the discrete time approximation of the mass-spring-damper system presented in class:

$$\frac{m}{T^2}y[k-2] - \left(\frac{2m}{T^2} + \frac{\mu}{T}\right)y[k-1] + \left(\frac{m}{T^2} + \frac{\mu}{T} + k\right)y[k] = u[k].$$

   The system parameters are $m = 1$, $\mu = 0.3$, and $k = 2$. The sampling rate is $T = 0.2$ seconds. For time $t < 0$, the output y is know to be zero. The input applied to the system is a unit impulse:

$$u[k] = \begin{cases} 1 & \text{if} \quad k = 0 \\ 0 & \text{if} \quad k \neq 0 \end{cases}$$

   Write a MATLAB script to compute the output y of the system for time ranging from 0 to 20 seconds. Plot y as a function of time. Attach a hardcopy of your MATLAB code.