Testing 123

10-601 HMM/CRF $\boxed{11/16/16}$

Inference Tasks

$$p(X_1, X_2, X_3) = \frac{1}{Z} q(X_1, X_2, X_3)$$

(2) Partition Function

X_1	X_2	X_3	$q(X_1,X_2,X_3)$
0	0	0	0.1
0	0	1	3
0	1	0	4
1	0	0	3
0	1	1	0.5
1	1	0	0.6
1	0	1	0.4
1	1	1	0.1

$$Z = \sum_{x_1} \sum_{x_2} \sum_{x_3} q(x_1, x_2, x_3)$$

$$\Rightarrow Z = 11.7$$

(1) Marginal Probabilities

$$p(X_1) = \sum_{X_2} \sum_{X_3} p(X_1, X_2, X_3) = \sum_{X_2} \sum_{X_3} \frac{1}{Z} q(X_1, X_2, X_3)$$

$$p(X_2) = \sum_{X_1} \sum_{X_3} p(X_1, X_2, X_3)$$

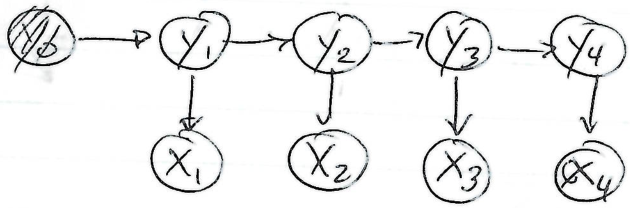
$$p(X_1 = 1) = 4.1$$

(3) Most Probable Assignment (MPA)

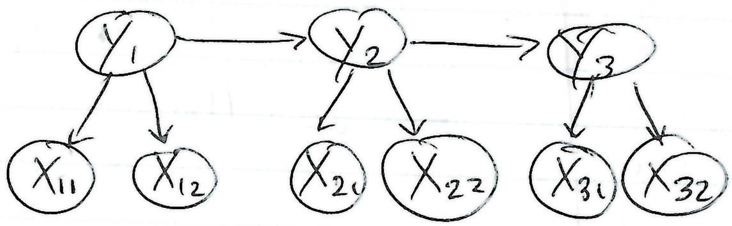
$$\vec{x} = \operatorname*{argmax}_{\vec{x}} p(x_1, x_2, x_3)$$

$$= \operatorname*{argmax}_{\vec{x}} \frac{1}{Z} q(x_1, x_2, x_3) = \operatorname*{argmax}_{\vec{x}} q(x_1, x_2, x_3)$$

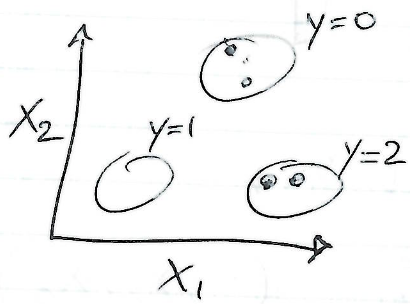
Samples for HMM

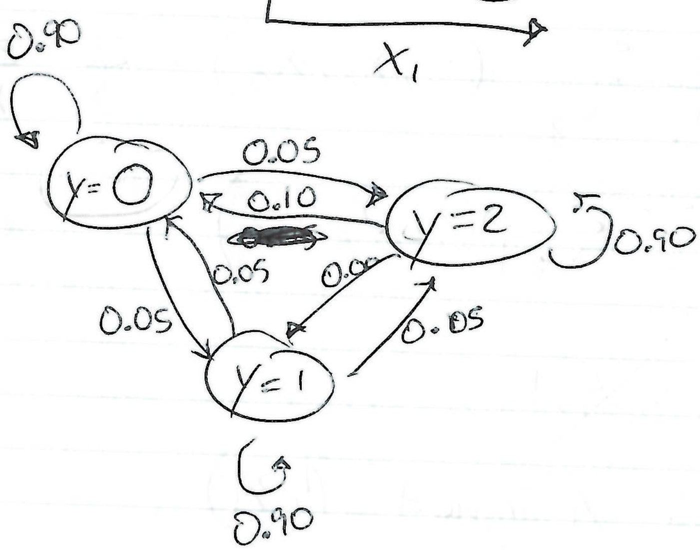


Gaussian NB for Time Series Data



$y \in \{0, 1, 2\}$





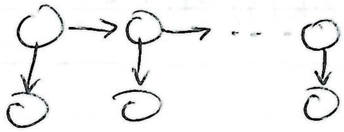
Supervised HMM Learning

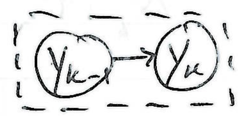
$$D = \{(\vec{x}^{(i)}, \vec{y}^{(i)})\}_{i=1}^{N}$$

$$\vec{\Theta} = (A, B)$$

HMM

$$\Theta^* = \underset{\Theta = A,B}{\operatorname{argmax}} \sum_{i=1}^{N} \log p_{A,B}(x^{(i)}, y^{(i)})$$





$$B^* = \underset{B}{\operatorname{argmax}} \sum_{i=1}^{N} \sum_{k=1}^{K} \log p(y_k^{(i)} \mid y_{k-1}^{(i)}) \quad \text{s.t.} \quad \sum_t B_{s,t} = 1$$

$$A^* = \underset{A}{\operatorname{argmax}} \sum_{i=1}^{N} \sum_{k=1}^{K} \log p(x_k^{(i)} \mid y_k^{(i)}) \quad \text{s.t.} \quad \sum_w A_{t,w} = 1$$

$$B^*_{s,t} = \frac{\sum_{i=1}^{N} \sum_{k=1}^{K} \mathbb{I}(y_k^{(i)} = t,\ y_{k-1}^{(i)} = s)}{\sum_{i=1}^{N} \sum_{k=1}^{K} \mathbb{I}(y_{k-1}^{(i)} = s)}$$

$\cancel{A^*_{t,k} =}$

$$A^*_{t,w} = \frac{\sum_{i=1}^{N} \sum_{k=1}^{K} \mathbb{I}(x_k^{(i)} = w,\ y_k^{(i)} = t)}{\sum_i \sum_k \mathbb{I}(y_k^{(i)} = t)}$$

Forward Algorithm

Define : $\alpha_k(t) \triangleq p(x_1, \dots, x_k, y_k = t)$

$$= \sum_{y_1, \dots, y_{k-1}} p(x_1, \dots, x_k, y_1, \dots, y_{k-1}, y_k = t)$$

Base Case : $\alpha_0(\text{START}) = 1$ $\qquad y_0 = \text{START}$

Recursion : $\alpha_k(t) = \sum_s p(x_k \mid y_k = t)\, \alpha_{k-1}(s)\, p(y_k = t \mid y_{k-1} = s)$

$$\alpha = \begin{array}{cc} & K \\ T & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

compute a single $\alpha_k(t)$ req's $\cancel{K}T$ additions

$$O(K T^2) \text{ - f-b}$$

$$O(\cancel{K} T^K) \text{ - naive giant table}$$