

# Histogram-Based Objective Function for Radiotherapy Planning

## Assumed functional form

For a given treatment plan  $P$  consisting of specified beam angles and individual beam doses, let  $D(P, x)$  be the represent the predicted dose per voxel as a function of  $P$  and voxel location  $x$ . We wish to measure the utility or “goodness” of  $D$  — denoted by  $U(D)$  — in a manner that is both clinically useful and computationally feasible.

We propose to assume  $U(D)$  is of the following form:

$$U(D) = U(H_1(D), \dots, H_N(D))$$

where each  $H_r(D)$  is a function of the values of  $D(P, x)$  restricted to some subset  $S_r$  of all possible voxels. Each voxel will be contained in exactly one subset  $S_r$ . Each subset  $S_r$  corresponds to a single tissue type, such as normal brain tissue, tumor, or a particular type of critical normal tissue.

$U(H_1(D), \dots, H_N(D))$  might, for example, combine  $H_1(D)$  through  $H_N(D)$  in some simple fashion such as a weighted arithmetic or geometric mean.

Every  $H_r(D)$  is assumed to be a function of a *histogram* of the values that  $D(x)$  takes on over the set of  $x \in S_r$ . Formally,

$$H_r(D) = F(c_1^r, \dots, c_M^r)$$

where

- the histogram has  $M$  bins;
- each  $c_i^r$  is the count of voxels  $x \in S_r$  such that  $l_i^r \leq D(x) < u_i^r$ ;
- $l_i^r$  and  $u_i^r$  are the dose values corresponding to the  $r^{th}$  histogram’s  $i^{th}$  bin’s lower and upper bounds, respectively.

For example, Figure 1 shows a crude representation of a cross-section of a possible brain scan. The tissue voxels have been partitioned into five different subsets:  $S_{norm}$  for normal brain tissue,  $S_{tumor}$  for tumor tissue,  $S_{eyes}$  for the

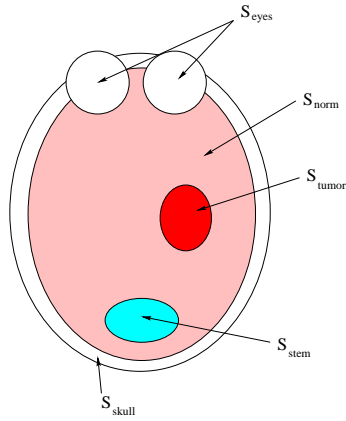


Figure 1: Example tissue partitions  $S_r$

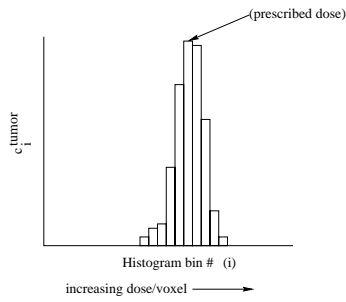


Figure 2: Example histogram for tumor tissue

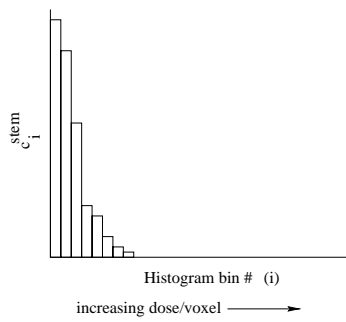


Figure 3: Example histogram for critical normal tissue (e.g., brain stem)

eyes,  $S_{stem}$  for the brain stem, and  $S_{skull}$  for the skull. Figure 2 shows a possible histogram for the radiation per voxel over the  $S_{tumor}$  voxel subset; Figure 3 shows a possible histogram for the  $S_{stem}$  subset.

One factor of possible clinical significance that the above assumptions may rule out modeling is the fact that it may be worse to have a few large “hot spots” within the tissue rather than many smaller “hot spots” with the same total volume and radiation intensity. However, such factors would be computationally expensive to model, and the assumption that each  $H_r$  is histogram-based still allows for much flexibility in terms of what objective functions can be modeled. For example:

## Penalties for Deviations from Prescribed Dose

The histogram-based approach may easily be used to implement an objective function that penalizes deviations from prescribed doses on a voxel-by-voxel basis. A weighted-sum-of-squared-deviations objective function could be implemented simply by specifying:

$$H_r(D) = w_r * \sum_{i=0}^M [c_i^r * (P_r - (\frac{u_i^r - l_i^r}{2})^2)]$$

where  $P_r$  is the prescribed dose,  $w_r$  is a weight indicating the relative importance of getting as close to the prescribed dose as possible for the particular tissue type  $r$ . (Note that we don’t actually need to use a finite number  $M$  of bins for this objective function; we could simply iterate over all the voxels in  $S_r$  instead.)

Such an objective function should cause the optimizer to:

- Keep the doses close to the prescribed doses, particularly in critical tissue or other tissue with high weights  $w_r$ .
- This should naturally lead to high dose gradients at tissue boundaries, particularly boundaries between tumor tissue and normal critical tissue.

Alternatively, we could formulate even simpler objective functions with penalties proportional to the amount of normal tissue dosed over a certain threshold, plus additional penalties proportional to the amount of tumor tissue receiving doses under another threshold. Or, we could allow the practitioner to graphically specify more complicated penalty functions for specific deviations from the prescribed dose via the use of splines.

Any of the above possible objective functions could be supplemented with an additional term that explicitly penalizes nonuniform dose distributions throughout the tumor. For example, some multiple of the standard deviation of the dose values found in the tumor tissue’s histogram could be subtracted from the rest of the objective function.

## Cell Response Models

Instead of an objective function based on penalizing deviations from a prescribed dose, one might wish for an objective function that attempted to explicitly model the probability of controlling the tumor without inducing serious complications in normal tissue. Most current theoretical models of tumor control probability (TCP) and normal tissue complication probability (NTCP) are based on dose histograms, largely since more complicated models would require more data than currently available. Thus, a histogram-based approach such as the one above would be sufficient to implement a biological-response-based objective function.

## Computational advantages

A histogram-based objective function of the form described above has the computational advantage that if a single beam weight or position is varied, one only need look at the voxels intersecting the old and new beam positions in order to update the value of the objective function. This may provide a significant computational advantage when using optimization algorithms that adjust only a few beams at a time while exploring the space of possible solutions.