System Design, Modelling, and Control of a Four-Wheel-Steering Mobile Robot*

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Abstract

This paper describes a dynamic model and cross-coupling control of a mobile robot with four independently steered and driven wheels. The dynamic model incorporates the components of the wheel mechanisms, backlash effects of the wheel actuators and non-linear forces produced by the tire-ground interaction. The problem of computationally efficient control of the vehicle is solved using a low-level control loop that cross-couples steering and driving motors and is utilized in conjunction with a kinematics-based trajectory-tracking controller. The results of simulation show significant reduction of the vehicle slip with moderate trajectory-tracking errors.

1 Introduction

Research on modelling and control of wheeled vehicles can be divided into two major categories: one that is oriented towards automobiles and terrain vehicles, the other is oriented towards indoor wheeled mobile robots (WMRs). This division is based on the differences in the design of the vehicles, operational and environmental conditions. To mention a few:

- wheels of WMRs are often independently actuated, as opposed to mechanically coupled pairs of steering wheels of automobiles.
- a wheel plane of a typical WMR is normally perpendicular to the ground plane, while a wheel plane of an automobile may have a nonzero inclination angle (camber) [2, 10].
- indoor WMRs usually operate within a lower speed range compared to automobiles.

Mechanical coupling of the wheels in automobiles often allows the use of a simplified, *single-track* model of two-wheel-steering and four-wheel-steering cars for the design of motion controllers [1, 4]. In modelling and control of mobile robots with independently steered and/or driven wheels it is required to explicitly account for each of the wheels of the vehicle.

Such complex models of four-wheel vehicles has been developed in works [3, 7, 8]. However the controllers in these works are still designed under the assumption that the wheels are steered in pairs. The models also utilize simplified wheel actuator dynamics that may not be appropriate for modelling of some types of WMRs, e. g. heavy vehicles with DC motor actuated wheels, for which the effects of backlash and wheel mechanism dynamics may be significant. Such WMRs are often utilized as autonomous guided vehicles (AGVs) in tight manufacturing environments having high requirements on maneuverability. These requirements imply the necessity of active utilization of the capability of such WMRs for independent wheel steering.

Kinematics-based control of WMRs with multiple steering wheels has received considerable attention over the last decade. Efficient trajectory-tracking controllers are developed using dynamic feedback [9]. To the authors' best knowledge, controllers that utilize complex dynamic models of the WMRs with multiple steering wheels are not used in practice due to their computational infeasibility and difficulties related to analytical investigation of their properties.

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One of the possible ways to deal with the trade-off between computational efficiency of control and a granulation of a corresponding model is to augment a kinematics-based controller with a controller that accounts for a particular dynamic effect. In this work we describe such a controller that aims specifically at minimization of slippage due to wheel misalignment.

The controller extends the approach presented in [6] by (i) cross-coupling all four wheels of the vehicle, instead of coupling them pairwise, (ii) incorporating not only steering but also driving actuator dynamics, (iii) generalizing the control objectives so that the wheels are able to track a linear approximation of the vehicle kinematic constraints during transient states of control. This coupling scheme allows the four wheels to be steered at arbitrary angles, not necessarily the same angles as in [6].

The performance of the cross-coupling controller is evaluated with the dynamic model developed for the AGV prototype of the project. The dynamic model of the vehicle differs from those described in [3, 7, 8] in that, in addition to nonlinear forces at the contact between the tire and the ground, it incorporates the dynamics of the components of the wheel mechanisms and a more realistic model of the wheel actuators including backlash effects of DC motors.

The paper is organized as follows. Section 2 describes the kinematics of the WMR. The dynamic model is presented in section 3. The cross-coupling controller and simulation results are addressed in section 4. Concluding remarks are given in section 5.

2 Kinematic model

A diagram of the vehicle is presented in figure 1. Assuming that the vehicle frame is a rigid body, the velocities v_i of the wheel base points W_i are related to the velocity v_c of the reference point C of the frame as

$$v_i = v_c + \dot{\theta}\mathbf{k} \times \overrightarrow{CW_i}, \qquad i = 1, \ldots, 4. \tag{1}$$

Pure rolling condition implies that the wheel's rotation rate $\dot{\phi}_i$ and steering angle ψ_i with respect to the frame coordinate system are such that

$$\dot{\phi}_i = \frac{|v_i|}{r}, \qquad \tan \psi_i = \frac{v_{iy}}{v_{ix}}, \qquad i = 1, \ldots, 4. \tag{2}$$

The kinematic equation relating acceleration a_i of the wheel base point W_i and acceleration a_c of the vehicle reference point C is

$$a_i = a_c + \ddot{\theta}\mathbf{k} \times \overrightarrow{CW_i} - \dot{\theta}^2 \overrightarrow{CW_i}. \tag{3}$$

Using the fact that $a_c = (\dot{v}_x - \dot{\theta} v_y)\mathbf{i} + (\dot{v}_y + \dot{\theta} v_x)\mathbf{j}$ we have $a_i = a_{xi}\mathbf{i} + a_{y_i}\mathbf{j}$, where

$$a_{xi} = \dot{v}_x - \dot{\theta} v_y - CW_{ix}\dot{\theta}^2 - CW_{iy}\ddot{\theta}, \tag{4}$$

$$a_{y_i} = \dot{v}_y + \dot{\theta} v_x - CW_{iy}\dot{\theta}^2 + CW_{ix}\ddot{\theta}. \tag{5}$$

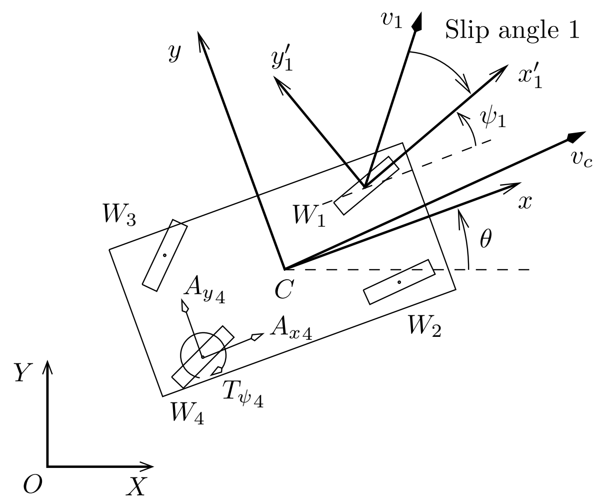

Figure 1: Notation of kinematics of the WMR and dynamics of the frame.

3 Dynamic model

The dynamic model of the vehicle is derived by considering dynamics of the vehicle frame, wheel struts, wheels, and wheel motors.

3.1 Frame dynamics

The frame lateral, longitudinal and yaw dynamic equations are:

$$ma_x = m(\dot{v}_x - \dot{\theta} v_y) = \sum_i A_{xi}, \tag{6}$$

$$ma_y = m(\dot{v}_y + \dot{\theta} v_x) = \sum_i A_{y_i}, \tag{7}$$

$$I\ddot{\theta} = \sum_i P_i \tag{8}$$

respectively, where A_{xi} and A_{y_i} are the x-component and the y-component of reaction at the steer joint W_i, $P_i = -T_{\psi i} + CW_{ix}A_{y_i} - CW_{iy}A_{xi}$ is the vertical component of the total torque applied to the frame via the steer joint W_i.

We neglect horizontal components P_{xi}, P_{y_i} of the torque applied to the vehicle frame via the steer joint, assuming that all wheels of the vehicle are always in contact with the ground and that the change of normal reaction at the wheel-ground contact point is negligible.

3.2 Wheel strut dynamics

We assume that for each wheel i gravity centers of the strut and the wheel, the point of the wheel contact with the ground, geometric center of the wheel, and steer joint W_i are all on a single vertical line. As with the frame, we neglect horizontal components of the total moment of the wheel strut.

$$m_s a_{xi} = S_{xi} - A_{xi}, \tag{9}$$
$$m_s a_{y_i} = S_{y_i} - A_{y_i}, \tag{10}$$
$$w_s = S_{zi} - A_{zi}, \tag{11}$$
$$I_s(\ddot{\theta} + \ddot{\psi}_i) = T_{\psi_i} - D_{zi}. \tag{12}$$

3.3 Wheel dynamics

The forces applied to the wheel are related as follows:

$$m_w a_{xi} = F_{xi} - S_{xi}, \tag{13}$$
$$m_w a_{y_i} = F_{y_i} - S_{y_i}, \tag{14}$$
$$w_w = N - S_{zi}. \tag{15}$$

The total moment produced by couples and forces applied to wheel i is

$$\begin{aligned} M_i &= T_{\phi_i} + D_{zi} - E_i + D_{x'i} - r\mathbf{k}' \times (F_{x'i}\mathbf{i}' + F_{y'_i}\mathbf{j}') \\ &= (D_{x'i} + rF_{y'_i})\mathbf{i}' + (T_{\phi_i} - rF_{x'i})\mathbf{j}' + (D_{zi} - E_i)\mathbf{k}'. \end{aligned} \tag{16}$$

We assume the wheel to be cylindrical so that the axes x'_i, y'_i, z'_i are its principal axes of inertia. The angular moments of the wheel i are:

$$M_{xi} = I_{x'x'}\dot{\omega}_{x'i} + (I_{z'z'} - I_{y'y'})\omega_{y'_i}\omega_{z'i}, \tag{17}$$
$$M_{y_i} = I_{y'y'}\dot{\omega}_{y'i} + (I_{x'x'} - I_{z'z'})\omega_{x'i}\omega_{z'i}, \tag{18}$$
$$M_{zi} = I_{z'z'}\dot{\omega}_{z'i} + (I_{y'y'} - I_{x'x'})\omega_{x'i}\omega_{y'_i}, \tag{19}$$

where $I_{x'x'} = I_{z'z'} = I_t$ is the transverse moment of inertia and $I_{y'y'} = I_a$ is the axial moment of inertia of the wheel. Assuming that the x'_i-component of the angular velocity of the wheel is zero (no camber), the angular velocity of the wheel i is

$$\omega_i = (\dot{\theta} + \dot{\psi}_i)\mathbf{k}' + \dot{\phi}_i\mathbf{j}'. \tag{20}$$

Substitution of (20) into (17)–(19) and using (16) and (12) gives us the expressions for the steering and driving torques respectively:

$$T_{\psi_i} = (I_s + I_t)(\ddot{\theta} + \ddot{\psi}_i) + E_i, \tag{21}$$
$$T_{\phi_i} = I_a\ddot{\phi}_i + rF_{x'i}. \tag{22}$$

The lateral and longitudinal tire friction forces $F_{y'}$, $F_{x'}$, and the aligning torque due to friction E_i are modelled as commonly done in the automotive industry [2], [10].

3.4 Dynamics of wheel motors

Assuming that the armature-winding inductance of the motors is negligible, the equations governing the dynamics of the (i) steering and (ii) driving motors of the wheel i are as follows:
(i) steering motors

$$k'_1 e_{\psi_i} = k'_3\dot{\psi}_i + R' i_{\psi_i}, \tag{23}$$
$$k'_2 i_{\psi_i} = (I_s + I_t)(\ddot{\theta} + \ddot{\psi}_i) + b'\dot{\psi}_i + E_i, \tag{24}$$

where $(I_s + I_t)(\ddot{\theta} + \ddot{\psi}) + E_i = T_{\psi_i}$ is the steering torque applied to the wheel, and $b'\dot{\psi}_i$ is the energy loss due to the friction in the motor and in the gear train;
(ii) driving motors

$$k''_1 e_{\phi_i} = k''_3\dot{\phi}_i + R'' i_{\phi_i}, \tag{25}$$
$$k''_2 i_{\phi_i} = I_a\ddot{\phi}_i + b''\dot{\phi}_i + rF_{x'i}, \tag{26}$$

where $I_a\ddot{\phi}_i + rF_{x'i} = T_{\phi_i}$ is the driving torque applied to the wheel, and $b''\dot{\phi}_i$ is the energy loss due to the friction in the motor and in the gear train.

3.5 Vehicle Dynamics

Combining (6)–(8), (9)–(12), (13), (14), (4), (5) with (23)–(26), (21), (22), the state-space model of the vehicle can be presented as:

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \ddot{\theta} \\ \ddot{\psi}_{1-4} \\ \ddot{\phi}_{1-4} \end{bmatrix} = \begin{aligned} M^{-1}Q(v_x, v_y, \dot{\theta}, \dot{\psi}_{1-4}, \dot{\phi}_{1-4}, \\ \psi_{1-4}, \phi_{1-4}, e_{\psi_{1-4}}, e_{\phi_{1-4}}), \end{aligned} \tag{27}$$

where matrices M and Q are specified in the Appendix.

4 Control

4.1 Cross-coupling controller

We consider the problem of control of the mobile robot equipped with multiple steering and driving wheels. The efficient kinematics-based trajectory-tracking control via *linearizing dynamic feedback* is proposed in [9]. To preserve the computational feasibility of kinematics-based control and to account for tire slip due to misalignment of the wheels, we propose to use a cross-coupling controller in conjunction with a kinematics-based trajectory-tracking controller.

The proposed cross-coupling controller is based on the idea of a piecewise linear approximation of the pure rolling condition (2) with the node points in the space of steering angles and driving rates generated by a higher level trajectory tracking controller [5, 6].

For example, the linear approximation of the curve in the space of steering angles is specified by the relationship

$$\frac{\psi_i - \psi_{i\,d}}{\delta_i} = \frac{\psi_j - \psi_{j\,d}}{\delta_j}, \quad \delta_i \neq 0, \quad \delta_j \neq 0, \tag{28}$$

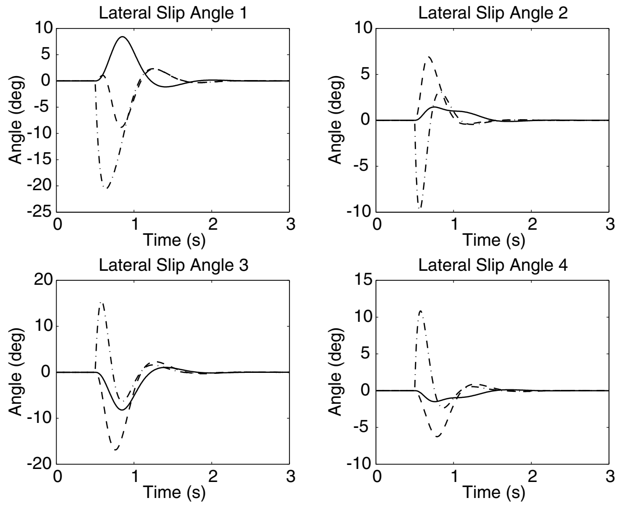

Figure 2: Lateral slip angles corresponding to the four wheels with the different control schemes: dash-dotted line — uncoupled control; dashed line — partially coupled control (uncoupled controller for the steering motors, with the driving motors coupled with the respective steering motors); solid line — cross-coupling control of the steering and driving motors.

where ψ_{i_s} and ψ_{i_d} are the initial and desired steering angles respectively, $\delta_i = \psi_{i_s} - \psi_{i_d}$ are the initial errors. We refer to (28) as *the condition of proportional errors* between the steering actuators i and j. A similar relationship can couple the driving and steering actuators.

The LQR-based controller that couples the steering and driving actuators according to the condition of proportional errors is described in detail in [5].

4.2 Simulation results

The performance of the cross-coupling controller is evaluated by simulation with the dynamic model implemented in Matlab. The reference trajectory consists of a straight-line interval and a circular arc of the radius $R = 2$ m with the center closer to wheels 1 and 4. The vehicle undergoes the turning maneuver so that during the line-interval following all reference steering angles are equal to zero, and during the circular-arc following the reference steering angles are related as $\psi_{1d} = -\psi_{3d} = 38.8°$, $\psi_{2d} = -\psi_{4d} = 20.1°$. Lateral slip angles obtained for different coupling schemes are presented in figure 2. The simulated and the ideal trajectory in the space of steering angles are shown in figure 3. The trajectory tracking errors under the cross-coupling control are larger than under the uncoupled control, but considered moderate (more details in [5]).

5 Conclusion

In this paper we develop a dynamic model of a mobile robot with four independently steered and driven wheels. Detailed modelling of wheel plants allows utilization of the model for simulation involving such dynamic effects as slippage due to wheel misalignment and driving rate errors.

To minimize slippage due to misalignment of the wheels, a cross-coupling controller is proposed to follow a piecewise linear approximation of the trajectory corresponding to the pure rolling condition in the space of steering and driving actuators. The simulation performed with the dynamic model shows significant reduction of lateral slip with moderate trajectory-tracking errors.

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Appendix

A Nomenclature

reaction force and torque at steer joint	A_i, T_{ψ_i}
mass and moment of inertia of the frame	m, I
reaction force and torque at wheel-strut joint	S_i, D_i
mass and weight of the strut	m_s, w_s
moment of inertia of the strut	I_s
lateral tire friction force	$F_{y'i}$
longitudinal tire friction (tractive) force	$F_{x'i}$
normal force at the tire-ground contact	N
aligning torque due to friction	E_i
steering torque	$T_{\psi i}$
driving torque	$T_{\phi i}$
mass and weight of the wheel	m_w, w_w
radius of the wheel	r
input voltage of the motors	$e_{\psi i}$, $e_{\phi i}$
amplifier constants	k_1', k_1''
motor torque constants	k_2', k_2''
back emf constants of the motors	k_3', k_3''
armature current of the motors	$i_{\psi i}$, $i_{\phi i}$
resistance of the motors	R', R''
viscous-friction coefficients	b', b''

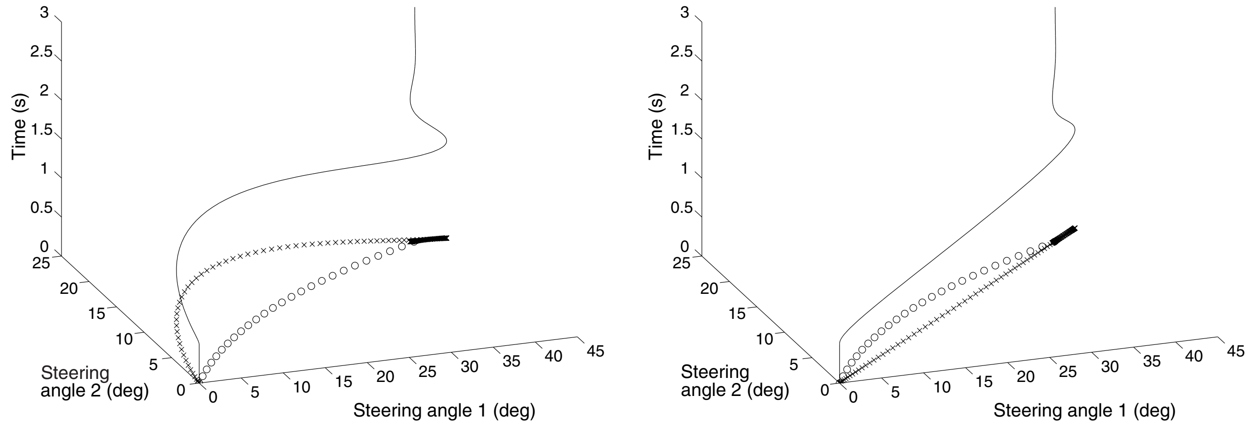
B Generalized mass and force matrices

The elements of matrix of generalized masses $M_{5\times 5}$ are as follows:

$$M_{1,1} = M_{2,2} = m + \sum_i (m_s + m_w),$$

$$M_{1,3} = M_{3,1} = -\sum_i (m_s + m_w) CW_{iy},$$

$$M_{2,3} = M_{3,2} = \sum_i (m_s + m_w) CW_{ix},$$



(a) Uncoupled control. (b) Coupled control.

Figure 3: Steering angles of wheels 1 and 2 under different control schemes. For each subfigure the solid line corresponds to the actual steering angles of the wheels, the x-marked line is the projection of the 3D trajectory represented by the solid line on the 2D space of steering angles (without time), and the o-marked line is the trajectory in the space of steering angles that corresponds to the no-slip steering.

$$
\begin{aligned}
M_{3,3} &= I + \sum_i (m_s + m_w) {CW_i}^2, \\
M_{4,3} &= M_{4,4} = I_s + I_t, \quad M_{5,5} = I_a,
\end{aligned}
$$

and the rest of elements are zeros. The elements of the vector of generalized forces $Q_{5\times 1}$ are as follows:

$$
\begin{aligned}
Q_{1,1} &= m v_y \dot{\theta} + \\
&\quad + \sum_i \left(F_{x\,i} + (m_s + m_w) \left(v_y \dot{\theta} + {CW_i}_x \dot{\theta}^2 \right) \right), \\
Q_{2,1} &= -m v_x \dot{\theta} + \\
&\quad + \sum_i \left(F_{y\,i} - (m_s + m_w) \left(v_x \dot{\theta} + {CW_i}_y \dot{\theta}^2 \right) \right), \\
Q_{3,1} &= -\sum_i \left({T_\psi}_i + ({CW_i}_x {F_y}_i - {CW_i}_y F_{x\,i}) - \right. \\
&\quad \left. - \dot{\theta} \, (m_s + m_w) \left({CW_i}_x v_x + {CW_i}_y v_y \right) \right), \\
Q_{4,1} &= {T_\psi}_i - E_i, \quad Q_{5,1} = {T_\phi}_i - r F_{x'\,i},
\end{aligned}
$$

where steering and driving torques are derived from (23)–(26) with the armature voltage e_ψ, e_ϕ of the steering and driving motors given.

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