Def Suppose $F: \mathbb{R}^m \to \mathbb{R}$ is C^1.

We say F is <u>positively homogeneous of degree d</u>

if $F(c\vec{x}) = c^d F(\vec{x})$ for all $c > 0$ & $\vec{x} \in \mathbb{R}^m$.

Euler's Th^m

F is positively homogeneous of degree d

iff

$$\vec{x} \cdot \nabla F(\vec{x}) = d F(\vec{x}) \quad \text{for all } \vec{x} \in \mathbb{R}^m.$$

Application

Kinetic energy is homogeneous of degree 2 in its velocity variables.

Thus $$\sum_j \frac{\partial T}{\partial \dot{q}_j} \dot{q}_j = 2T.$$

$$\frac{\partial T}{\partial \dot{q}_j} = \frac{\partial L}{\partial \dot{q}_j} = p_j$$

So $$\sum_j p_j \dot{q}_j = 2T.$$