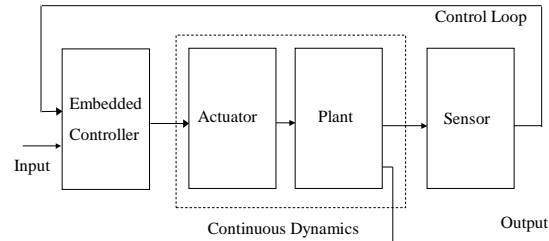# Model Reduction for Verification of Hybrid Systems

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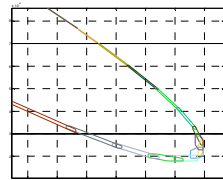zhih@andrew.cmu.edu

Objective

Verification of Embedded Control Systems

Verification Methods

- Computing reachable sets in continuous state space (difficult for system with order >7)
- Verifying safety properties using conservative approximation
- Using counterexamples to guide the refinement procedure

Flow-pipe* – The approximation of reachable states

*Using oriented rectangular hull approximation: B. H. Krogh and O. Stursberg, *On efficient representation and computation of reachable sets for hybrid systems*, in Hybrid Systems: Computation and Control (HSCC'03), Lecture Notes in Computer Science (LNCS), Springer

Model Reduction

- Approximating a high order component with a lower order one
- The error of approximation is **bounded**** for Linear Time Invariant Systems (LTI)
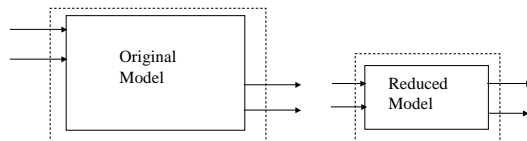
**Keith Glover and Jonathan R. Parrington, *Bounds on the Achievable Accuracy in Model Reduction*, Modelling, Robustness and Sensitivity Reduction in Control Systems,95-199, 1987, Springer
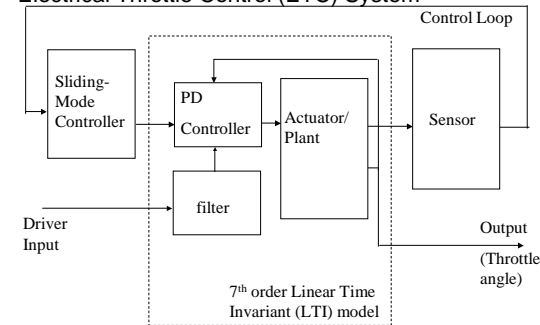
Applying Model Reduction in Verification

- Reachable sets are restricted to the states of interests (output states + states used in control loop)
- Computing reachable sets in reduced state space, then projecting to the states of interests
- Including the error introduced by model reduction in the results
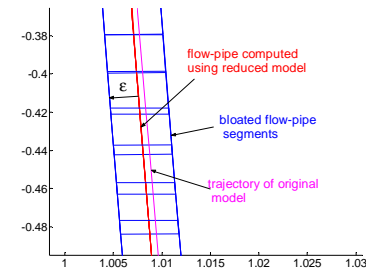
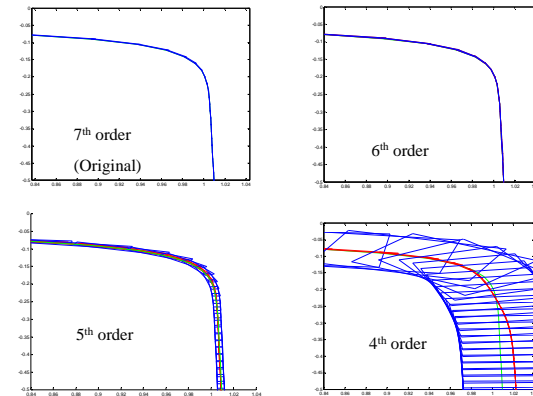## Case Study

Electrical Throttle Control (ETC) System

2 inputs: set-point, sliding mode signal

2 outputs: throttle angle, sliding surface

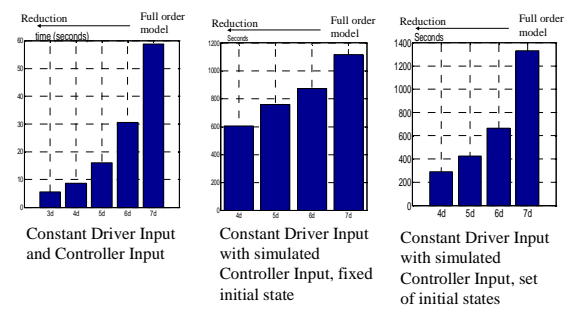| Order	7 (Original)	6	5	4
Error bounds	~	4.53×10^{-8}	3.18×10^{-3}	4.78×10^{-2}

Error Bounds for Different Reduced Models

Conservative Flow-pipes for different reduced models (Using balanced truncation*** method)

***B. C. Moore, *Principle Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction*, IEEE Transaction on Automatic Control, vol. AC-26, no. 1, Feb 1981, pp. 17-32

Computation time of flow-pipe segments for different reduced models

Constant Driver Input and Controller Input

Constant Driver Input with simulated Controller Input, fixed initial state

Constant Driver Input with simulated Controller Input, set of initial states

Future Work

- Implementation as subroutines in CheckMate/VTB
- Applying Model Reduction in Counterexample-Guided Verification scheme
- Composition of reduced models