10. Kinematics and Inverse Kinematics of the Rhino XR-3

Topics

Be able to

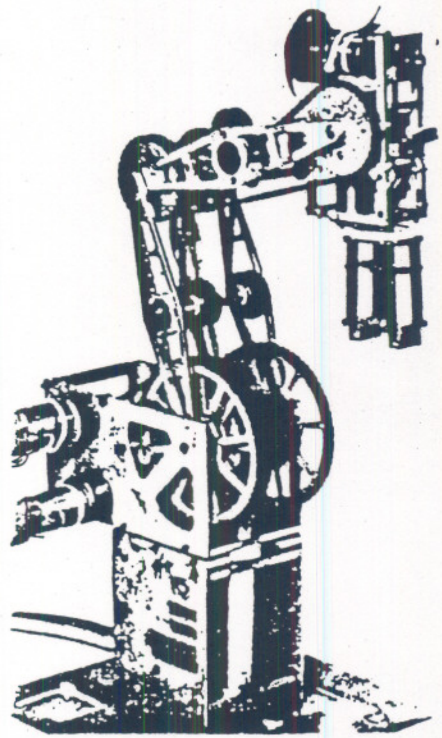
- find the kinematics and inverse kinematics of a *five-axis articulated* robot.

- find the *tool-configuration vector* for this case.

Ref.: Schilling, pp. 62-68, 90-96.

The Rhino XR-3

has five revolute joints. Identify the joints from the figure below.

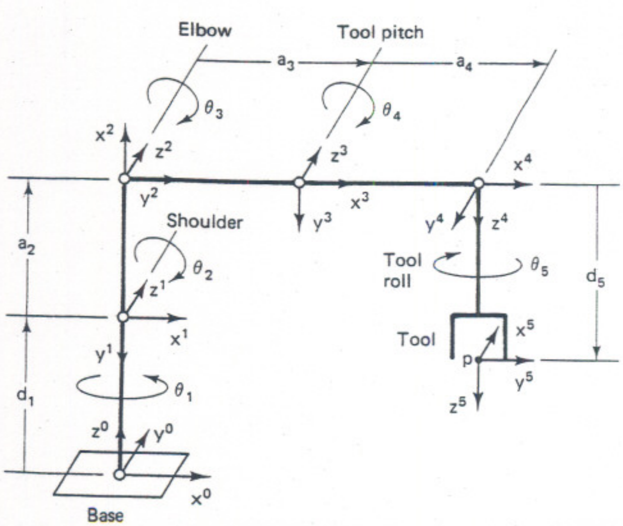


The Link Coordinate Diagram

Link Parameters

Axis	θ	d	a	α	*Home*
1					
2					
3					
4					
5					

$\theta's$ satisfy the right hand rule
$d's$ are translations along $z's$
$a's$ are translations along $x's$
$\alpha's$ are rotations about x



The Arm Matrix

$$T_{base}^{tool} = T_{base}^{wrist}\, T_{wrist}^{tool}$$

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find T_{base}^{wrist} ,

Example

Where is the wrist frame in base coordinates when $q = [0, -90, 90, 0, -90]^T$? What is the meaning of this q?

Solution

$$T_{base}^{wrist}(q) =$$

This is the orientation and position of L_3 in base coordinates.

Interpretation

arm matrix (cont)

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find $T_{wrist}^{tool}(q)$,

Final Arm Matrix

$$\begin{bmatrix} C_1 C_{234} C_5 + S_1 S_5 & -C_1 C_{234} S_5 + S_1 C_5 & -C_1 S_{234} & \vdots & C_1(a_2 C_2 + a_3 C_{23} + a_4 C_{234} - d_5 S_{234}) \\ S_1 C_{234} C_5 - C_1 S_5 & -S_1 C_{234} S_5 - C_1 C_5 & -S_1 S_{234} & \vdots & S_1(a_2 C_2 + a_3 C_{23} + a_4 C_{234} - d_5 S_{234}) \\ -S_{234} C_5 & S_{234} S_5 & -C_{234} & \vdots & d_1 - a_2 S_2 - a_3 S_{23} - a_4 S_{234} - d_5 C_{234} \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}$$

Example

What is the position and orientation of the tool in base coordinates when

$$q = [0, -90, 90, 0, -90]^T?$$

Solution

$$T_{base}^{tool}(q) =$$

Where is the tool tip located in base coordinates?

Inverse Kinematics

Find the *Tool Configuration Vector.*

Here, the approach vector r^3 is along z^5. What is it in base coordinates?

This leads to the tool configuration vector,

$w(q) =$

Base Joint

Note: $w_2/w_1 =$

Elbow Joint, q_3

Start with $q_{234} = q_2 + q_3 + q_4 =$

Use $C_1 w_4 + S_1 w_5 =$

define: (to find q_3)

substitute for the w's

$b_1 =$

$b_2 =$

Now use $b_1^2 + b_2^2 =$

Elbow-up solution kee
workspace. Joint limit
down solution.

Shoulder Joint, q_2

we know q_3, so solve

$b_1 = a_2 C_2 + a_3 C_{23} =$

$b_2 = a_2 S_2 + a_3 S_{23} =$

for C_2 and S_2.

Tool Pitch Joint, q_4

$$q_{234} = q_2 + q_3 + q_4$$

Tool Roll Joint Angle, q_5

$$q_5 =$$