Let $W$ be the clause list $[w_1, w_2, w_3, w_4, w_5, w_6, w_7]$ where

$$
\begin{align*}
  w_1 &= x_1 \lor x_2 \lor x_3 \lor x_4 \\
  w_2 &= x_1 \lor \neg x_2 \lor x_4 \\
  w_3 &= x_1 \lor \neg x_3 \lor x_4 \\
  w_4 &= x_5 \lor x_6 \\
  w_5 &= \neg x_5 \lor x_6 \\
  w_6 &= \neg x_4 \lor x_7 \lor x_1 \\
  w_7 &= \neg x_4 \lor \neg x_7 \lor x_1
\end{align*}
$$

1 Decision Points

At each decision point GRASP must pick a new variable assignment. GRASP does this by:

At each node in the decision tree evaluate the number of clauses directly satisfied by each assignment to each variable. Choose the variable and the assignment that directly satisfies the largest number of clauses.


(a) What assignment would GRASP choose first for the clauses above?

(b) After making that assignment, what assignment would it choose second?

2 Implication Graphs and Conflict Clauses

Assume that while doing Boolean constraint propagation (unit propagation), GRASP looks at clauses from the first clause in the list to the last and looks at the literals in clauses from left to right.

(a) Suppose instead that the GRASP decision procedure was set up to assign false to $x_1$ at the first decision node and assign true to $x_5$ at the second decision node. Draw the implication graph that results from these assignments.

(b) Suppose that it assigns false to $x_4$ at the third decision node. This produces a conflict. Draw the new implication graph including the conflict node $k_1$. Provide the conflict clause produced for this conflict node.

(c) This conflict will force the algorithm to next try the assignment of true to $x_4$. A new conflict $k_2$ arises. Draw the implication graph including the new conflict $k_2$. Provide the conflict clause produced for this conflict node. (Remember that the conflict clause produced for $k_1$ will be added to the front of the list of clauses.)

(d) To what decision level does the algorithm backtrack after finding $k_2$? Why?