The k-center problem is a canonical and long-studied facility location and clustering problem with many applications in both its symmetric and asymmetric forms. Both versions of the problem have tight approximation factors on worst case instances: a 2-approximation for symmetric k-center and an $O(\log^*k)$-approximation for the asymmetric version.

In this work, we go beyond the worst case and provide strong positive results both for the asymmetric and symmetric k-center problems under a very natural input stability (promise) condition called $\hat{\alpha}$-perturbation resilience (Bilu & Linial 2012), which states that the optimal solution does not change under any alpha-factor perturbation to the input distances. We show that by assuming 2-perturbation resilience, the exact solution for the asymmetric k-center problem can be found in polynomial time. To our knowledge, this is the first problem that is hard to approximate to any constant factor in the worst case, yet can be optimally solved in polynomial time under perturbation resilience for a constant value of alpha. Furthermore, we prove our result is tight by showing symmetric k-center under $(2\hat{\alpha}^{-\epsilon})$-perturbation resilience is hard unless NP=RP. This is the first tight result for any problem under perturbation resilience, i.e., this is the first time the exact value of alpha for which the problem switches from being NP-hard to efficiently computable has been found.

Our results illustrate a surprising relationship between symmetric and asymmetric k-center instances under perturbation resilience. Unlike approximation ratio, for which symmetric k-center is easily solved to a factor of 2 but asymmetric k-center cannot be approximated to any constant factor, both symmetric and asymmetric k-center can be solved optimally under resilience to 2-perturbations.

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