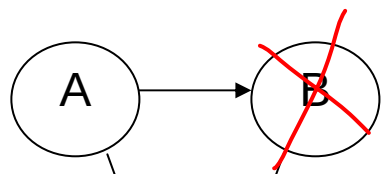


10708 Recitation

Variable Elimination

October 12, 2006

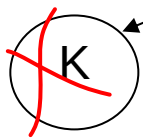
T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

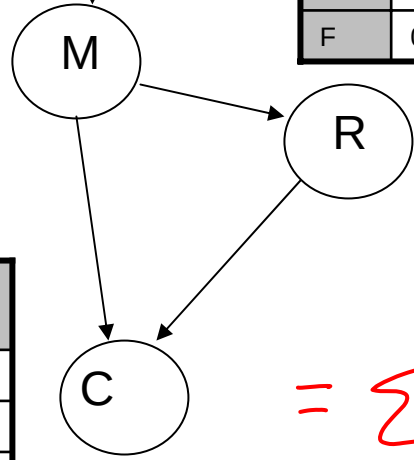
K G	T	F
T	0.7	0.3
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9



M G	T	F
T	0.6	0.4
F	0.5	0.5

R M	T	F
T	0.7	0.3
F	0.1	0.9



C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

$$P(C=T|G=T) \propto P(C=T, G=T)$$

$$= \sum_{a,b,k,m,r} P(a,b,G=T,k,m,r,c=T)$$

2^5

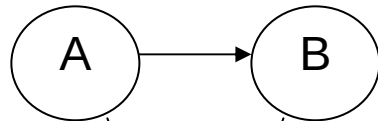
$$= \sum_{\substack{a,b,k, \\ m,r}} P(a)P(b|a)P(G=T|a,b)P(k|G=T)P(m|G=T)P(C=T|m,r)P(r|m)$$

$$= \sum_{a,b,m,r} P(a)P(b|a)P(G=T|a,b)P(m|G=T)P(C=T|m,r)P(r|m) \sum_k P(k|G=T)$$

elim K



T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

R M	T	F
T	0.7	0.3
F	0.1	0.9

$$= \sum_{a,m,r} P(a) P(m|G=T) P(r|m) p(c=T|m,r) \sum_b p(b|a) P(G=T|a,b)$$

$$f_1(a,b) = p(b|a) P(G=T|a,b)$$

	B	$\bar{B}$
A	0.8	0.2
$\bar{A}$	0.6	0.4

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
	0.9	0.7	0.5	0.1

	B	$\bar{B}$
A	0.72	0.14
$\bar{A}$	0.3	0.04

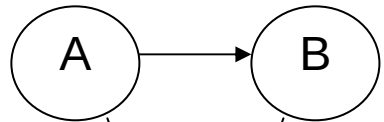
$$g_1(a) = \sum_b f_1(a,b)$$

	0.86
A	0.86
$\bar{A}$	0.34

elim B



T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

R M	T	F
T	0.7	0.3
F	0.1	0.9

K G	T	F
T	0.7	0.3
F	0.6	0.4

M G	T	F
T	0.6	0.4
F	0.5	0.5

C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

$$= \sum_{a,m,r} p(a) p(m|G=T) p(r|m) p(c=T|m,r) g_c(a)$$

$$= \sum_{m,r} p(m|G=T) p(r|m) p(c=T|m,r) \sum_a p(a) g_c(a)$$

$p(a)$

elim A

A	0.1
$\bar{A}$	0.9

 $\rightsquigarrow$

	0.86
	0.34

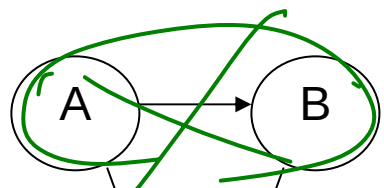
$$f_2(a) = \begin{array}{|c|} \hline .086 \\ \hline .306 \\ \hline \end{array}$$

$$g_2(\cdot) = 0.392$$

$$= 0.392 \sum_{m,r} p(m|G=T) p(r|m) p(c=T|m,r)$$



T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

$$= 0.392 \sum_m P(m | G=T) \sum_r p(r | m) \underline{p(c=T | r, m)}$$

K G	T	F
T	0.7	0.3
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

$$f_3(r, m) =$$

	R	$\bar{R}$
M	0.7	0.3
$\bar{M}$	0.1	0.9

MR	0.7
$M\bar{R}$	0.5
$\bar{M}R$	0.6
$\bar{M}\bar{R}$	0.05

M G	T	F
T	0.6	0.4
F	0.5	0.5

R M	T	F
T	0.7	0.3
F	0.1	0.9

	R	$\bar{R}$
M	0.49	0.3 · 0.5
$\bar{M}$	0.1 · 0.6	0.9 · 0.05

$$g_3(m) = \sum_r f_3(r, m)$$

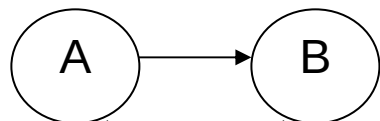
	0.64
$\bar{M}$	0.105

$P(M|G=T)$

elim R

C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

R M	T	F
T	0.7	0.3
F	0.1	0.9

K G	T	F
T	0.7	0.3
F	0.6	0.4

M G	T	F
T	0.6	0.4
F	0.5	0.5

C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

$$= \underbrace{0.392}_{m} \cdot P(m|G=T) g_3(m)$$

m	0.6
$\bar{m}$	0.4

m	0.64
$\bar{m}$	0.105

$$= 0.426$$

$$= 0.392 \cdot 0.426 = 0.167$$

$$P(C=T, G=T) = 0.167$$

$$P(C=F, G=T) = 0.225$$

$$f_3' = p(r|m)p(C=F|r,m)$$

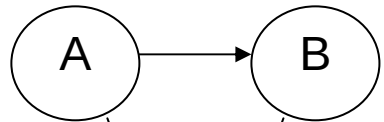
$$g_3'(m) = \sum_r f_3'(m,r) = m$$

m	0.36
$\bar{m}$	0.895

$$= \frac{0.167}{0.167 + 0.225} = 0.426$$



T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

K G	T	F
T	0.7	0.3
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

M G	T	F
T	0.6	0.4
F	0.5	0.5

R M	T	F
T	0.7	0.3
F	0.1	0.9

C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

$$0.392 = \sum_{a,b} P(a) P(b|a) P(G=T|a,b)$$

$$= \sum_{a,b} P(a,b) P(G=T|a,b)$$

$$= \sum_{a,b} P(G=T, a, b)$$

$$= P(G=T)$$

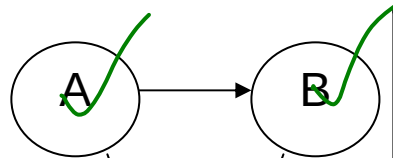
$$P(C=T|G=T) \text{ } P(C=T, G=T)$$

$$\text{Since } P(C=T|G=T) = \frac{P(C=T, G=T)}{P(G=T)},$$

if we had dropped the 0.392 in our calculation, we'd have been done!



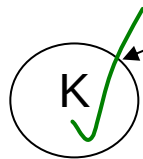
T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

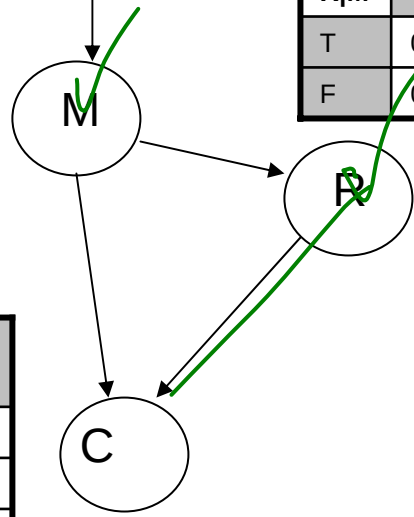
K G	T	F
T	0.7	0.3
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9



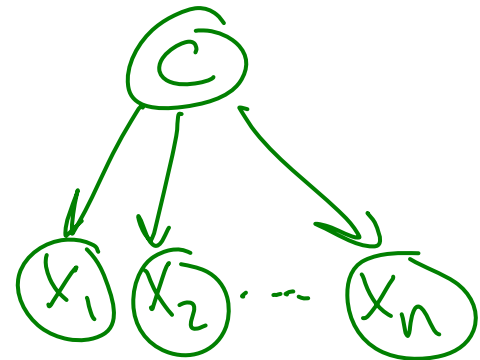
M G	T	F
T	0.6	0.4
F	0.5	0.5

R M	T	F
T	0.7	0.3
F	0.1	0.9

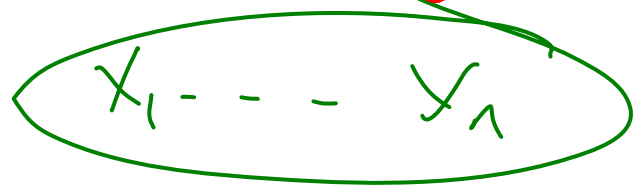


C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

K, B, A, R, M



BAD
naive Bayes
elim order



Variable elimination algorithm

- Given a BN and a query $P(X|e) \propto P(X,e)$
- Instantiate evidence e
- Prune non-active vars for $\{X, e\}$
- Choose an ordering on variables, e.g., $X_1, \dots, X_n$
- Initial *factors* $\{f_1, \dots, f_n\}$: $f_i = P(X_i | \mathbf{Pa}_{X_i})$ (CPT for X_i)
- For $i = 1$ to n , If $X_i \notin \{X, \mathbf{E}\}$
 - Collect factors $f_1, \dots, f_k$ that include X_i
 - Generate a new factor by eliminating X_i from these factors

IMPORTANT!!!

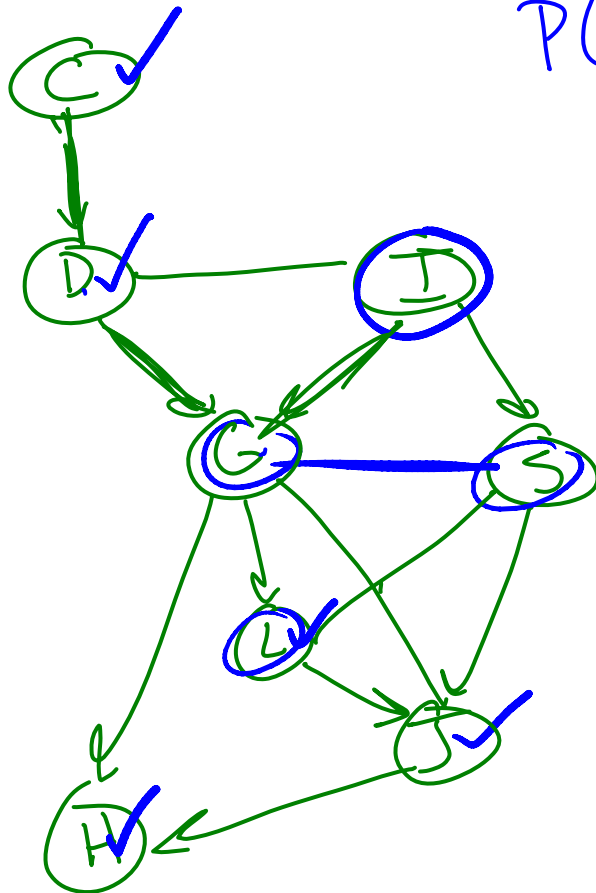
$$g = \sum_{X_i} \prod_{j=1}^k f_j$$

- Variable X_i has been eliminated!
- Normalize $P(X, e)$ to obtain $P(X|e)$

$P(D|C)$

$P(G|I,D) \Rightarrow$

scope =
D, C, G, I



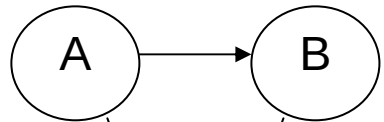
$P(H=\tau)$

C, D, H, J

$\text{argmax}_{x_1, \dots, x_n} P(x_1, \dots, x_n | e)$

2^n

T	F
0.1	0.9



B A	T	F
T	0.8	0.2
F	0.6	0.4

① max $P(K|G) = 0.7$
 a,b,g,m,r,c

K G	T	F
T	0.7	0.3
F	0.6	0.4

G A,B	T	F
TT	0.9	0.1
TF	0.7	0.3
FT	0.5	0.5
FF	0.1	0.9

②
 forward pass
 backward pass

M G	T	F
T	0.6	0.4
F	0.5	0.5

R M	T	F
T	0.7	0.3
F	0.1	0.9

C M,R	T	F
TT	0.7	0.3
TF	0.5	0.5
FT	0.6	0.4
FF	0.05	0.95

MPE $\Rightarrow$
Tomorrow in class