#### **ABSTRACT**

This thesis presents a framework for low-latency interactive simulation of linear elastostatic models and other systems arising from linear elliptic partial differention equations. This approach makes it feasible to interactively simulate large-scale physical models.

Linearity is exploited by formulating the BVP solution in terms of Green's functions (GFs) which may be precomputed to provide speed and cheap lookup operations. Runtime BVPs are solved using a collection of Capacitance Matrix Algorithms (CMAs) based on the Sherman-Morrison-Woodbury formula. Temporal coherence is exploited by caching and reusing, as well as sequentially updating, previous capacitance matrix inverses.

Multiresolution enhancements make it practical to simulate and store very large models. Efficient compressed representations of precomputed GFs are obtained using second-generation wavelets defined on surfaces. Fast inverse wavelet transforms allow fast summation methods to be used to speedup runtime BVP solution. Wavelet GF compression factors are directly related to interactive simulation speedup, and examples are provided with hundredfold improvements at modest error levels. Furthermore, hierarchical constraints are defined using hierarchical basis functions, and related hierarchical GFs are then used to construct an hierarchical CMA. This direct solution approach is suitable for hard real time simulation since it provides a mechanism for gracefully degrading to coarser resolution approximations, and the wavelet representations allow for runtime adaptive multiresolution rendering.

These GF CMAs are well-suited to interactive haptic applications since GFs allow random access to solution components and the capacitance matrix is the contact compliance used for high-fidelity force-feedback rendering. Examples are provided for distributed and point-like interactions.

Precomputed multizone kinematic GF models are also considered, with examples provided for character animation in computer graphics.

Finally, we briefly discuss the generation of multiresolution GF models using either numerical precomputation methods or reality-based robotic measurement.

#### **BIOGRAPHICAL NOTES**

Born: July 20, 1972, London, Ontario, Canada

Academic Studies: B. Sc. University of Western Ontario, 1995

M. Sc. University of British Columbia, 1997

#### **GRADUATE STUDIES**

Field of Study: Interactive multi-modal simulation of continuous systems, algorithms for real time elastostatics, multiresolution analysis on manifolds, numerical analysis, numerical solution of boundary integral equations, computer graphics and haptics.

Courses		Instructors
CPSC 510	Multigrid and Multilevel Methods	Dr. U. Ascher
CPSC 515	Computational Robotics	Dr. D. Pai
CPSC 517	Sparse Matrix Computation	Dr. J. Varah
ELEC 592	Achitectures for Learning Systems	Dr. S. Sarkaria
MATH 521	Numerical Analysis I	Dr. B. Wetton
MATH 550	Asymptotic and Perturbation Methods	Dr. A. Peirce
MATH 551	Perturbation Methods for Diff. Equations	Dr. A. Peirce
MATH 552	Introduction to Dynamical Systems	Dr. W. Nagata
MATH 560	Math Biology I	Dr. R. Miura
PHYS 500B	Elementary Quantum Mechanics	Dr. M. McMillan
PHYS 505	Nuclei and Particles	Dr. G. Jones & Dr. J. Iqbal

#### **AWARDS**

2001	1 <sup>st</sup> prize for technology demonstration, 11 <sup>th</sup> Annual Precarn-IRIS			
	(Institute for Robotics and Intelligent Systems) Conference			
2000	1 <sup>st</sup> prize for poster presentation, 10 <sup>th</sup> Annual Precarn-IRIS			
	(Institute for Robotics and Intelligent Systems) Conference			
1999-2001	University Graduate Fellowship, Math, UBC.			
1997-1999	NSERC PGSb (Ph.D.), IAM, UBC.			
1995-1997	NSERC PGSa (M.Sc.), IAM, UBC.			
1995	Gold medal for highest standing in UWO Applied Math theoretical			
	physics program.			
1991-1995	Canada Scholar (four years, continuing).			
1991-1995	UWO Scholarship (four years, continuing).			
1994,1995	Two NSERC undergraduate research awards.			
1990	Canadian Governor General's Academic Medal.			

#### **PUBLICATIONS**

Doug L. James and Dinesh K. Pai, A Unified Treatment of Elastostatic and Rigid Contact for Real Time Haptics, *Haptics-e, The Electronic Journal of Haptics Research* (www.haptics-e.org), 2001. (To appear)

Dinesh K. Pai, Kees van den Doel, Doug L. James, Jochen Lang, John E. Lloyd, Joshua L. Richmond, Som H. Yau, Scanning Physical Interaction Behavior of 3D Objects, *In SIGGRAPH 01 Conference Proceedings, Annual Conference Series, ACM SIGGRAPH*, August, 2001.

Doug L. James and Dinesh K. Pai, Pressure Masks for Point-like Contact with Elastic Models, *In Proceedings of the Fifth Phantom User Group Workshop*, J.K. Salisbury and M.A. Srinivasan (Eds), 2000.

Doug L. James and Dinesh K. Pai, ARTDEFO: Accurate Real Time Deformable Objects, *In SIGGRAPH 99 Conference Proceedings, Annual Conference Series, ACM SIGGRAPH*, August, 1999.

J.-P. St.-Maurice, W. Kofman, and D. James, In-situ generation of intense parallel fields in the lower ionosphere, *J. Geophys. Res.*, 101: 335-356, 1996.

# **PRESENTATIONS**

Scanning Physical Interaction Behavior of 3D Objects, with Dinesh Pai et al., *ACM SIGGRAPH 01 Annual Conference*, Los Angeles, CA, USA, August, 2001.

Multiresolution Methods for Force-feedback Simulation of Elastic Objects, International Conference on Scientific Computation And Differential Equations (SciCADE), Vancouver, BC, Canada, July 2001.

Visual Acquisition and Haptic Interaction with Deformable Models, Interactive demonstration, with Jochen Lang and Dinesh Pai, *International Conference on Computer Vision (ICCV)*, Vancouver, BC, Canada, July 2001.

Force-feedback Simulation of Reality-based Elastic Models, with Jochen Lang et al., 11<sup>th</sup> Annual Precarn-IRIS (Institute for Robotics and Intelligent Systems) Conference, Ottawa, ON, Canada, June, 2001.

Interactive Elastic Modeling, Invited exhibitor, *ACM1: Beyond Cyberspace*, San Jose, CA, USA, March, 2001.

Fast Simulation of Elastostatic Deformable Models, Invited speaker, *Game Technology Seminars 2001*, chaired by Chris Hecker and Jeff Lander, San Francisco, CA, USA, January, 2001.

Pressure Masks for Point-like Contact with Elastic Models, *Fifth Phantom User Group Workshop*, Aspen, CO, USA, October, 2000.

Interactive Elastic Modeling, Force-feedback demonstration, ACM SIGGRAPH 00 Exhibition, New Orleans, LA, USA, July 2000.

Interactive Elastic Modeling, Force-feedback demonstration with poster, 10<sup>th</sup> Annual Precarn-IRIS (Institute for Robotics and Intelligent Systems) Conference, Montreal, QC, Canada, June, 2000.

ARTDEFO: Accurate Real Time Deformable Objects, with Dinesh Pai, ACM SIGGRAPH 99 Annual Conference, Los Angeles, CA, USA, August, 1999.

# SUPERVISORY COMMITTEE

Dr. Dinesh K. Pai, Research Supervisor (Computer Science)

Dr. Alain Fournier (Computer Science, passed away August 2000)

Dr. Wolfgang Heidrich (Computer Science)

Dr. Anthony Peirce (Mathematics)

Dr. Reza Vaziri (Civil Engineering)



# **PROGRAMME**

The Final Oral Examination For the Degree of

DOCTOR OF PHILOSOPHY (Department of Mathematics, Institute of Applied Mathematics)

# **Douglas Leonard James**

B. Sc. University of Western Ontario, 1995M. Sc. University of British Columbia, 1997

Tuesday, September 25, 2001, 1:00 pm Room 201, Graduate Student Centre

"Multiresolution Green's Function Methods for Interactive Simulation of Large-scale Elastostatic Objects and Other Physical Systems in Equilibrium"

#### **EXAMINING COMMITTEE**

# Chair:

Matthew Choptuik (Physics and Astronomy)

# Supervisory Committee:

- Dr. Dinesh K. Pai, Research Supervisor (Computer Science)
- Dr. Anthony Peirce (Mathematics)
- Dr. Wolfgang Heidrich (Computer Science)

# University Examiners:

- Dr. Tim Salcudean (Electrical and Computer Engineering)
- Dr. Jim Varah (Computer Science)

#### External Examiner:

John Canny (Computer Science, UC Berkeley)