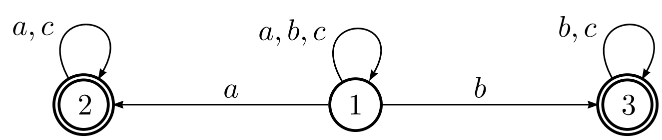
15-354: Computational Discrete Math	**K. Sutner**
Assignment 7	Due: November 5, 2009

Problem 1: Determinization of Büchi Automtata (50 pts.)

Background

Below is the natural Büchi automaton $\mathcal{A}$ for the language $L \subseteq \{a, b, c\}^\omega$ of all words that contain either at least one a but finitely many b's or at least one b but finitely many a's (look at the machine if this makes no sense).



Here $I = \{1\}$ and $F = \{2, 3\}$.

Task

A. Write a "regular expression" for this language.

B. Run Safra's algorithm on $\mathcal{A}$ to obtain a Rabin automaton for L.

C. Construct a Büchi automaton for the complement of L.

Comment

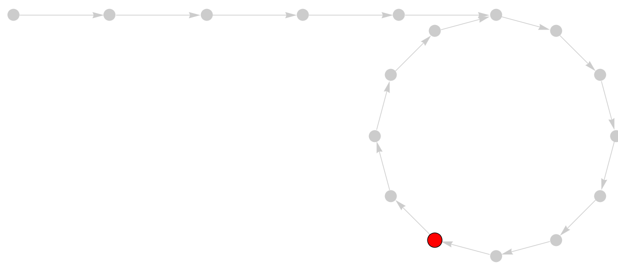
For the last part, make sure to draw a picture. It's not at all bad if you pick the right layout. Step 1 is to draw a nice picture for the Rabin automaton in part (B). Think linear.

Problem 2: Floyd goes Algebraic (50 pts.)

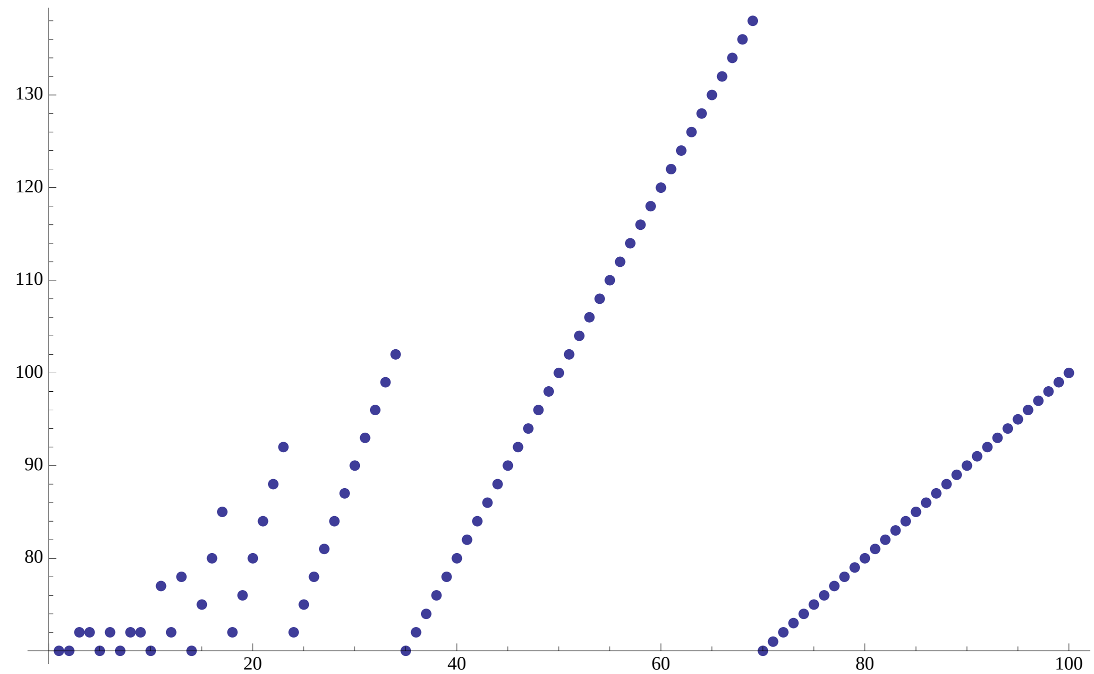
Background

One of the most elementary results about finite semigroups is that every element in the semigroup has an idempotent power. Recall that x is idempotent if $x^2 = x$. Then the claim is that for S any finite semigroup and $a \in S$, there exists some integer $r \geq 1$ such that a^r is idempotent.

Ignoring algebraic aspects for a moment, note that the powers of a (the points a^i, $i \geq 0$) must form a lasso since S is finite. Hence we can associate a with a transient $t = t(a) \geq 0$ and a period $p = p(a) \geq 1$.



More importantly, we can apply Floyd's trick to find a point on the loop (the big, red dot above). Let's call the time when the algorithm finds this point the Floyd time, an integer in the range 0 to $t+p-1$. The next picture shows the Floyd times for $t = 70$ and $p = 1, \ldots, 100$.



Inquisitive minds will wonder if there is any connection between the Floyd point and the idempotent from above.

Task

A. Prove the claim about idempotent powers. Hint: think about the Floyd point.

B. Explain the plot of the Floyd times above.

C. Describe the exponent r in the claim in terms of the transient and period of a.

D. Show that the powers of a that lie on the loop form a group with the idempotent a^r as identity.

Comment

For extra credit you might (re-)consider the question of what happens in Floyd's algorithm when one changes the velocities of the particles to $1 \leq u < v$.