Correlates of homicide: new space/time interaction tests for spatiotemporal point processes

Seth Flaxman
Carnegie Mellon University

Spatial Statistics Conference
June 7, 2013
Joint work with....

Daniel Neill\textsuperscript{1}, Wilpen Gorr\textsuperscript{1}, Alex Smola\textsuperscript{2}

1. H. J. Heinz III College, Carnegie Mellon University
2. School of Computer Science, Carnegie Mellon University

This work was partially supported by the National Science Foundation, grants IIS-0916345, IIS-0911032, and IIS-0953330.
Research question: what predicts homicides?

Background: space-time interaction tests

Methods: kernel-based measures of independence

Applications: 911 call data, crime offense reports from Chicago
Chicago

- Population: 2.7 million
- Area: 234 square miles
- **Research question**: Which types of calls to 911 are predictive of homicides and shootings nearby?
Space-time interaction

- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.
Space-time interaction

- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.
- If two events are close in space, they are likely to be close in time [Diggle, 1995].
Space-time interaction

- Residual space-time dependence, after controlling for purely spatial and purely temporal dependence.
- If two events are close in space, they are likely to be close in time [Diggle, 1995].

**Data:** point processes

\[ P_1 = \{(x_i^1, y_i^1, t_i^1), i \in 1, \ldots, n_1\}, \quad P_2 = \{(x_j^2, y_j^2, t_j^2), j \in 1, \ldots, n_2\} \]
**Knox test [1964]**

Put the $N = n_1 \cdot n_2$ pairs of points into a contingency table:

<table>
<thead>
<tr>
<th></th>
<th>close in space</th>
<th>far in space</th>
<th>$= N_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>close in time</td>
<td>$X$</td>
<td>$a$</td>
<td></td>
</tr>
<tr>
<td>far in time</td>
<td>$b$</td>
<td>$c$</td>
<td></td>
</tr>
<tr>
<td>$= N_s$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Statistical tests for space-time interaction

**Knox test [1964]**

Put the $N = n_1 \cdot n_2$ pairs of points into a contingency table:

<table>
<thead>
<tr>
<th></th>
<th>close in space</th>
<th>far in space</th>
<th>$= N_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>close in time</td>
<td>$X$</td>
<td>$a$</td>
<td>$= N_t$</td>
</tr>
<tr>
<td>far in time</td>
<td>$b$</td>
<td>$c$</td>
<td></td>
</tr>
<tr>
<td>$= N_s$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test statistic: $\frac{X}{N} - \frac{N_t}{N} \cdot \frac{N_s}{N}$
Statistical tests for space-time interaction

Knox test [1964]

Put the \( N = n_1 \cdot n_2 \) pairs of points into a contingency table:

<table>
<thead>
<tr>
<th></th>
<th>close in space</th>
<th>far in space</th>
<th>( = N_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>close in time</td>
<td>( X )</td>
<td>( a )</td>
<td></td>
</tr>
<tr>
<td>far in time</td>
<td>( b )</td>
<td>( c )</td>
<td></td>
</tr>
<tr>
<td>( = N_s )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test statistic: \( \frac{X}{N} - \frac{N_t}{N} \cdot \frac{N_s}{N} \)

(or just \( X \))
Mantel test [1967]

Put the $N$ pairs of points into two matrices:

$$K = \begin{pmatrix}
0 & \|s_1 - s_2\| & \cdots & \|s_1 - s_n\| \\
\|s_2 - s_1\| & 0 & \cdots & \|s_2 - s_n\| \\
\vdots & \ddots & \ddots & \vdots \\
\|s_n - s_1\| & \cdots & \|s_n - s_{n-1}\| & 0
\end{pmatrix}$$

$$L = \begin{pmatrix}
0 & |t_1 - t_2| & \cdots & |t_1 - t_n| \\
|t_2 - t_1| & 0 & \cdots & |t_2 - t_n| \\
\vdots & \ddots & \ddots & \vdots \\
|t_n - t_1| & \cdots & |t_n - t_{n-1}| & 0
\end{pmatrix}$$

Test statistic: $\sum_{i,j} K_{i,j} L_{i,j}$
Mantel test [1967]

Put the $N$ pairs of points into two matrices:

**space:** $K = \begin{pmatrix} 0 & \|s_1 - s_2\| & \cdots & \|s_1 - s_n\| \\ \|s_2 - s_1\| & 0 & \cdots & \|s_2 - s_n\| \\ \|s_3 - s_1\| & \|s_3 - s_2\| & \cdots & 0 \end{pmatrix}$
Mantel test [1967]

Put the $N$ pairs of points into two matrices:

space: $K = \begin{pmatrix}
0 & \|s_1 - s_2\| & \cdots & \|s_1 - s_n\| \\
\|s_2 - s_1\| & 0 & \cdots & \|s_2 - s_n\| \\
& \ddots & & \\
\|s_n - s_1\| & \|s_n - s_2\| & \cdots & 0
\end{pmatrix}$

time: $L = \begin{pmatrix}
0 & |t_1 - t_2| & \cdots & |t_1 - t_n| \\
|t_2 - t_1| & 0 & \cdots & |t_2 - t_n| \\
& \ddots & & \\
|t_n - t_1| & |t_n - t_2| & \cdots & 0
\end{pmatrix}$
Mantel test [1967]

Put the $N$ pairs of points into two matrices:

**space:** $K = \begin{pmatrix}
0 & \|s_1 - s_2\| & \ldots & \|s_1 - s_n\| \\
\|s_2 - s_1\| & 0 & \ldots & \|s_2 - s_n\| \\
\|s_3 - s_1\| & \|s_3 - s_2\| & \ldots & 0 \\
\|s_n - s_1\| & \|s_n - s_2\| & \ldots & 0
\end{pmatrix}$

**time:** $L = \begin{pmatrix}
0 & |t_1 - t_2| & \ldots & |t_1 - t_n| \\
|t_2 - t_1| & 0 & \ldots & |t_2 - t_n| \\
|t_3 - t_1| & |t_3 - t_2| & \ldots & 0 \\
|t_n - t_1| & |t_n - t_2| & \ldots & 0
\end{pmatrix}$

Test statistic: $\sum_{i,j} K_{i,j} L_{i,j}$
Shortcomings

- Knox: discretizes using pre-specified cutoffs
- Mantel: linear measure of independence (correlation)
- Focus is exclusively on interpoint (Euclidean) distances
- No way to include covariates, more spatial or temporal structure
A kernel is a real-valued paired similarity function: \( k(x, y) \in \mathcal{R} \). Larger values \( \Rightarrow \) more similar.

You’ve heard of kernels: they’re a generalization of covariance functions \( C(x, y) \! \).

Example: Gaussian \( k(x, y) = e^{-\|x-y\|^2} \)

Mathematical theory: kernels turn points into infinite dimensional vectors, i.e. functions:
The “kernel trick”

- Take a standard statistical tool (e.g. clustering, PCA, SVMs), replace dot products or similarity metrics $\langle x, y \rangle$ with kernels $k(x, y)$ throughout.
- Enables the application of simple, linear methods in non-linear settings
Mantel test [1967]

Put the \( N \) pairs of points into two matrices:

**space:** \[ K = \begin{pmatrix} 
0 & \|s_1 - s_2\| & \cdots & \|s_1 - s_n\| \\
\|s_2 - s_1\| & 0 & \cdots & \|s_2 - s_n\| \\
\|s_3 - s_1\| & \|s_3 - s_2\| & \cdots & 0 \\
\|s_n - s_1\| & \|s_n - s_2\| & \cdots & 0 \\
\end{pmatrix} \]

**time:** \[ L = \begin{pmatrix} 
0 & |t_1 - t_2| & \cdots & |t_1 - t_n| \\
|t_2 - t_1| & 0 & \cdots & |t_2 - t_n| \\
|t_3 - t_1| & |t_3 - t_2| & \cdots & 0 \\
|t_n - t_1| & |t_n - t_2| & \cdots & 0 \\
\end{pmatrix} \]

Test statistic: \( \sum_{i,j} K_{i,j} L_{i,j} \)
Mantel test: transformed

\[ k(s) := \frac{1}{s + \epsilon_1}, \ell(t) := \frac{1}{t + \epsilon_2} \]
Mantel test: transformed

\[ k(s) := \frac{1}{s + \epsilon_1}, \quad \ell(t) := \frac{1}{t + \epsilon_2} \]

\[ K = \begin{pmatrix}
0 & k(\|s_1 - s_2\|) & \ldots & k(\|s_1 - s_n\|) \\
k(\|s_2 - s_1\|) & 0 & \ldots & k(\|s_2 - s_n\|) \\
\vdots & & \ddots & \vdots \\
k(\|s_n - s_1\|) & k(\|s_n - s_2\|) & \ldots & 0
\end{pmatrix} \]

\[ L = \begin{pmatrix}
0 & \ell(|t_1 - t_2|) & \ldots & \ell(|t_1 - t_n|) \\
\ell(|t_2 - t_1|) & 0 & \ldots & \ell(|t_2 - t_n|) \\
\vdots & & \ddots & \vdots \\
\ell(|t_n - t_1|) & \ell(|t_n - t_2|) & \ldots & 0
\end{pmatrix} \]
Mantel test: transformed

\[ k(s) := \frac{1}{s + \epsilon_1}, \quad \ell(t) := \frac{1}{t + \epsilon_2} \]

\[
K = \begin{pmatrix}
0 & k(\|s_1 - s_2\|) & \cdots & k(\|s_1 - s_n\|) \\
k(\|s_2 - s_1\|) & 0 & \cdots & k(\|s_2 - s_n\|) \\
\vdots & \vdots & \ddots & \vdots \\
k(\|s_n - s_1\|) & k(\|s_n - s_2\|) & \cdots & 0
\end{pmatrix}
\]

\[
L = \begin{pmatrix}
0 & \ell(|t_1 - t_2|) & \cdots & \ell(|t_1 - t_n|) \\
\ell(|t_2 - t_1|) & 0 & \cdots & \ell(|t_2 - t_n|) \\
\vdots & \vdots & \ddots & \vdots \\
\ell(|t_n - t_1|) & \ell(|t_n - t_2|) & \cdots & 0
\end{pmatrix}
\]

Test statistic: \( \sum_{i,j} K_{i,j}L_{i,j} \)
“Kernelized” Mantel test

Choose kernels $k$ and $\ell$:

$$K = \begin{pmatrix}
0 & k(s_1, s_2) & \cdots & k(s_1, s_n) \\
k(s_2, s_1) & 0 & \cdots & k(s_2, s_n) \\
\vdots & & \ddots & \vdots \\
k(s_n, s_1) & k(s_n, s_2) & \cdots & 0
\end{pmatrix}$$

$$L = \begin{pmatrix}
0 & \ell(t_1, t_2) & \cdots & \ell(t_1, t_n) \\
\ell(t_2, t_1) & 0 & \cdots & \ell(t_2, t_n) \\
\vdots & & \ddots & \vdots \\
\ell(t_n, t_1) & \ell(t_n, t_2) & \cdots & 0
\end{pmatrix}$$

Test statistic:

$$\sum_{i, j} k(s_i, s_j) \ell(t_i, t_j)$$
“Kernelized” Mantel test

Choose kernels $k$ and $\ell$:

$$K = \begin{pmatrix}
    0 & k(s_1, s_2) & \ldots & k(s_1, s_n) \\
    k(s_2, s_1) & 0 & \ldots & k(s_2, s_n) \\
    \vdots & \vdots & \ddots & \vdots \\
    k(s_n, s_1) & k(s_n, s_2) & \ldots & 0
\end{pmatrix}$$

$$L = \begin{pmatrix}
    0 & \ell(t_1, t_2) & \ldots & \ell(t_1, t_n) \\
    \ell(t_2, t_1) & 0 & \ldots & \ell(t_2, t_n) \\
    \vdots & \vdots & \ddots & \vdots \\
    \ell(t_n, t_1) & \ell(t_n, t_2) & \ldots & 0
\end{pmatrix}$$

Test statistic: $\sum_{i,j} k(s_i, s_j) \ell(t_i, t_j)$
Given points $\mathcal{P} = \{ p_i = (s_i, t_i) \}$ we have two ways of measuring similarity:

\[
k(p_i, p_j) := k(s_i, s_j) \quad \text{(similarity in space)}
\]

\[
\ell(p_i, p_j) := \ell(t_i, t_j) \quad \text{(similarity in time)}
\]
Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:
  \[
  k(p_i, p_j) := k(s_i, s_j) \quad \text{(similarity in space)}
  \]
  \[
  \ell(p_i, p_j) := \ell(t_i, t_j) \quad \text{(similarity in time)}
  \]

- Are these two notions of similarity independent?
Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:
  
  \[ k(p_i, p_j) := k(s_i, s_j) \]  
  (similarity in space)

  \[ \ell(p_i, p_j) := \ell(t_i, t_j) \]  
  (similarity in time)

- Are these two notions of similarity independent?

- Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?
Theory

- Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:
  $$k(p_i, p_j) := k(s_i, s_j) \text{ (similarity in space)}$$
  $$\ell(p_i, p_j) := \ell(t_i, t_j) \text{ (similarity in time)}$$

- Are these two notions of similarity independent?
- Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?
- Mantel test only picks up linear dependence (measured as correlation):
  $$\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$$
Given points $\mathcal{P} = \{p_i = (s_i, t_i)\}$ we have two ways of measuring similarity:

$$k(p_i, p_j) := k(s_i, s_j) \ (\text{similarity in space})$$

$$\ell(p_i, p_j) := \ell(t_i, t_j) \ (\text{similarity in time})$$

Are these two notions of similarity independent?

Equivalently: given a random point $p \sim \mathcal{P}$, are the Hilbert space representations $k(p, \cdot)$ and $\ell(p, \cdot)$ independent?

Mantel test only picks up linear dependence (measured as correlation):

$$\sum_{i,j} k(s_i, s_j)\ell(t_i, t_j)$$

Statistical machine learning says there’s a better way...
Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
- Given observations \((s_i, t_i) \sim S \times T\), is \(P(S, T) = P(S)P(T)\)?
Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)

- Given observations \((s_i, t_i) \sim S \times T\), is \(P(S, T) = P(S)P(T)\)?

- Looks for a function \(f\):

\[
\text{HSIC} = \sup_{f} \left( \mathbb{E}_{(s, t) \sim S \times T} f(s, t) - \mathbb{E}_{s \sim S, t \sim T} f(s, t) \right)^2
\]
Kernel measures of independence

- Hilbert-Schmidt Independence Criterion (Gretton et al. 2012)
- Given observations \((s_i, t_i) \sim S \times T\), is \(P(S, T) = P(S)P(T)\)?
- Looks for a function \(f\):

\[
\text{HSIC} = \sup_f \left( \mathbb{E}_{(s, t) \sim S \times T} f(s, t) - \mathbb{E}_{s \sim S, t \sim T} f(s, t) \right)^2
\]

- Intuition: \(f\) is a “witness” function, meant to find discrepancies between \(P(S, T)\) and \(P(S)P(T)\).
Kernel measures of independence

- For $f$ sufficiently complex, can do much more than distinguish linear dependence:

**Theorem.** $\text{HSIC} = 0$ if and only if $P(S, T) = P(S)P(T)$

- We show that these results hold for testing whether $k(p, \cdot)$ and $\ell(p, \cdot)$ are independent ⇒ new test for space-time interaction
Kernel measures of independence

- For $f$ sufficiently complex, can do much more than distinguish linear dependence:

  **Theorem.** \( \text{HSIC} = 0 \) if and only if \( P(S, T) = P(S)P(T) \)

- For $f$ in a Hilbert space and bounded, $f^*$ can be found in closed form:
Kernel measures of independence

- For $f$ sufficiently complex, can do much more than distinguish linear dependence:

  **Theorem.** $\text{HSIC} = 0$ if and only if $P(S, T) = P(S)P(T)$

- For $f$ in a Hilbert space and bounded, $f^*$ can be found in closed form:

\[
\text{HSIC} = \frac{1}{n^2} \sum_{i,j} k(s_i, s_j)\ell(t_i, t_j) - \frac{2}{n^3} \sum_{i,j,r} k(s_i, s_j)\ell(t_i, t_r) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i, s_j)\ell(t_q, t_r)
\]
Kernel measures of independence

- For \( f \) sufficiently complex, can do much more than distinguish linear dependence:

  **Theorem.** \( \text{HSIC} = 0 \) if and only if \( P(S, T) = P(S)P(T) \)

- For \( f \) in a Hilbert space and bounded, \( f^* \) can be found in closed form:

  \[
  \text{HSIC} = \frac{1}{n^2} \sum_{i,j} k(s_i, s_j)\ell(t_i, t_j) - \frac{2}{n^3} \sum_{i,j,r} k(s_i, s_j)\ell(t_i, t_r) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i, s_j)\ell(t_q, t_r)
  \]

- We show that these results hold for testing whether \( k(p, \cdot) \) and \( \ell(p, \cdot) \) are independent \( \Rightarrow \) new test for space-time interaction
Space-time interaction statistics

HSIC:

\[
\frac{1}{n^2} \sum_{i,j} k(s_i, s_j) \ell(t_i, t_j) - \frac{2}{n^3} \sum_{i,j,r} k(s_i, s_j) \ell(t_i, t_r) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i, s_j) \ell(t_q, t_r)
\]

Kernelized Mantel:

\[
\sum_{i,j} k(s_i, s_j) \ell(t_i, t_j)
\]

Notice: missing terms! Mantel is like HSIC, but with some non-optimal choice of \( f \) \((\Rightarrow \text{less power})\).
Our Contributions

- New way of thinking about space-time interaction in terms of kernels ⇒ new interpretation of HSIC ⇒ new test for space-time interaction
- Extensions to bivariate, forward in time cases
- Interesting connections with Mantel test, showing its shortcomings and a possible fix
- More flexible test: kernels can encode more than just distance between points. HSIC can test for non-linear dependencies.
Experimental Setup

Synthetic data: draw $n = 40$ or $n = 100$ random cluster centers, draw $k = 5$ or 1 children with locations displaced $N(0, \sigma)$ from parent in every direction.

Easy example: $\sigma = .025$
Experimental Setup

Synthetic data: draw $n = 40$ or $n = 100$ random cluster centers, draw $k = 5$ or 1 children with locations displaced $N(0, \sigma)$ from parent in every direction.

Hard example: $\sigma = .2$
Synthetic Data: Results

mean cluster size 1

mean cluster size 5

Diggle's test: 5 x 5
10 x 10
15 x 15
20 x 20
25 x 25
HSIC
Question: which types of calls to 911 predict homicides and aggravated battery with a handgun ("shootings")?
Experimental Setup: Crime Data

Question: which types of calls to 911 predict homicides and aggravated battery with a handgun ("shootings")?

Data:

- Dispatcher calls from January 2007-May 2010, coded by one of 271 types (≈ 9 million):
  - "01-01-2010", "12:25:00", "ARSON", 1172456, 1834562
  - "01-02-2010", "19:55:00", "THEFT", 1173123, 1831123
- All shootings / homicides from January 2007-May 2010 (9,087 total):
  - "01-01-2010", "19:00:37", "HOMICIDE", 1172001, 1834023
  - "01-07-2010", "19:55:00", "HOMICIDE", 1173934, 1831384
Experimental Setup: Algorithms

- Calculate p-values for directional, bivariate space-time interaction between each 911 call type and shootings
- Compare Knox and HSIC
- Knox: cutoff for close: 500 feet, 14 days
- HSIC: Gaussian RBF kernels, with equivalent bandwidth
- Permutation testing 1000 times to calculate p-values
## Results

<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>
### Results

<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>

**Expert rating: 5 (most reasonable)**
## Results

<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>

**Expert rating: 5 (most reasonable), 4**
## Results

<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>

**Expert rating:** 5 (most reasonable), 4, 3
## Results

<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>

Expert rating: 5 (most reasonable), 4, 3, 2
<table>
<thead>
<tr>
<th>HSIC &lt; .01</th>
<th>HSIC and Knox &lt; .01</th>
<th>Knox &lt; .01</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto accident pd</td>
<td>10-1</td>
<td>arson report</td>
</tr>
<tr>
<td>battery jo</td>
<td>death removal</td>
<td>auto theft ip</td>
</tr>
<tr>
<td>battery victim inj.</td>
<td>evidence technician (pri. 1)</td>
<td>criminal tres. (ov)</td>
</tr>
<tr>
<td>beat team meeting [ov]</td>
<td>evidence technician (pri. 2)</td>
<td>evidence technician (pri. 3)</td>
</tr>
<tr>
<td>crim dam. to prop rpt</td>
<td>gambling</td>
<td>found property</td>
</tr>
<tr>
<td>mental unauth absence</td>
<td>gang disturbance</td>
<td>k9 request</td>
</tr>
<tr>
<td>mission</td>
<td>outdoor roll call</td>
<td>kidnapping report</td>
</tr>
<tr>
<td>person with a gun</td>
<td>person shot</td>
<td>notify</td>
</tr>
<tr>
<td>robbery victim injured</td>
<td>plan 1-5</td>
<td>person stabbed</td>
</tr>
<tr>
<td>theft ip</td>
<td>shots fired</td>
<td>pick up car</td>
</tr>
<tr>
<td></td>
<td>shots fired (ov)</td>
<td>polling place check</td>
</tr>
<tr>
<td></td>
<td></td>
<td>suspicious person (ov)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>transport</td>
</tr>
</tbody>
</table>

**Expert rating:** 5 (most reasonable), 4, 3, 2, 1 (least reasonable)
Conclusions

- New data-driven formulation of “leading indicators” question as space-time interaction between pairs of point processes
- Defined a new kernel-based space-time interaction test
- HSIC performance was comparable to classical tests, parameter choices less critical
- Applied to large, real, and important dataset: shootings in Chicago
Thank you! Questions?¹

Seth Flaxman - flaxman@cmu.edu

¹Thank you to the Chicago Police Department for sharing data. Points of view or opinions contained within this presentation are those of the author and do not necessarily represent the official position or policies of the Chicago Police Department. Title page photo by Palsson on Flickr.
Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:
Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:

\[
\frac{1}{n^2} \sum_{i,j} k(s_i, s_j^2) \ell(t_i^1, t_j^2) - \frac{2}{n^3} \sum_{i,j,r} k(s_i, s_j^2) \ell(t_i^1, t_r^2) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i, s_j^2) \ell(t_q^1, t_r^2)
\]

- Interesting interpretation in Hilbert space
Extensions

- Bivariate case: for test statistic, restrict sums to pairs of points of different types:

\[
\frac{1}{n^2} \sum_{i,j} k(s_i^1, s_j^2) \ell(t_i^1, t_j^2) - \frac{2}{n^3} \sum_{i,j,r} k(s_i^1, s_j^2) \ell(t_i^1, t_r^2) + \frac{1}{n^4} \sum_{i,j,q,r} k(s_i^1, s_j^2) \ell(t_q^1, t_r^2)
\]

- Interesting interpretation in Hilbert space

- Only predict forward in time: restrict sums to pairs of points where \( t_i < t_j \).
Excess risk attributable to space-time interaction

\[ D(s, t) = \frac{F_{S,T}(s, t) - F_S(s)F_T(t)}{F_S(s)F_T(t)} \]

Given that we see an event of type 1, proportional increase (excess risk) of seeing an event of type 2.
Shots fired and shootings

HSIC: 0.001, Knox: 0.001.

Carnegie Mellon University
K9 REQUEST and shootings

HSIC: 0.69, Knox: 0.002.
PERSON WITH A GUN and shootings

HSIC: 0.001, Knox: 0.045.

Carnegie Mellon University
"Witness" $\hat{f}^*$:

$$\hat{f}^*(x, y) \propto \sum_{i} k(x, x_i) - \sum_{j} k(y, y_j)$$
False positive rate

HSIC: 4.07% false positive

Frequency

0 20 40 60 80 100 120

0.0 0.2 0.4 0.6 0.8 1.0

p-value

32 Carnegie Mellon University
False positive rate

Mantel (kernelized): 4.37% false positive

Frequency

p-value

0.0 0.2 0.4 0.6 0.8 1.0

0 20 40 60 80 100
False positive rate

Knox: 5.69% false positive

Frequency

0.0 0.2 0.4 0.6 0.8 1.0
0 20 40 60 80 100

p-value
HSIC: 0.001, Knox: 0.595.