An Example of the GRASP SAT Solver

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with help from Himanshu Jain

15-414: Bug Catching
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Carnegie Mellon University
2007

GRASP

- GRASP is SAT solver
- Created by

  Joan P. Marques Silva and

  Karem A. Sakallah
- See “GRASP – A New Search Algorithm for Satisfiability” by Silva and Sakallah for details
- This just presents an example of the algorithm in action

Simplifying Assumption

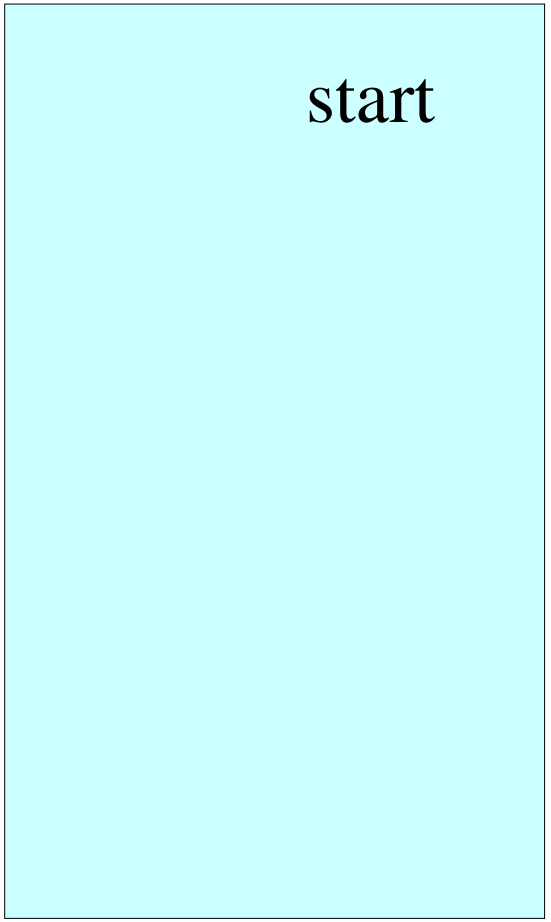
- I will act as though GRASP selects the next variable to assign a value to in numerical order and always tries false first
- In reality, GRASP selects the assignment that would satisfy as many clauses as possible
- I could add clauses to my example to avoid this simplification, but I like small examples

The Clauses

- $w1 = x1 + x2 + x5 + x4$
- $w2 = x1 + x2 + \neg x5 + x4$
- $w3 = \neg x3 + x6$
- $w4 = \neg x4 + x7 + x1$
- $w5 = \neg x4 + \neg x7 + x2$

Starting State

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



Assign False to x1

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
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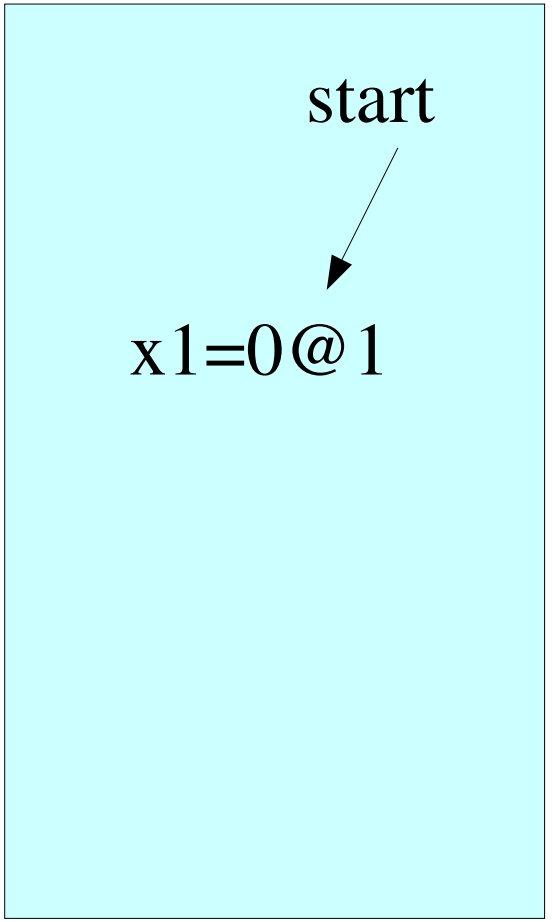

Assign False to x1

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
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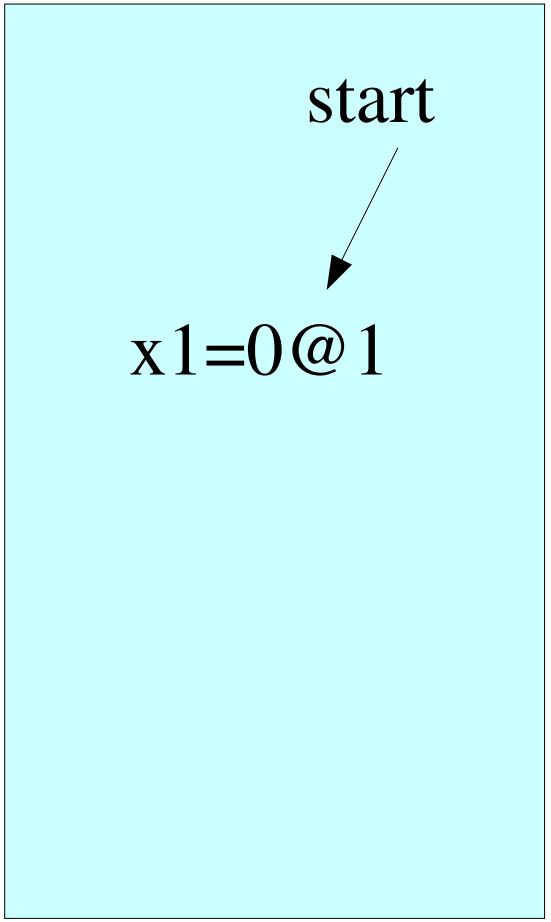
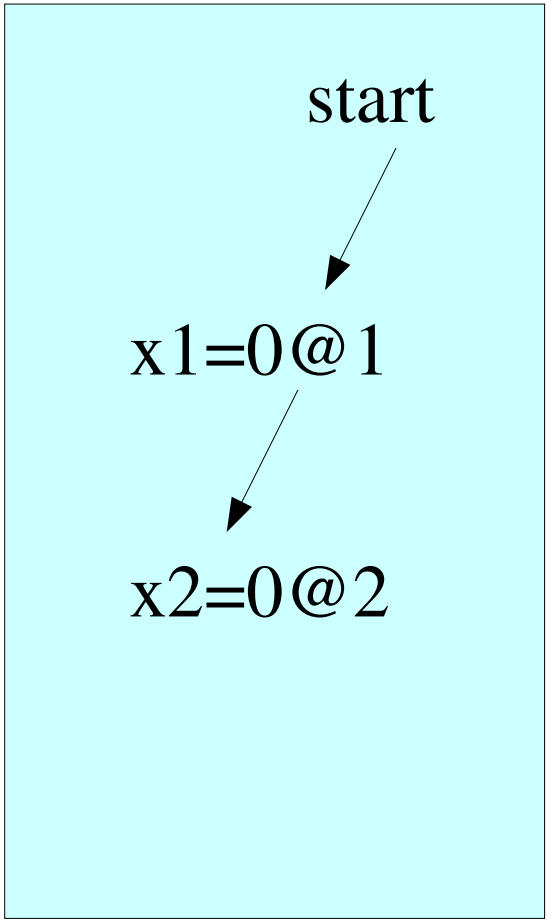


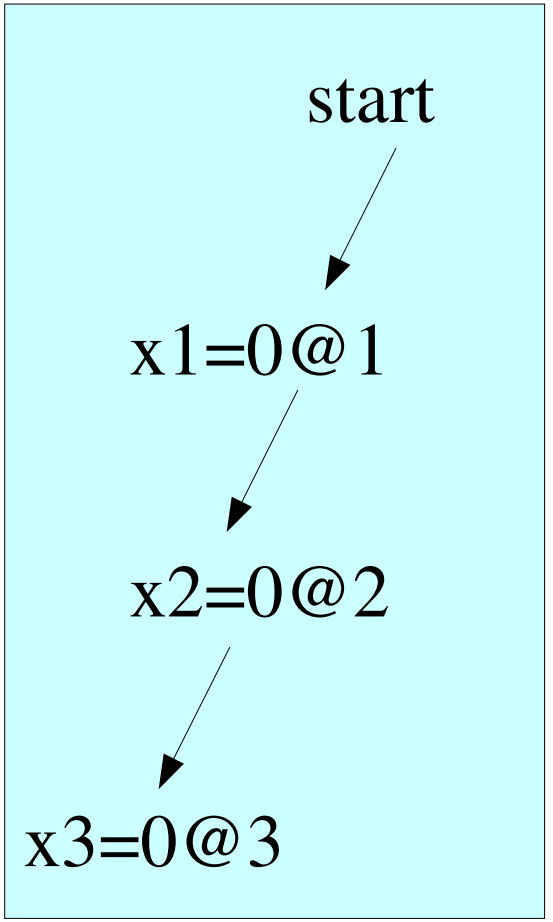
Assign False to x2

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



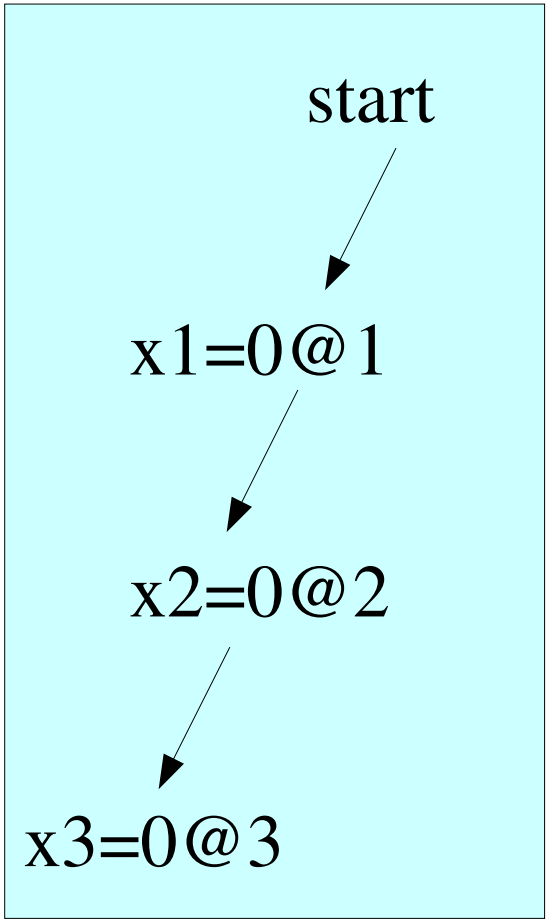
Assign False to x3

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



Assign False to x3

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



We have a unit clause! x6 must be set to true.

Assign False to x3

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
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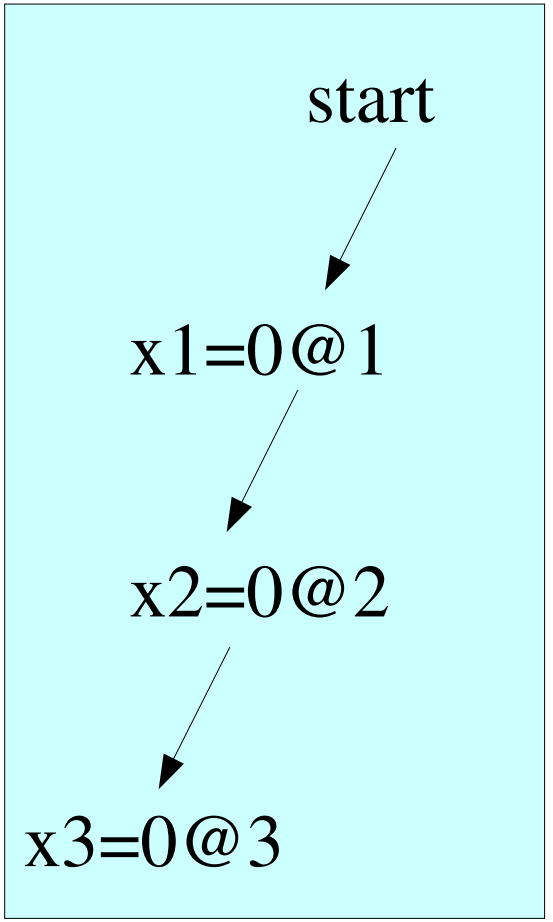


The x6 is set to true because x3 was set to false.

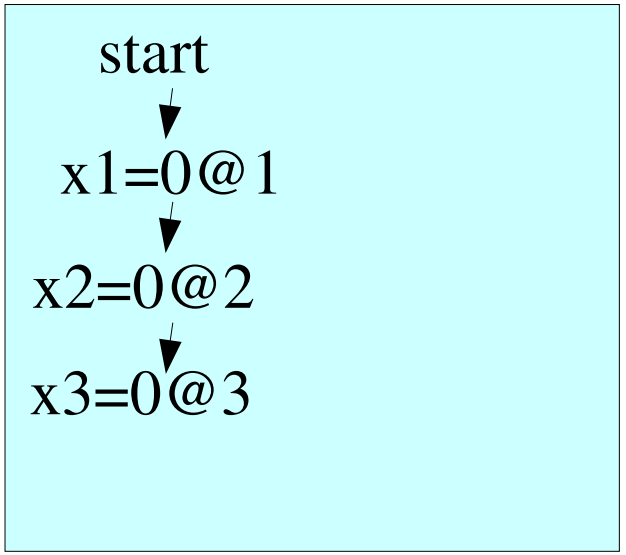
This might be useful info. Let's remember it.

Implication Graph

- An **implication graph** keeps track of why variables are assigned the value that they are
- Each assigned variable has a node
- Variables that force another variable to have an certain assignment point to it

Implication Graph

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



Let's make an implication graph.

Implication Graph

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
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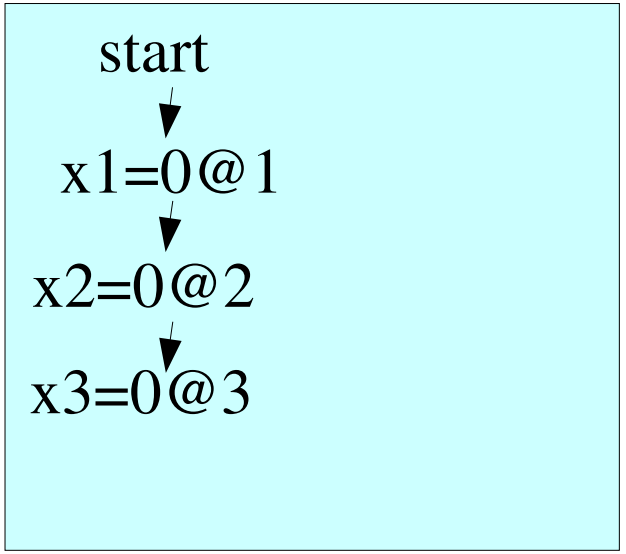


x1=0@1

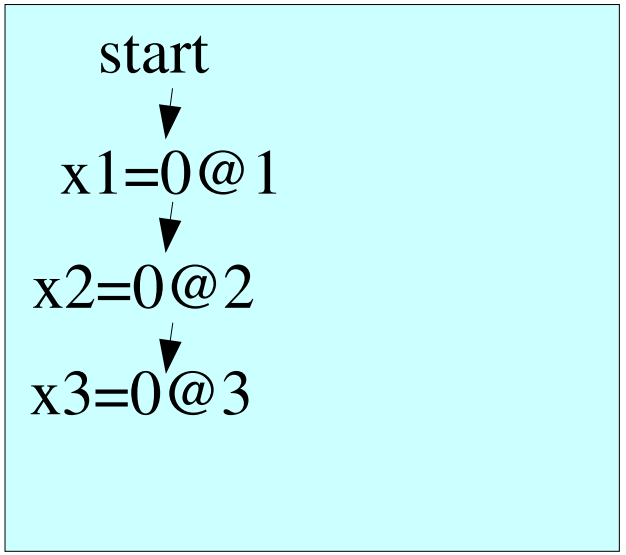
x2=0@2

x3=0@3 x6=1@3

Each assignment gets a node.

Implication Graph

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



x1=0@1

x2=0@2

x3=0@3 x6=1@3

Edges connect a node to why it was assigned the way it was

Implication Graph

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4

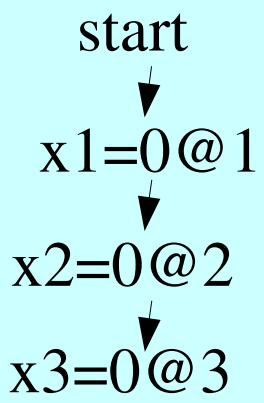
x1=0@1

x2=0@2

x3=0@3 $\xrightarrow{w3}$ x6=1@3

Because x3 = 0, x6 must be 1.

We know this from w3



Edges connect a node to why it was assigned the way it was

Implication Graph

- w1 = x1 + x2 + x5 + x4
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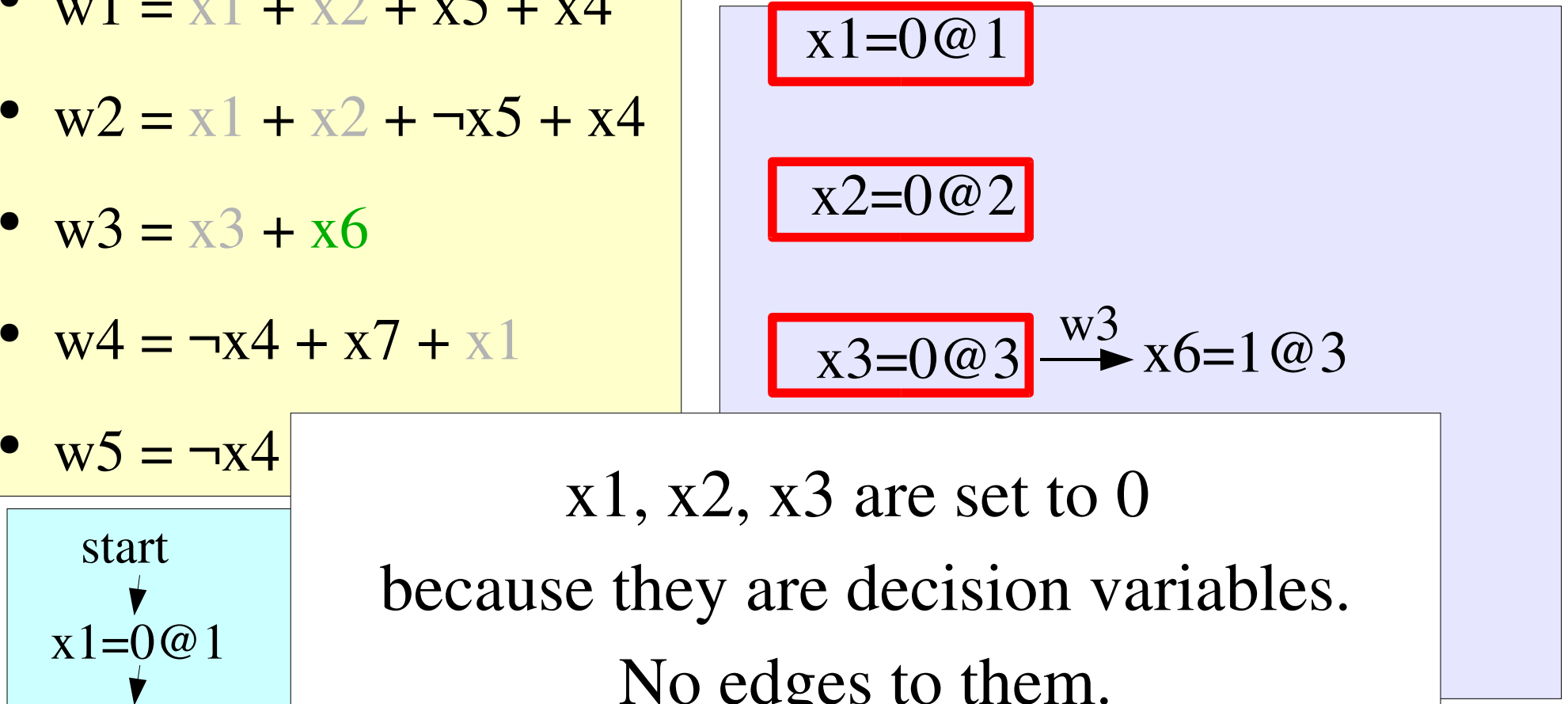
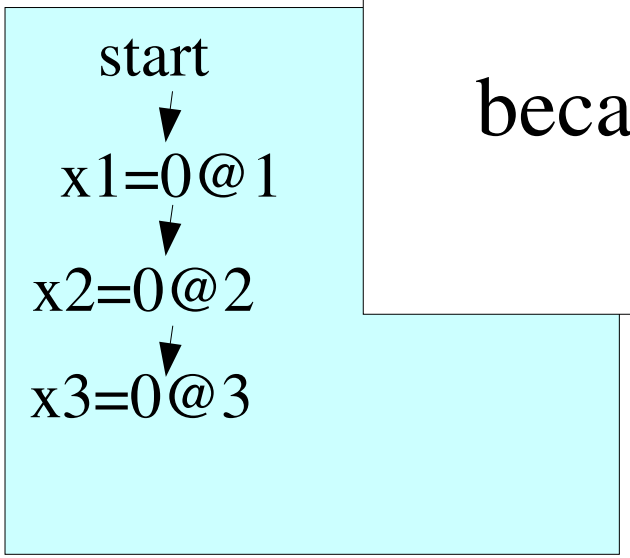
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4





x1, x2, x3 are set to 0
because they are decision variables.
No edges to them.

Edges connect a node to why it was assigned the way it was

Implication Graph

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
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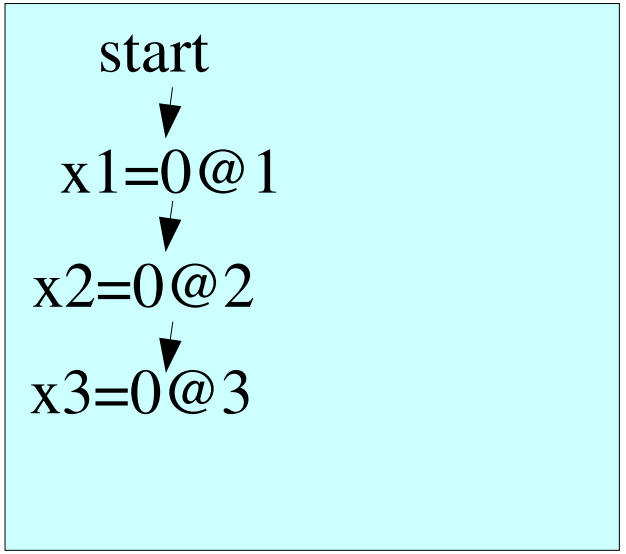
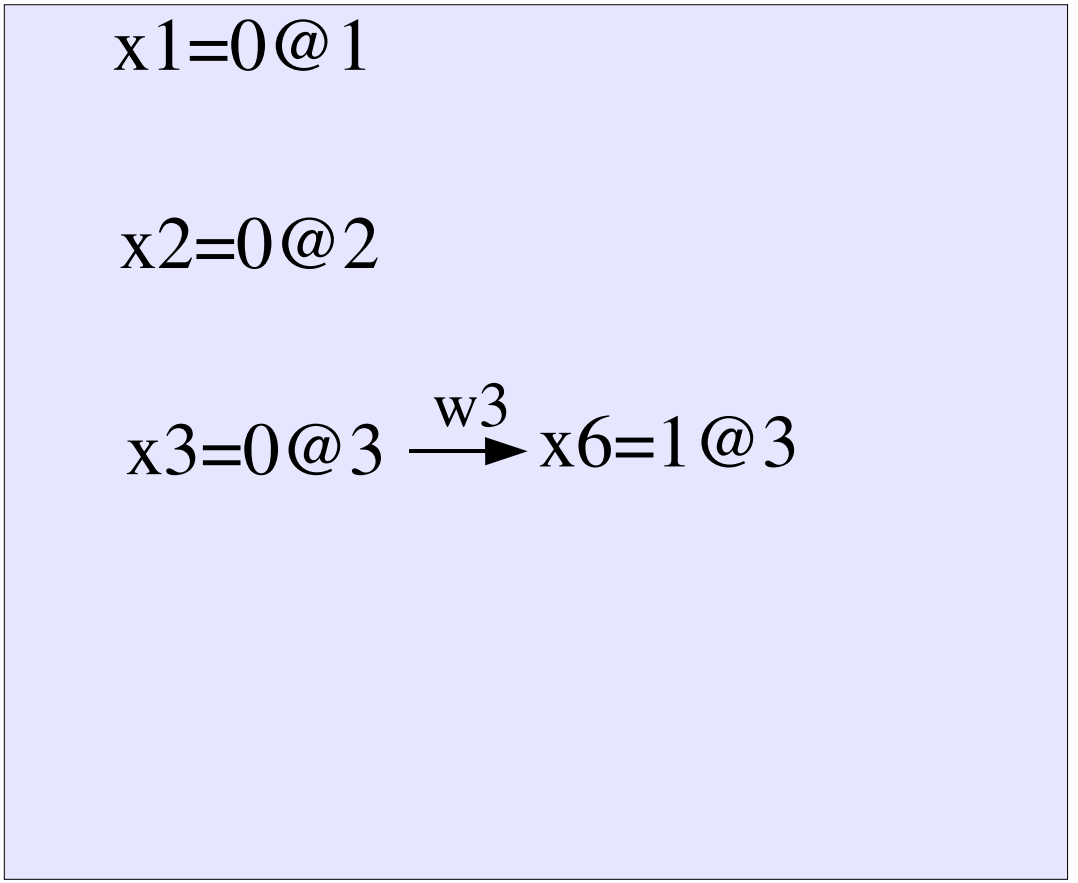
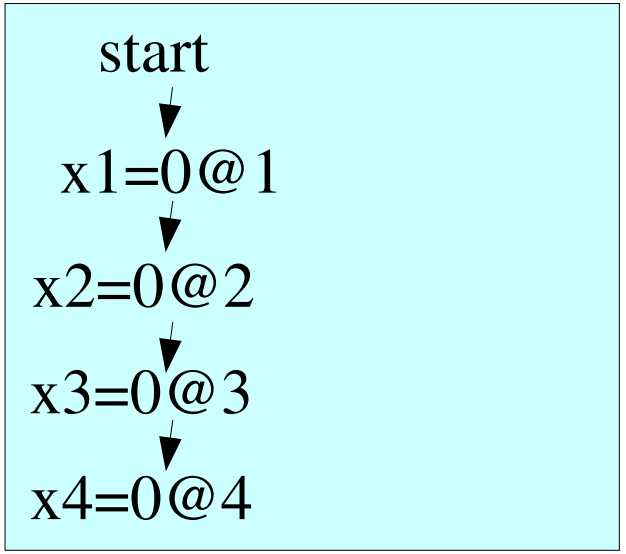




The finished graph

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



x1=0@1

x2=0@2

x3=0@3 $\xrightarrow{w3}$ x6=1@3

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
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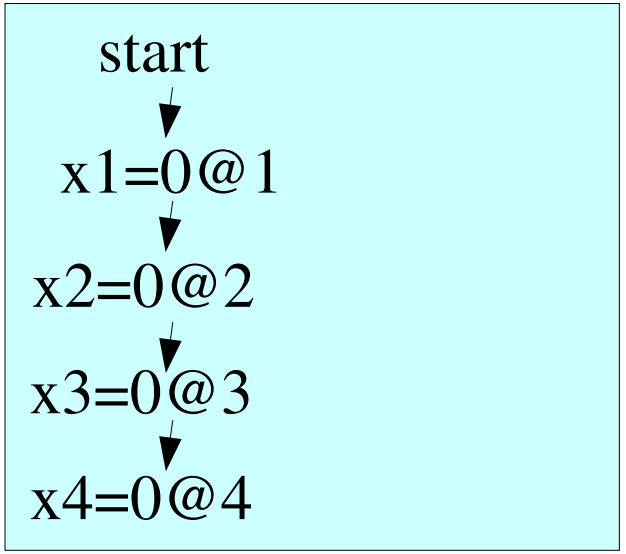


x1=0@1

x2=0@2

x4=0@4

x3=0@3 —w3→ x6=1@3

Add new node to graph

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2

start

x1=0@1

x2=0@2

x3=0@3

x4=0@4

x1=0@1

x2=0@2

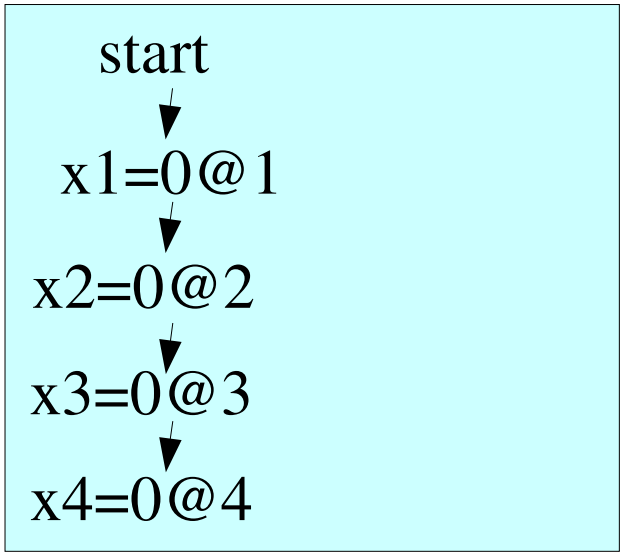
x4=0@4

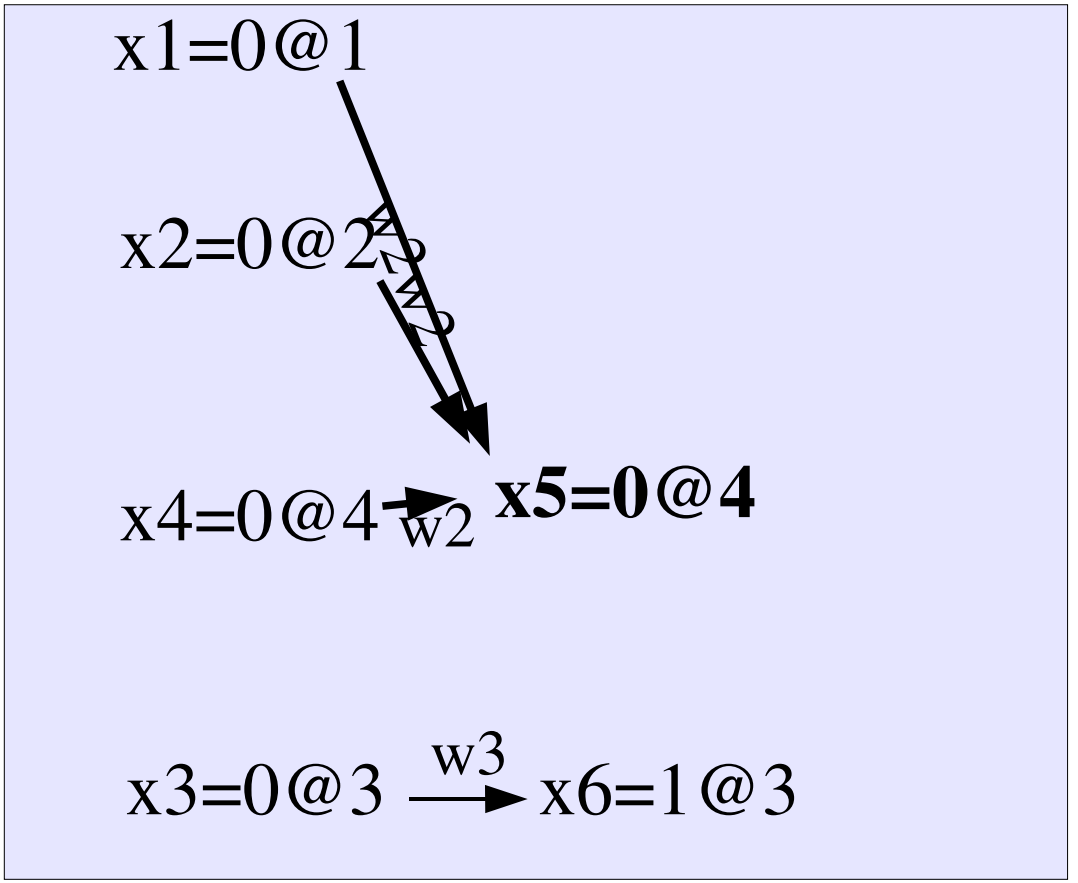
x3=0@3 —w3→ x6=1@3

Forces x5 to be false.

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2

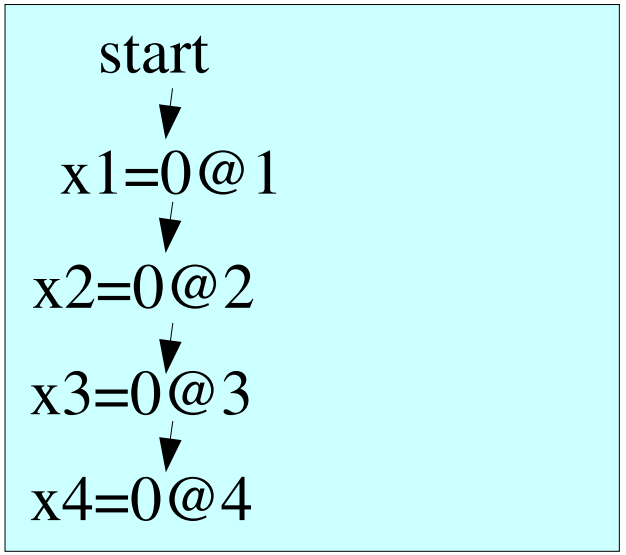


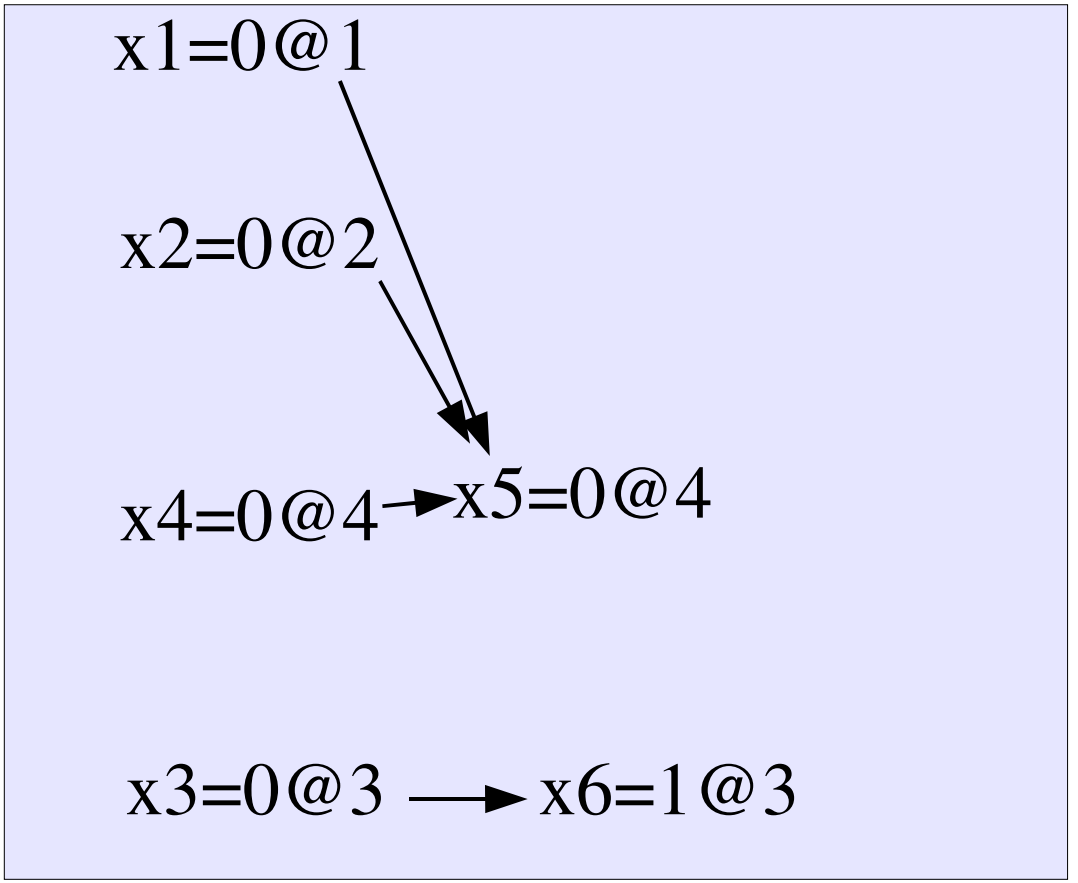


Add node and edges

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



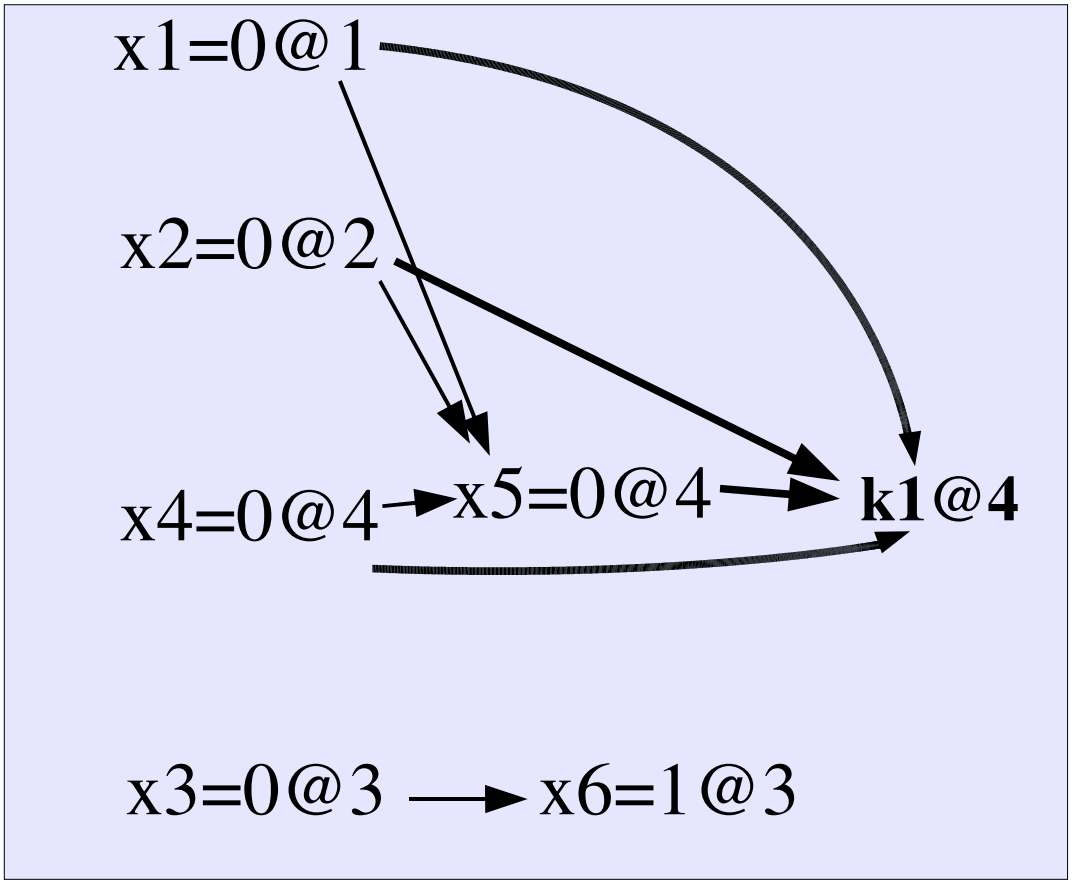


Leads to conflict!

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2

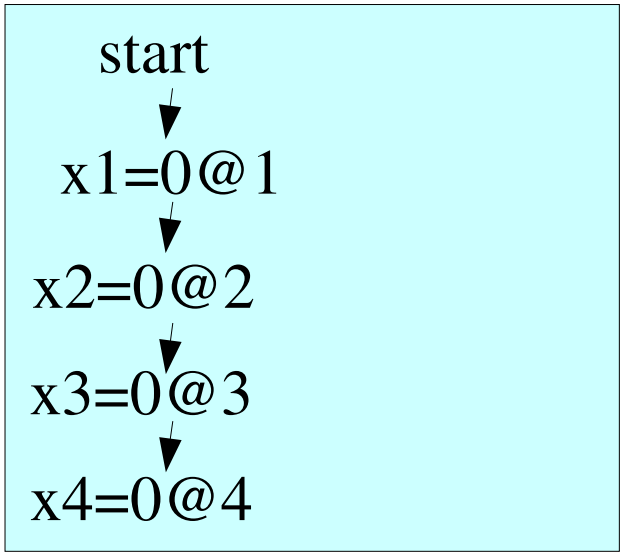
start
↓
x1=0@1
↓
x2=0@2
↓
x3=0@3
↓
x4=0@4

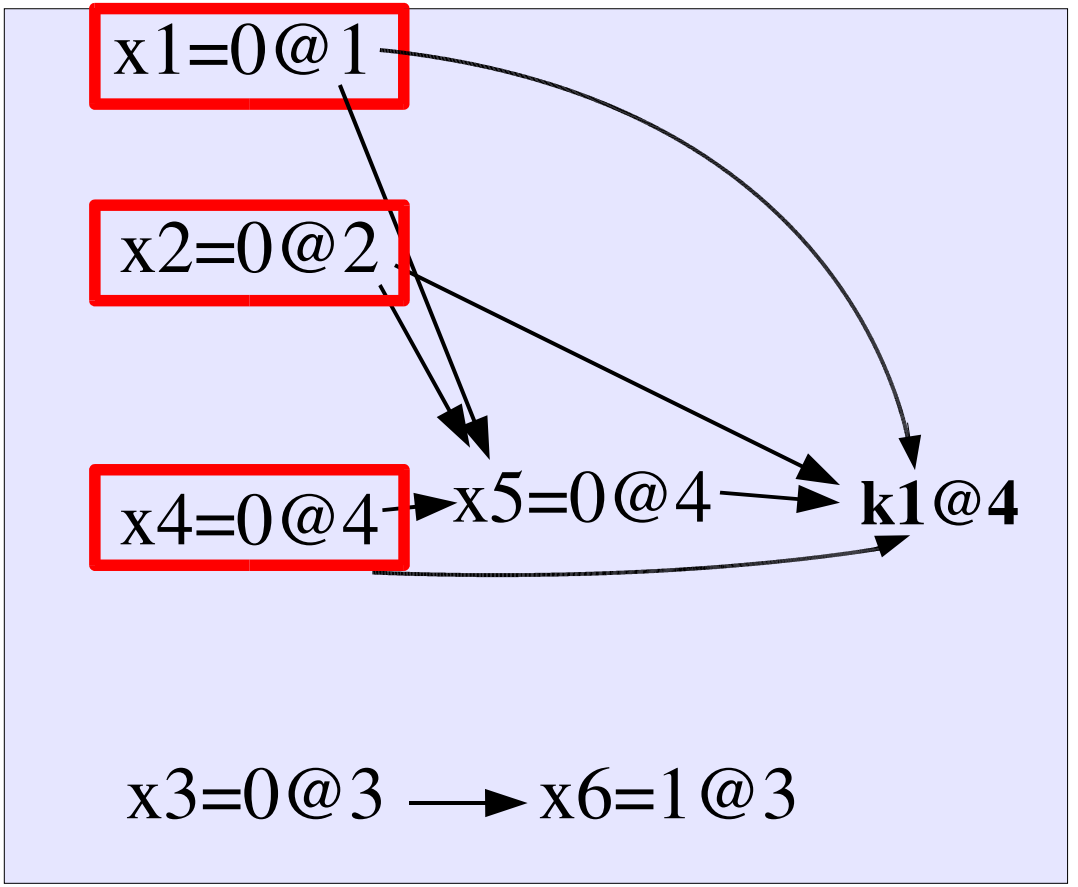


Add conflict node k1

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



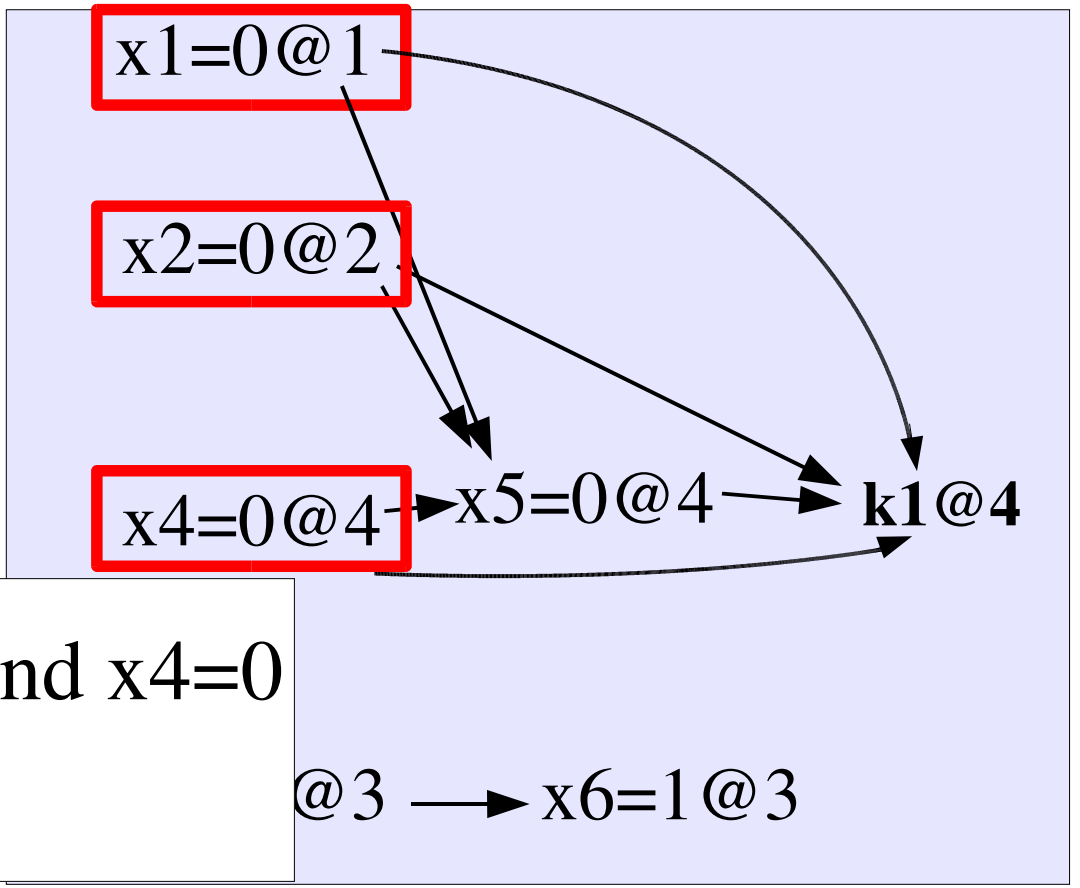


What caused conflict k1?

Follow the edges back to see

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2



It was that x1=0, x2=0, and x4=0

i.e., ¬x1 & ¬x2 & ¬x4

x2=0@2

x3=0@3

x4=0@4

What caused conflict k1?

Follow the edges back to see

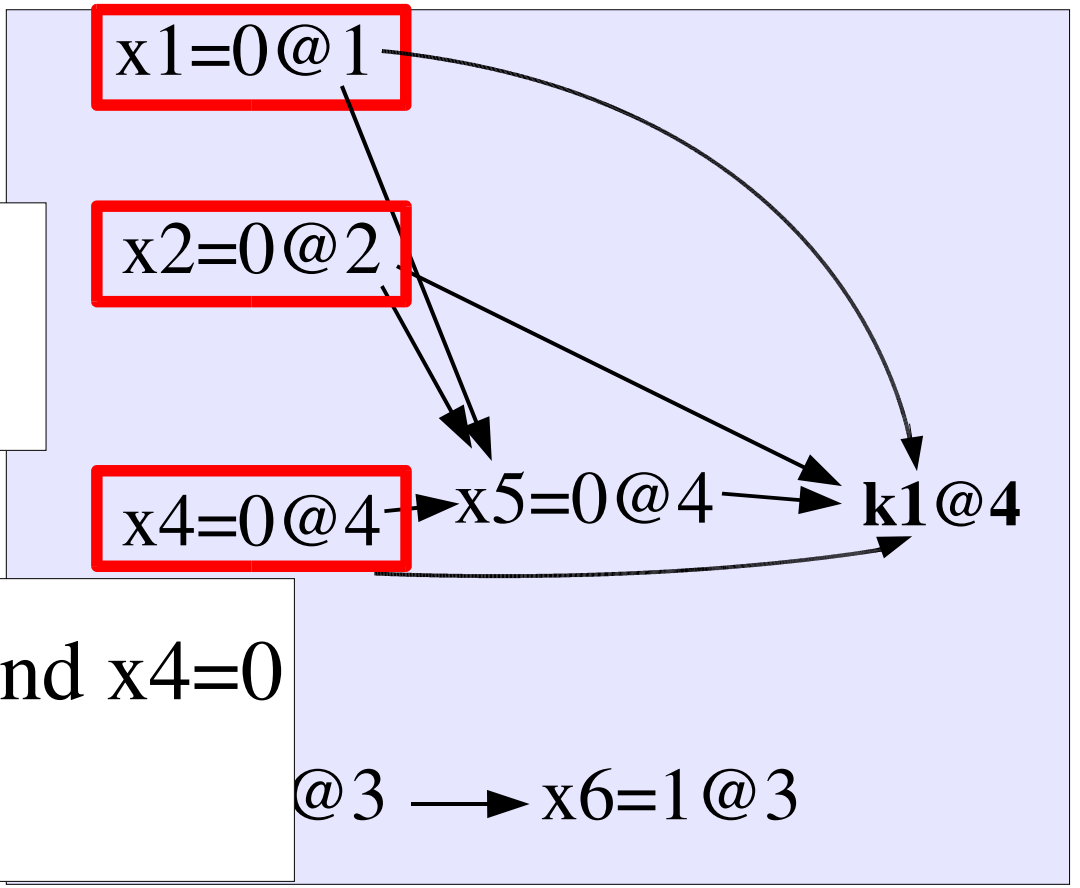
Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4

We don't need x5=0
since it is implied

- w5 = ¬x4 + ¬x7 + x2

It was that x1=0, x2=0, and x4=0
i.e., ¬x1 & ¬x2 & ¬x4



x2=0@2
x3=0@3
x4=0@4

What caused conflict k1?
Follow the edges back to see

Assign False to x4.

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
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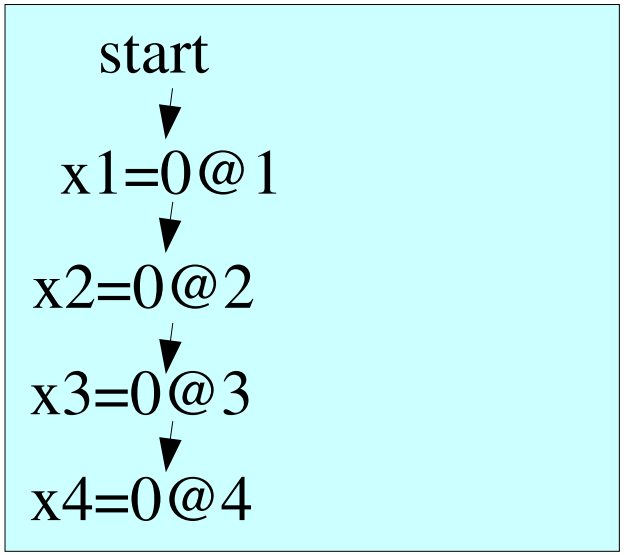
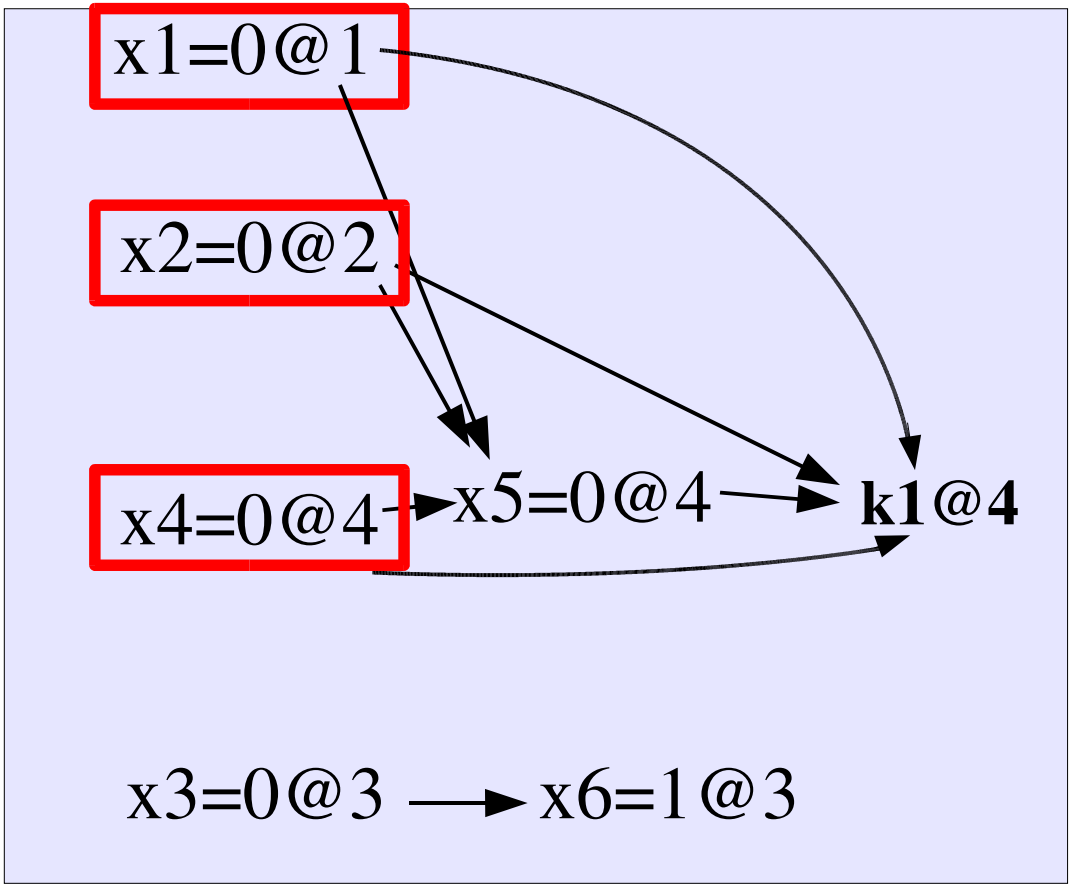




How can we avoid k1?

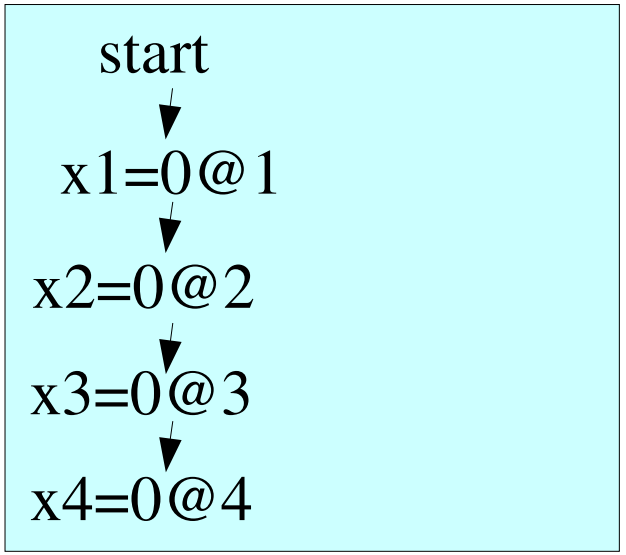
Conflict Clauses

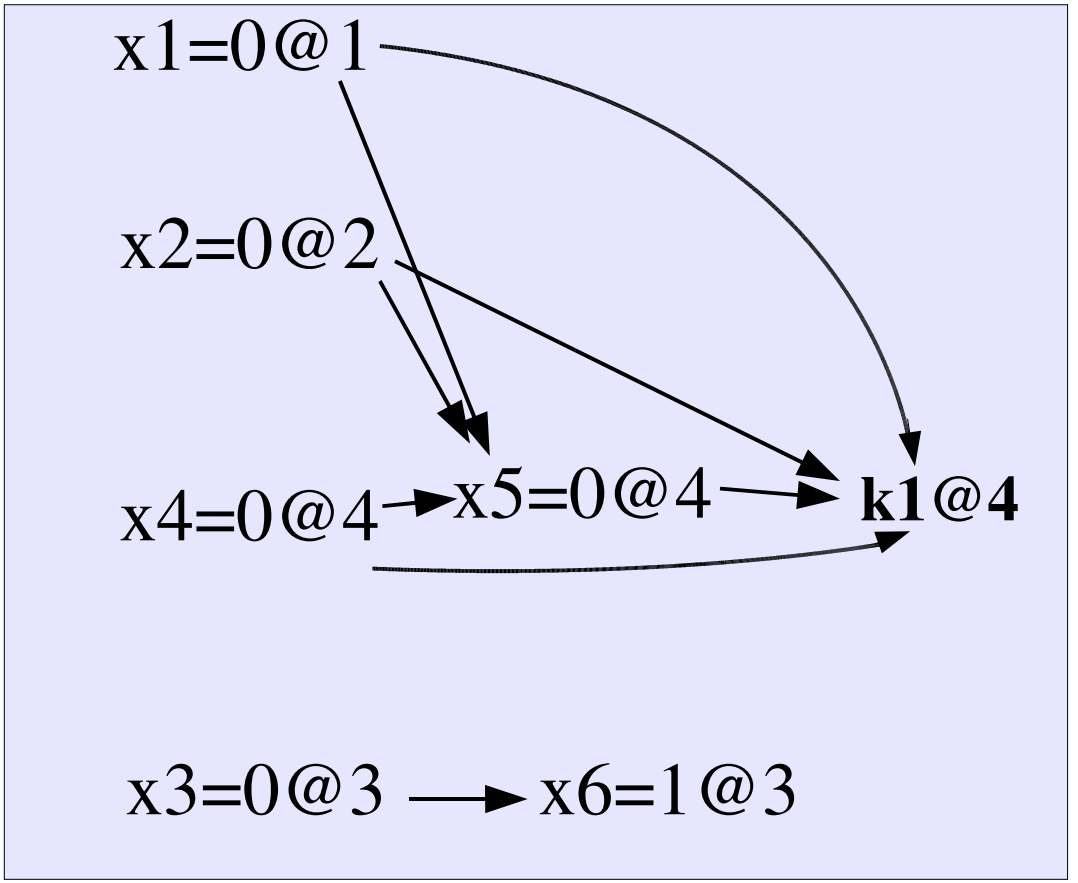
- k1 was caused by¬x1 & ¬x2 & ¬x4
- To avoid k1, force this not to be true
- Add conflict clause

$$w(k1) = \neg(\neg x1 \,\&\, \neg x2 \,\&\, \neg x4)$$
$$= x1 + x2 + x4$$

Add Conflict Clause

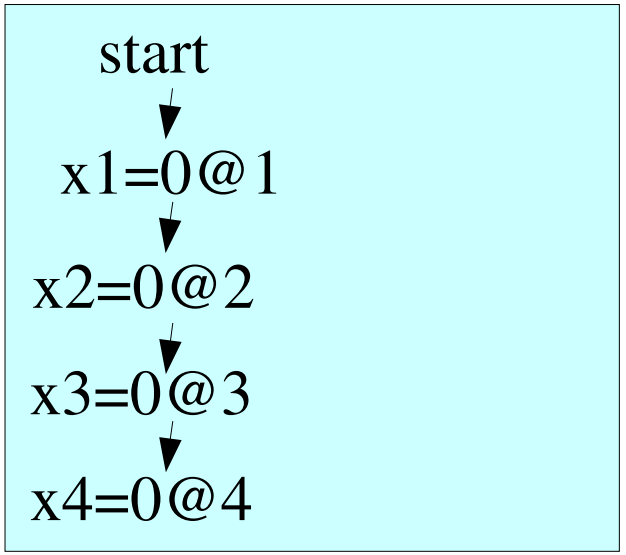
- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

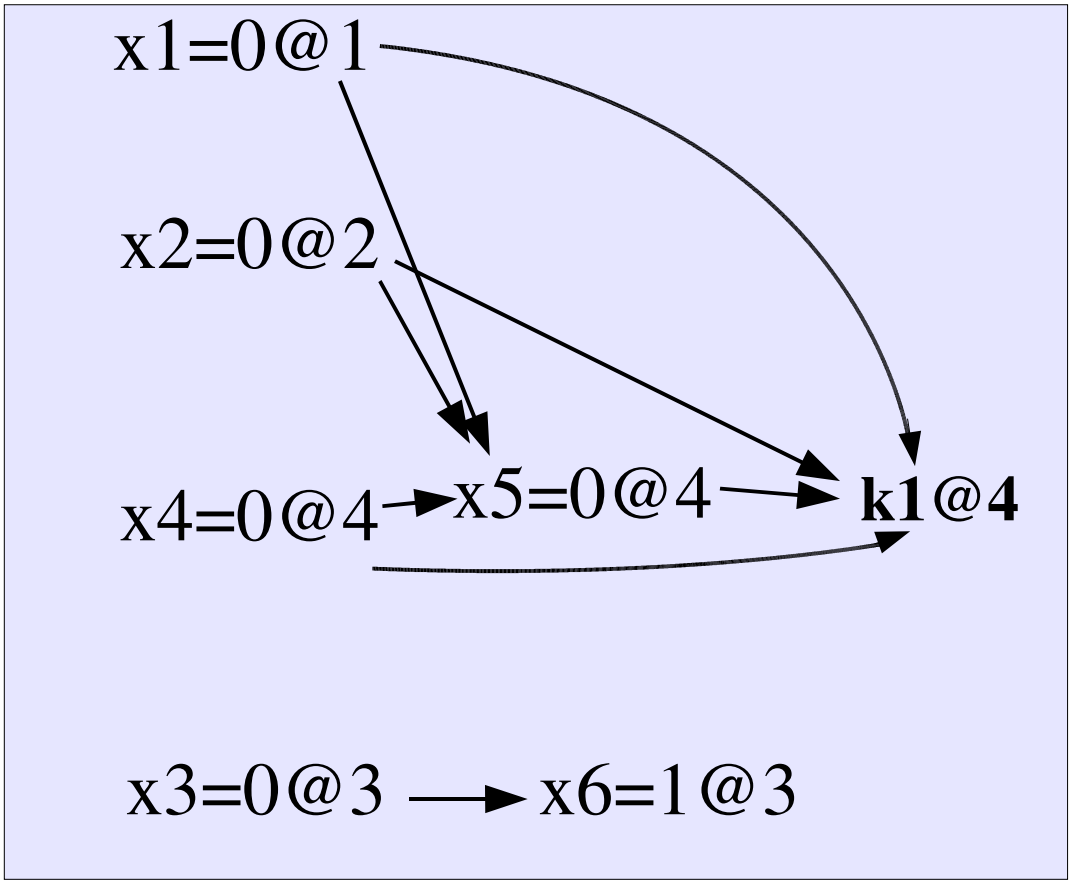




Add Conflict Clause

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4





w(k1) forces x4 to be true

Undo x4=0@4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
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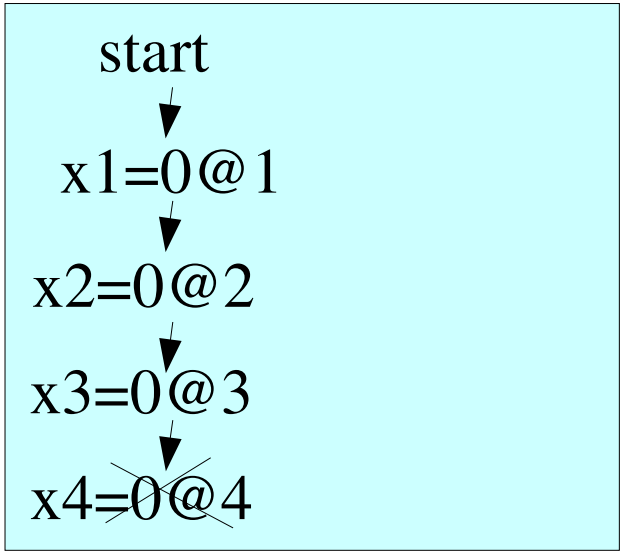
- w(k1) = x1 + x2 + x4



x1=0@1

x2=0@2

x3=0@3 ⟶ x6=1@3

Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
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- w4 = ¬x4 + x7 + x1
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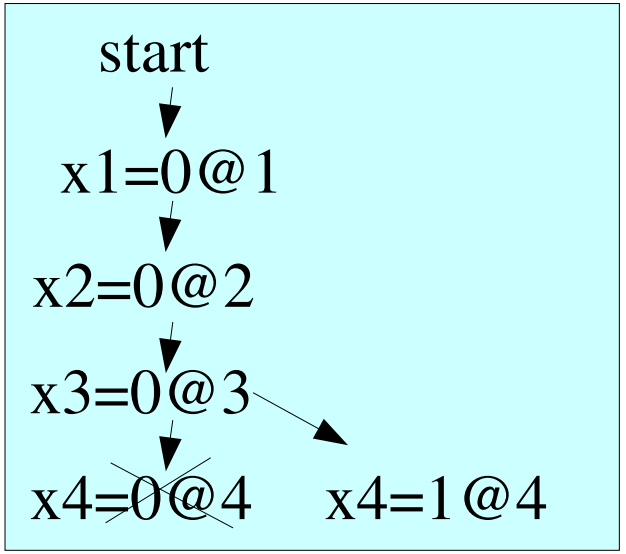
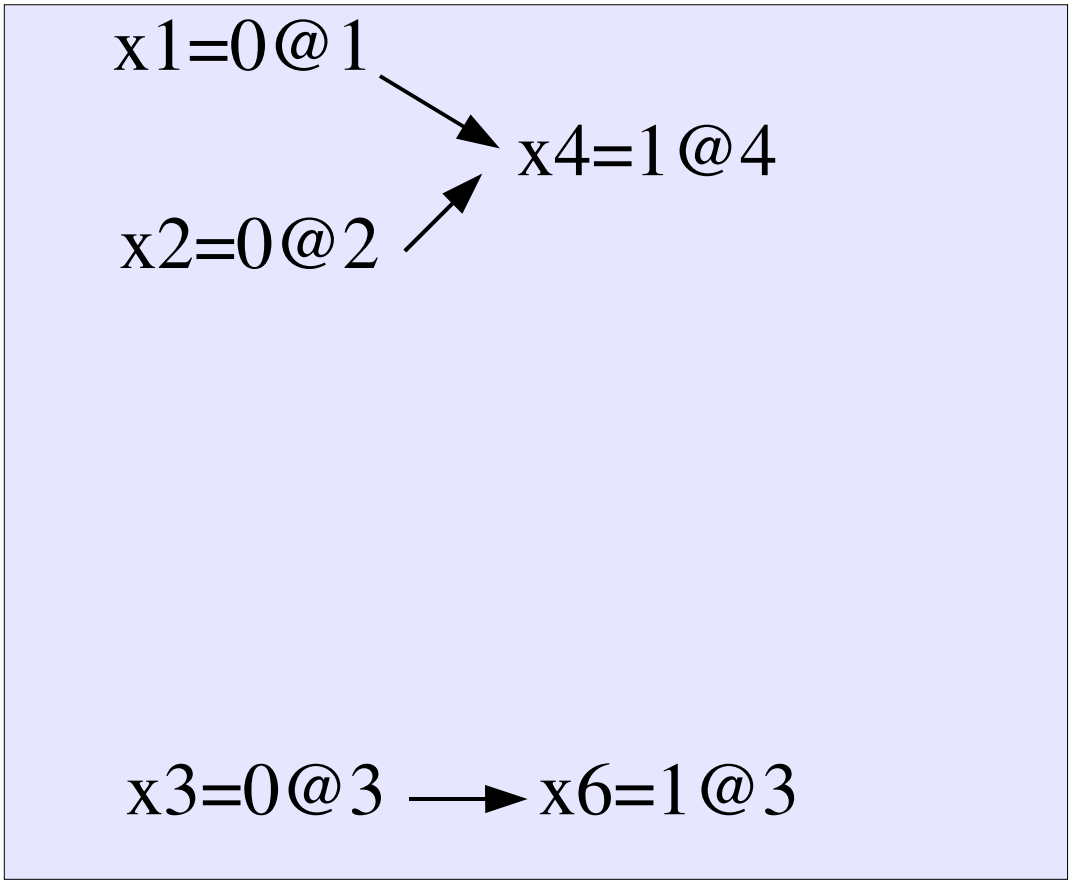
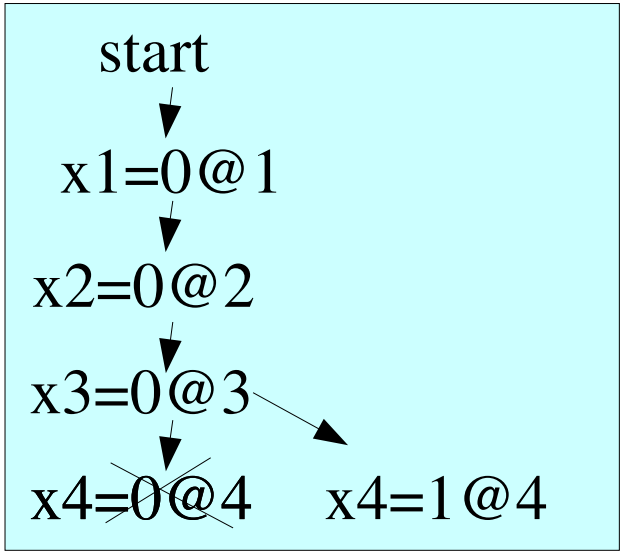
- w(k1) = x1 + x2 + x4

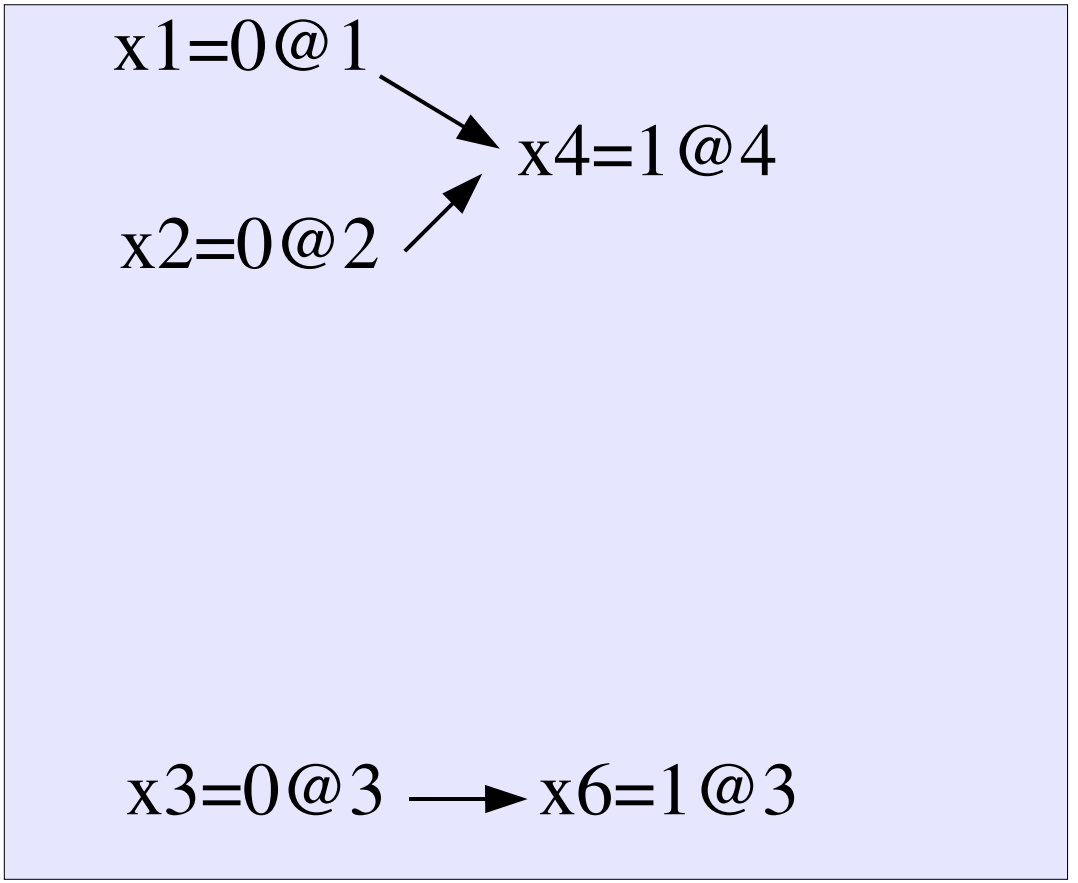




Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

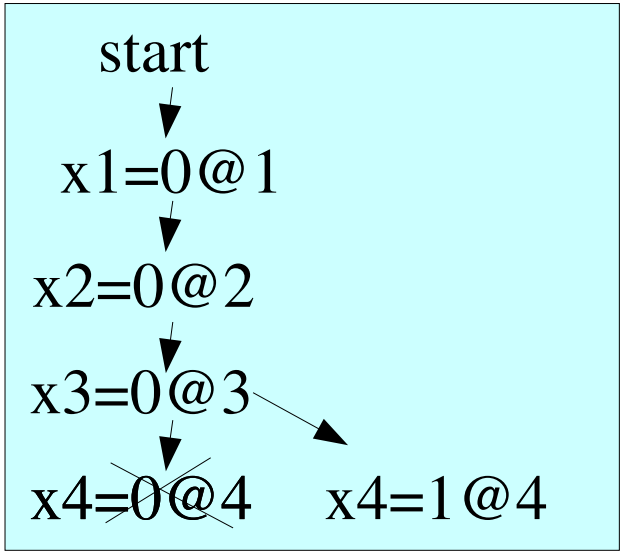


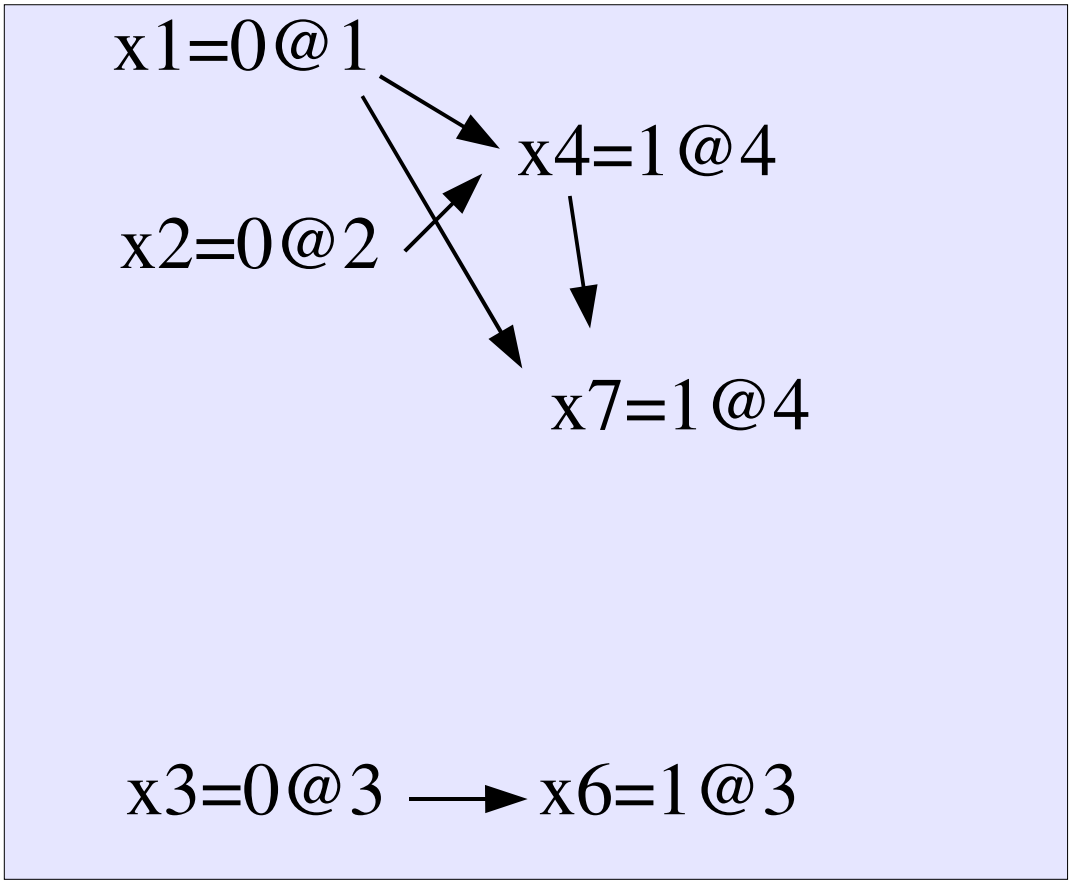


Forces x7 to be true

Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

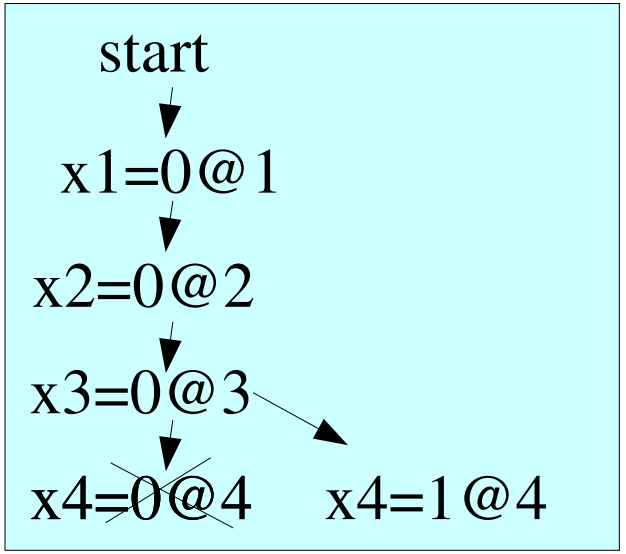


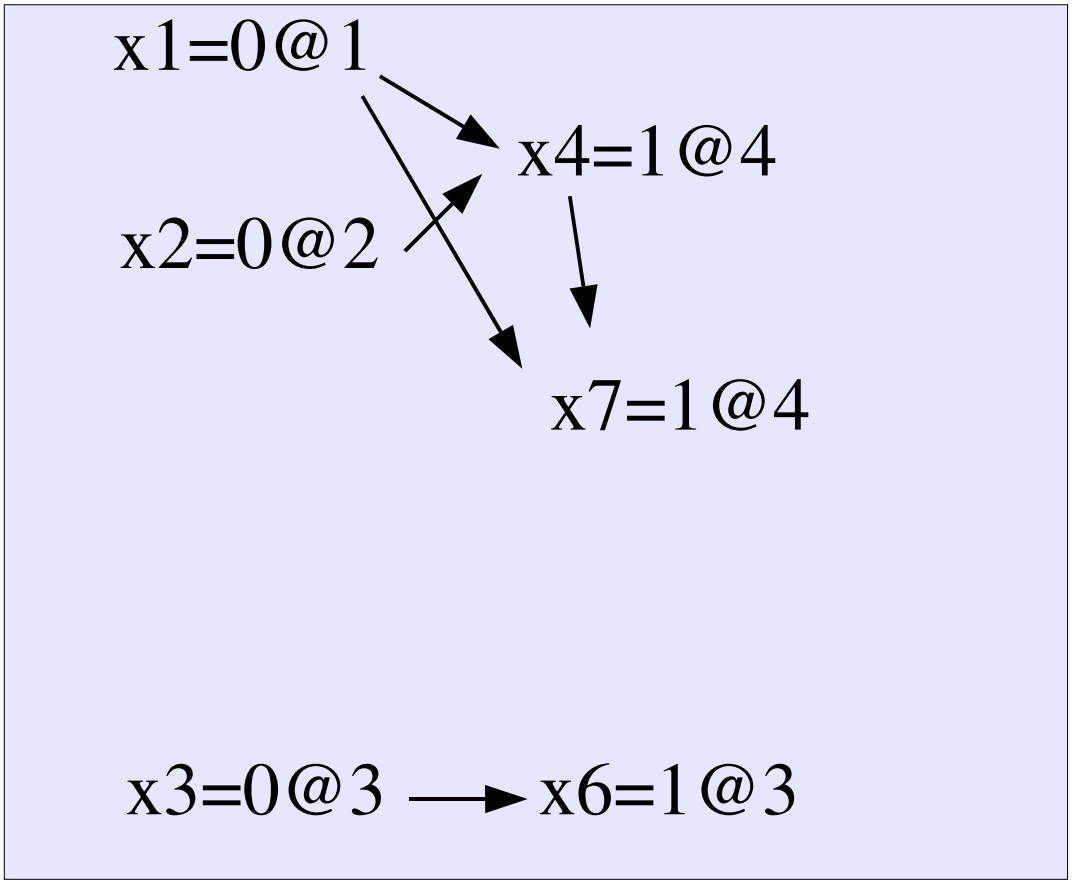


Add node and edges

Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

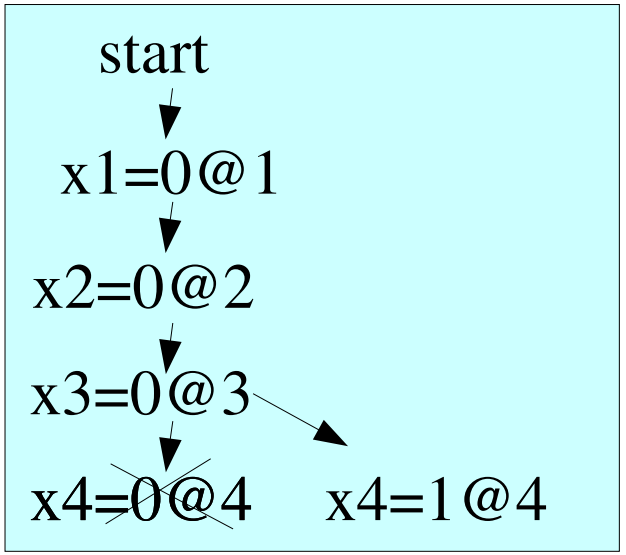


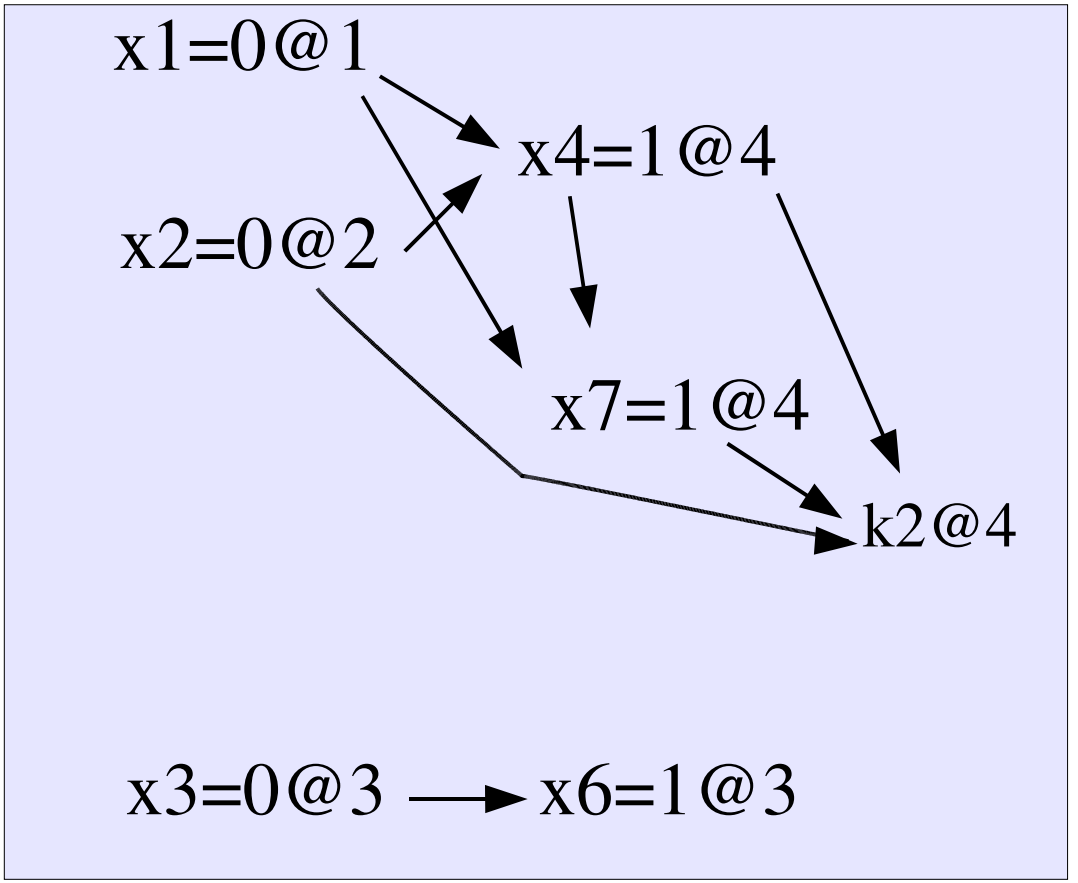


Leads to conflict!

Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

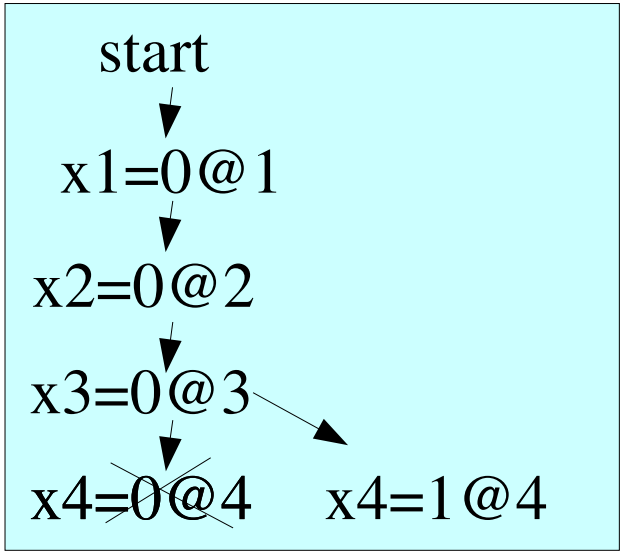


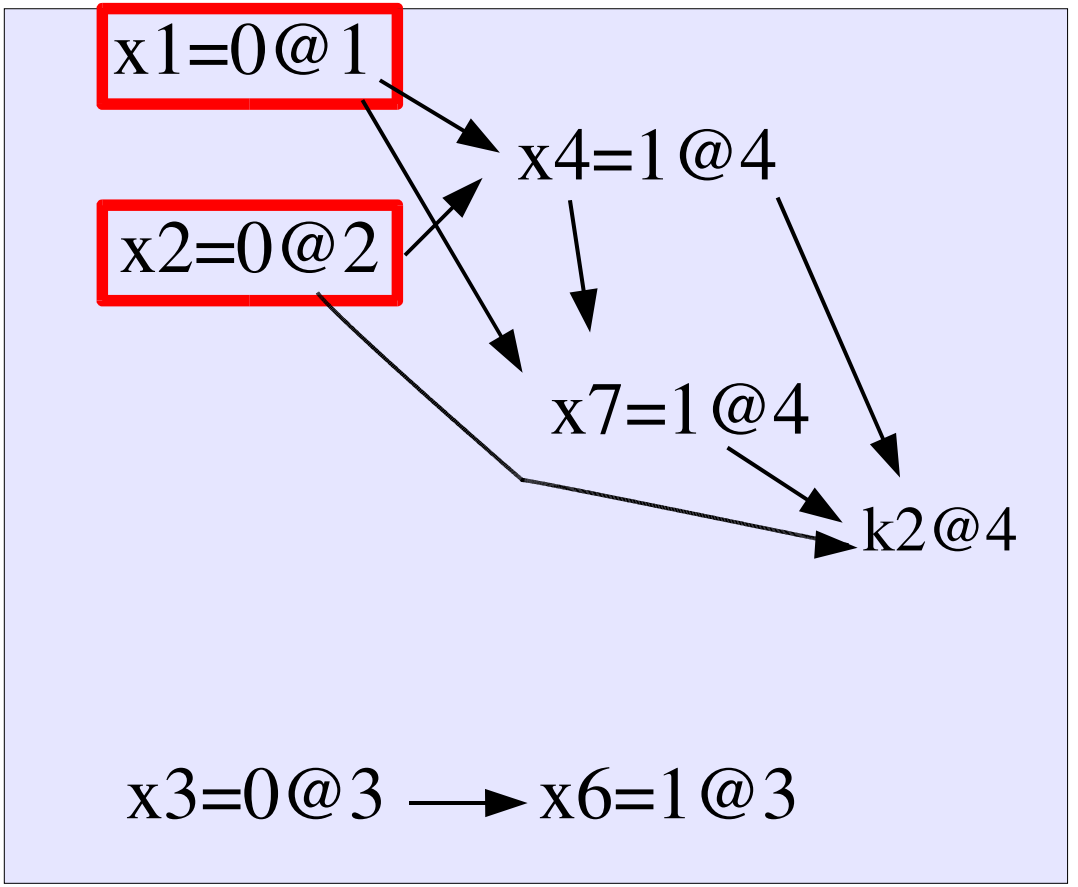


Add conflict node k2

Assign True to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4

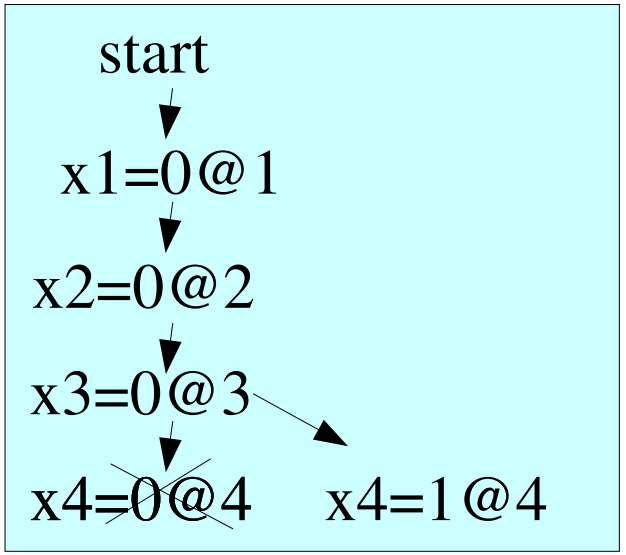


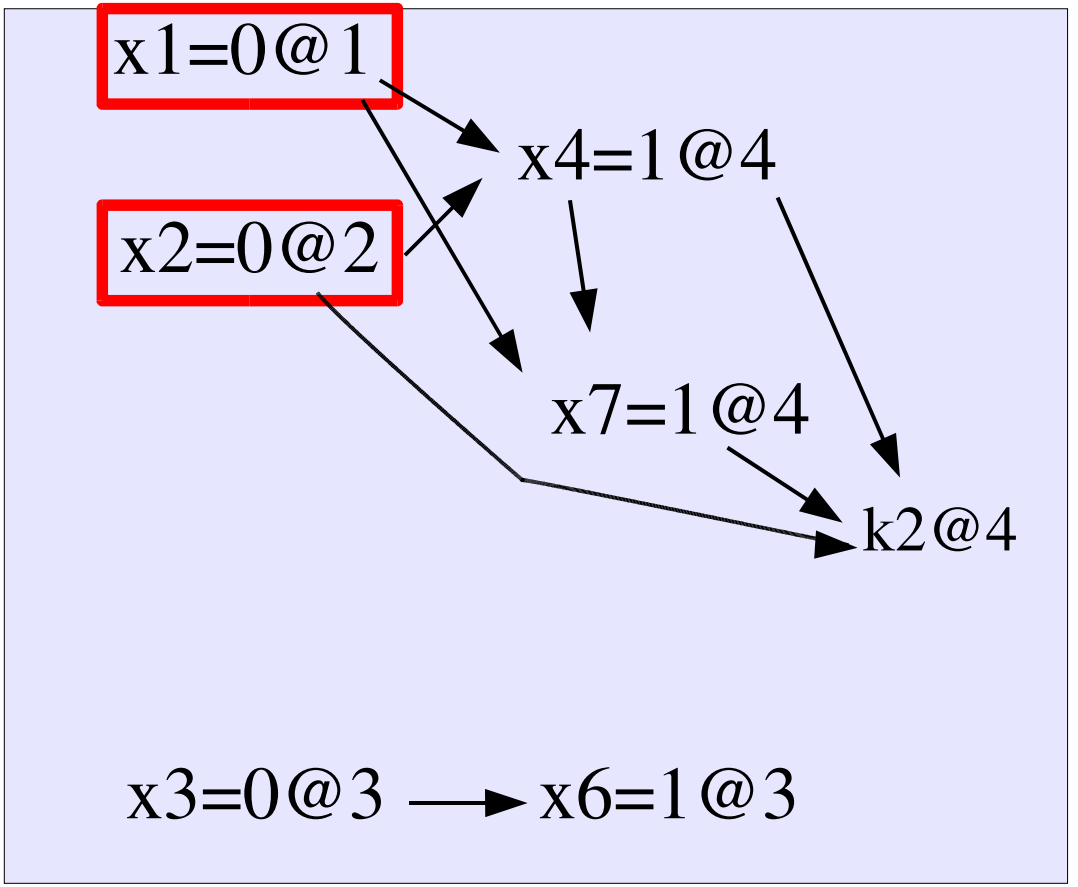


Go backwards from k2 to find root causes

Add Conflict Clause w(k2)

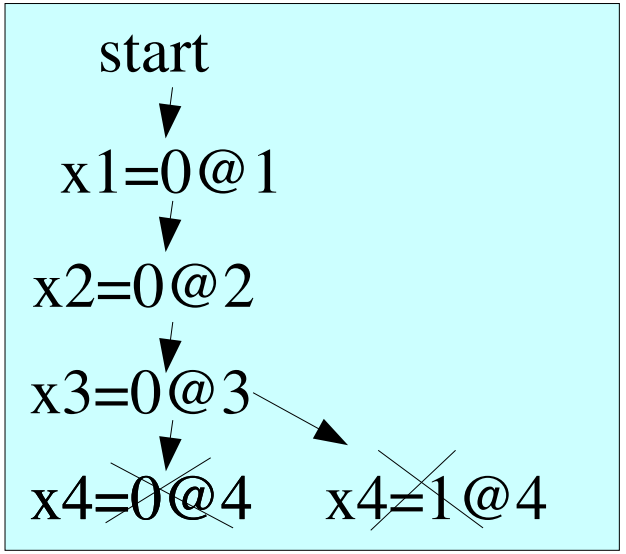
- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2





Backtracking

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



x1=0@1

x2=0@2

x3=0@3 → x6=1@3

We need to undo

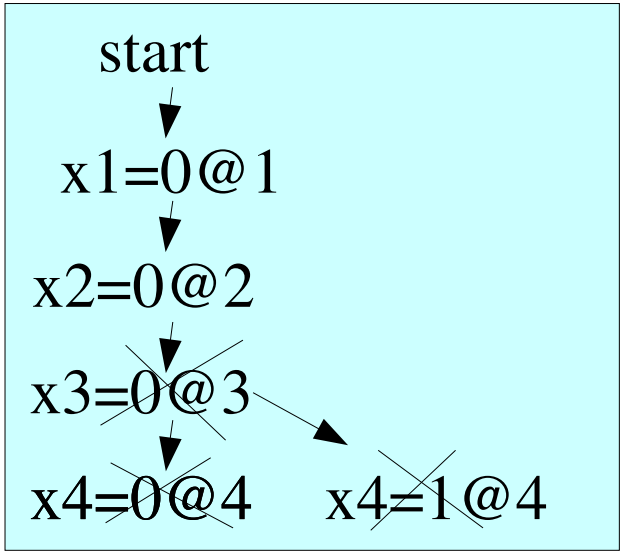
x4=1@4

Backtracking

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2

x1=0@1

x2=0@2

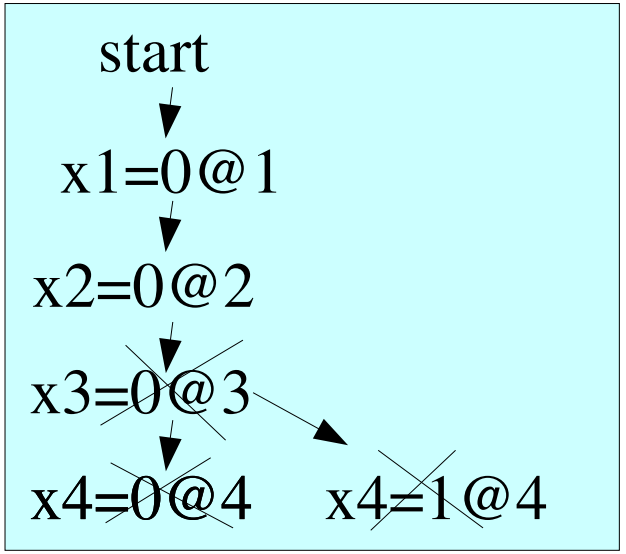


Since we tried both x4=0 and x4=1,

we'll also have to undo x3=0

Backtracking

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



x1=0@1

x2=0@2

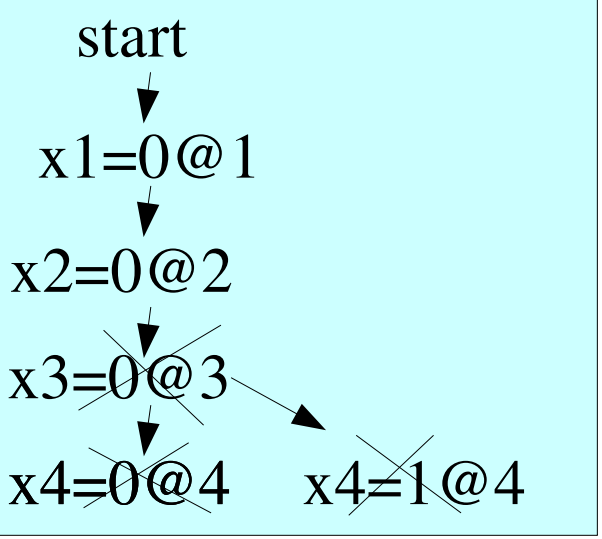
Now we try x3=1@3

Backtracking

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2

x1=0@1

x2=0@2



But wait!

x3 had nothing to do with k2.

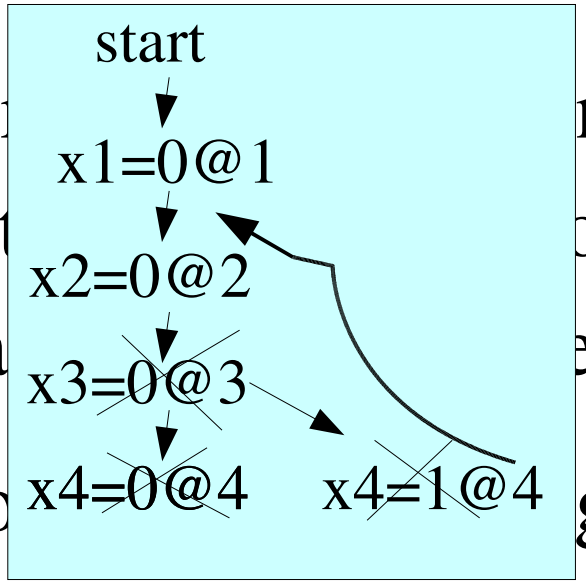
We can skip trying x3=1@3

and backtrack up to x2=0@2

Non-Chronological Backtracking

- SAT solvers like Davis-Putnam undo assignments in the reserve order in which they are made and always try the other assignment
- GRASP does non-chronological backtracking: it will skip assignment points all together and jump up higher in the decision tree

Non-Chronological Backtracking



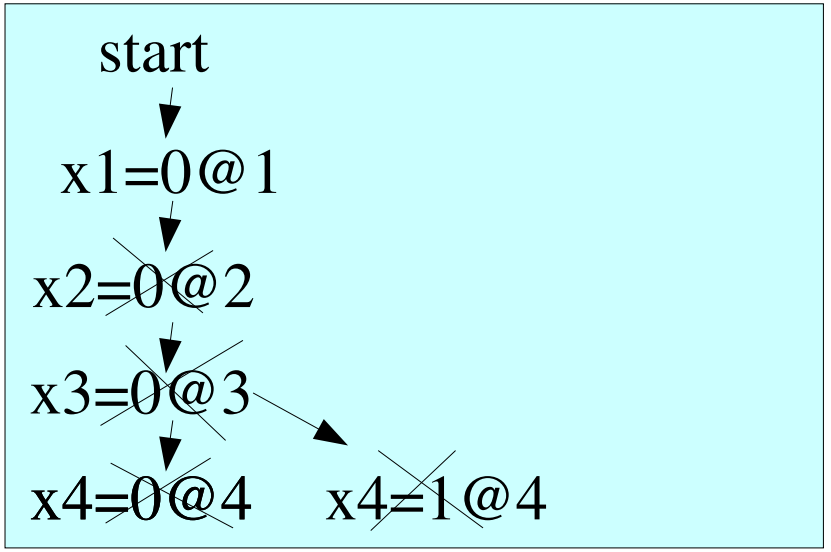
- SAT solve nam undo assignment order in which they are made a e other assignment
- GRASP do gical backtracking: it will skip assignment points all together and jump up higher in the decision tree

Non-Chronological Backtracking

- GRASP gets away with this because the conflict clause made from the implication graph allows it to know what assignments did not play a role in the conflict
- GRASP then jumps up to the latest assignment that did play a role
- This called **conflict-driven** backtracking

Backtracking

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



x1=0@1

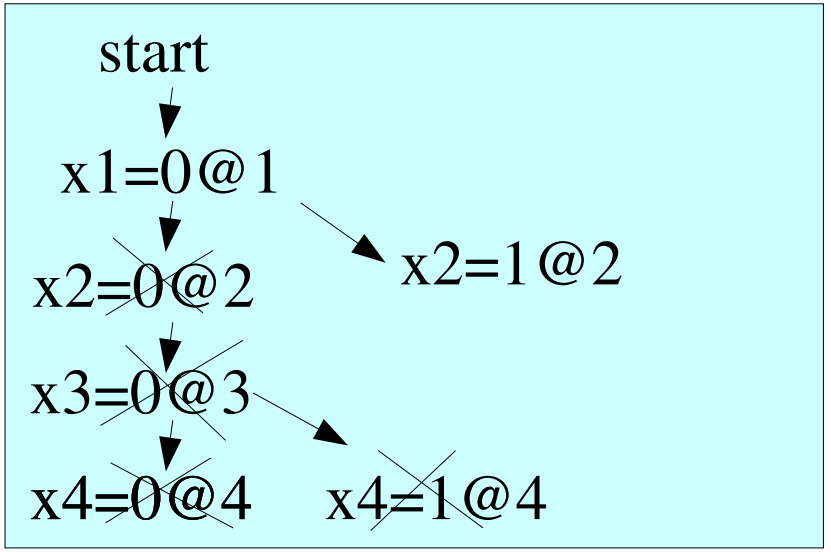
Undo x2=0@2

Assign True to x2

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2

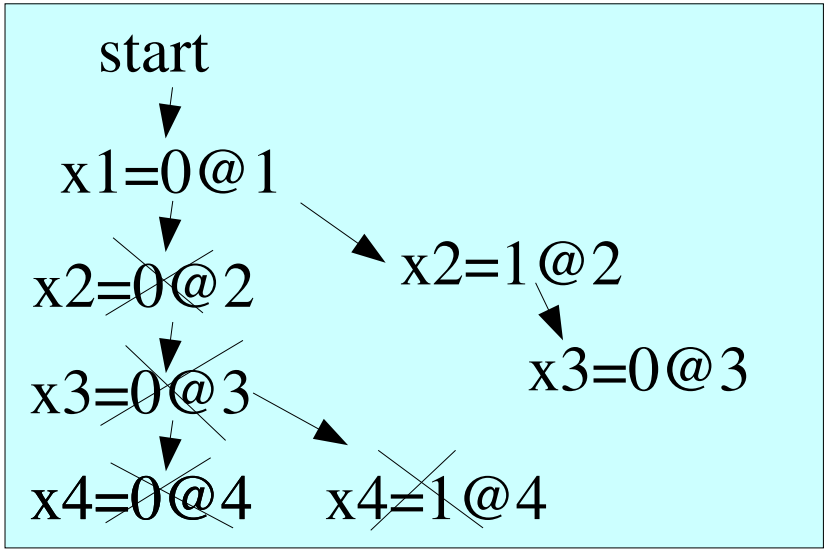
x1=0@1

x2=1@2



Assign False to x3

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



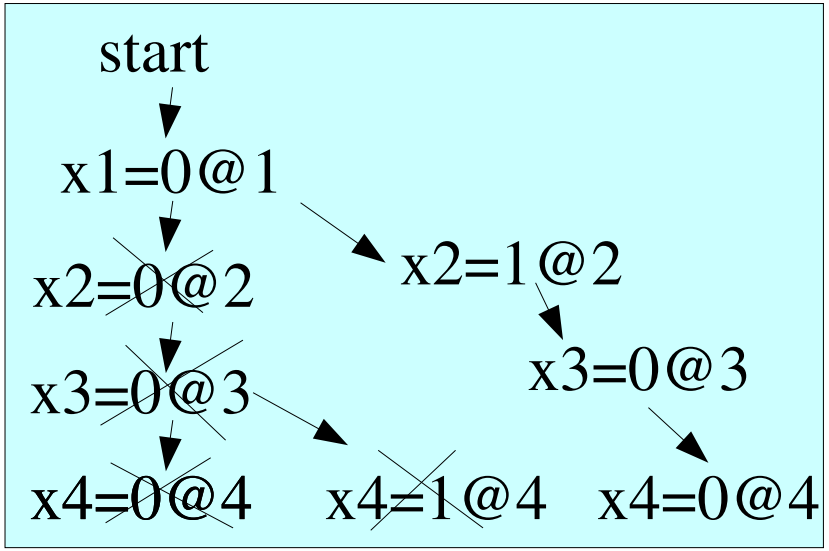
x1=0@1

x2=1@2

x3=0@3 ⟶ x6=1@3

Assign False to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



x1=0@1

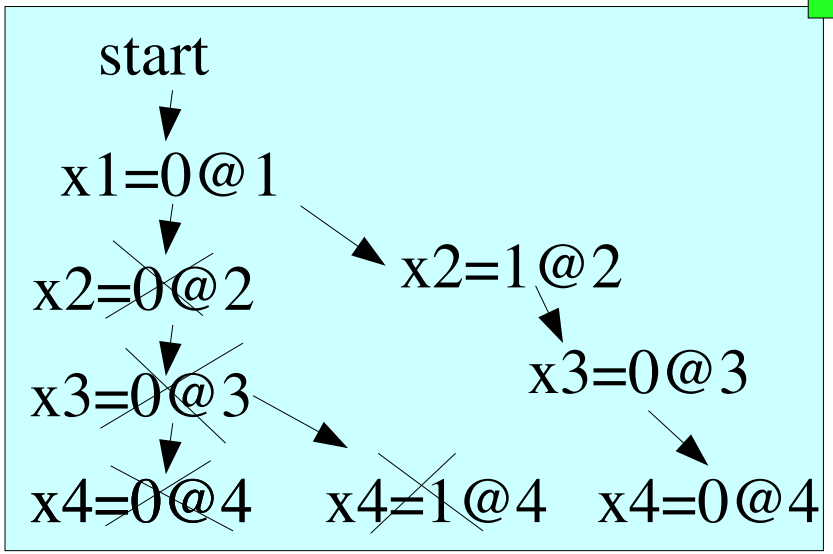
x2=1@2

x4=0@4

x3=0@3 ⟶ x6=1@3

Assign False to x4

- w1 = x1 + x2 + x5 + x4
- w2 = x1 + x2 + ¬x5 + x4
- w3 = x3 + x6
- w4 = ¬x4 + x7 + x1
- w5 = ¬x4 + ¬x7 + x2
- w(k1) = x1 + x2 + x4
- w(k2) = x1 + x2



x1=0@1

All Clauses Satisfied
We Are Done!

x3=0@3 → x6=1@3

Key Points

- The **implication graph** allows us to know why a conflict forms
- A **conflict clause** summarizes how to avoid a conflict
- Knowing this allows for **conflict-driven non-chronological backtracking**

Something to Ponder

- Keeping an implication graph takes both time and memory
- Is the ability to do conflict-driven backtracking worth this cost?
- How would you find out?