15-414-F06:

Midterm

Name:
Andrew ID:

Tuesday 17, 2006

Instructions

- Fill in the box above with your name, your Andrew ID, and your section. **Do it, now!**
- Clearly mark your answers in the allocated space. If need be, use the back of a page for scratch space. If you have made a mess, cross out the invalid parts of your solution, and circle the ones that should be graded.
- Scan the test first to make sure that none of the 10 pages are missing. The problems are of varying difficulty, you might wish to pick off the easy ones first.
- You have 75 minutes. Good luck.
- If you wish you can submit the solution to problem 3 on Thursday in class. You can collect a description of problem 3 before leaving the class. So first try to solve problems 1,2,4,5.

1	20	
2	20	
3	20	
4	20	
5	20	
Σ	100	

Problem 1: Propositional logic (20 pts)

(a) Find a disjunctive normal form for the well formed formula (wff) $(a \wedge b) \Leftrightarrow c$.

(b) Find a wff equivalent to $p \rightarrow q$ in which only propositional connective is the NAND ($\bullet$) operator ($a \bullet b$ means $\neg(a \wedge b)$).

(c) Is the following statement correct: *The preferred technique for converting Boolean circuits or formulas (that are not in CNF) to CNF is by adding new variables*? Give a short justification for your answer.

In the following three parts report whether the given formula is a tautology. Otherwise, give a falsifying assignment.

(d) $((p \rightarrow q) \rightarrow \neg p) \rightarrow \neg q$

(e) $((p \rightarrow q) \rightarrow p) \rightarrow p$

(f) $r \vee p \vee \neg((p \Leftrightarrow q) \wedge (q \Leftrightarrow \neg r))$

Problem 2: SAT related trivia (20 pts)

(a) List the name of three fast SAT solvers (CNF based)?

(b) What is the name of algorithm used in the above SAT solvers?

(c) List two ideas that are common in fast SAT solvers? Explain these ideas in one or two sentences.

(d) Name the SAT solver that introduced watched literal scheme?

(e) Where does a fast SAT solver spend most of its time?

Problem 3: Encoding N queens puzzle as a SAT formula (20 pts)

In chess, a queen can move as far as she pleases, horizontally, vertically, or diagonally. A chess board has 8 rows and 8 columns. The standard 8 by 8 Queen's problem asks how to place 8 queens on an ordinary chess board so that none of them can hit any other in one move. An obvious modification of the 8 by 8 problem is to consider an N by N chess board and ask if one can place N queens on such a board.

(a) Is there a solution to N queens puzzle for $N = 3$? If yes, draw a solution.

(b) Is there a solution to N queens puzzle for $N = 4$? If yes, draw a solution.

(c) In the following parts we will encode this problem as a CNF formula. We denote a cell in row i and column j of chess board as (i, j). Let x_{ij} be a propositional variable that is true if and only if cell (i, j) has a queen where $1 \leq i \leq N, 1 \leq j \leq N$.

Express the constraint that row i has atmost one queen using a set of clauses.

(d) What is the condition for two distinct cells (i, j) and (k, l) to lie diagonally in terms of i, j, k, l? We will refer to this condition as $onDiagonal((i, j), (k, l))$.

(e) Express the constraint that no two queens lie diagonally using a set of clauses. You may use the $onDiagonal((i, j), (k, l))$ predicate when writing the answer.

Problem 4: Binary Decision Diagrams (20 pts)

(a) Given a Boolean formula $x.(y+\bar{z})$, compute its reduced OBDD for the following orderings.
$x < y < z$

(b) $y < x < z$

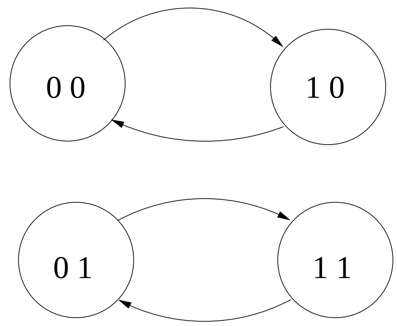

Figure 1: A small hardware design with four states

Consider a hardware design shown in Figure. 1. Each state is made of two bits v_1, v_2. The transitions between various states is shown in the figure.

(c) Suppose states 00 and 01 are the initial states. Represent the initial set of states as a Boolean formula using v_1, v_2.

(d) Write the transition relation for the design as a Boolean formula using v_1, v_2 for the current state and v_1', v_2' for the next state.

(e) We refer to the above initial set of states as $I(v_1, v_2)$ and transition relation as $T(v_1, v_2, v_1', v_2')$. What set of states are represented by the following formula: $\exists v_1, v_2[I(v_1, v_2) \wedge T(v_1, v_2, v_1', v_2')]$.

Problem 5: Quantified Boolean Formulas (20 pts)

(a) In the following two parts report whether the given Quantified Boolean formula is true or false.
$\exists x.\forall y.((x \wedge y) \vee (\neg x \wedge \neg y))$

(b) $\forall x.\exists y.((x \wedge y) \vee (\neg x \wedge \neg y))$

(c) Let γ be a valuation $(x, y, z) := (0, 1, 1)$. Report whether $\gamma \models f$ holds for the following parts.

f is $\exists x.(y.(x + z + \bar{y}) + x.\bar{y})$

(d) f is $\exists z.(x.\bar{z} + \forall x.((y + ((x + \bar{x}).z))))$

(e) f is $\exists x.\exists y.\forall z.(\bar{x} \Leftrightarrow (z \wedge y))$