Example <)><

Smelly	Color	Temperature	Goodness
yes	green	cool	bad
no	white	cold	good
yes	red	cool	sellable
yes	red	cold	good
yes	white	cool	good
yes	red	hot	bad

Multinomial Naive Bayes - "Categorical"

Parameters to learn

$$\theta_b \equiv P(Y = \text{bad}) = \frac{\sharp(Y = \text{bad})}{\sharp\,\text{examples}} = \frac{2}{6} = \frac{1}{3}$$

$$\theta_s = P(Y = \text{sellable}) = \frac{1}{6}$$

$$\theta_g = P(Y = \text{good}) = \frac{3}{6} = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}$$

$$P(\text{Smelly} = \text{yes} \mid Y = \text{bad}) = \frac{2}{2} = 1$$

$$P(\text{Smelly} = \text{no} \mid Y = \text{bad}) = 0$$

$$P(\text{Smelly} = \text{yes} \mid Y = \text{sellable}) = 1$$

$$P(\text{Smelly} = \text{no} \mid Y = \text{sellable}) = 0$$

$$P(\text{Smelly} = \text{yes} \mid Y = \text{good}) = \frac{2}{3}$$

$$P(\text{Smelly} = \text{no} \mid Y = \text{good}) = \frac{1}{3}$$

Summary:

□	bad	sellable	good
P (Y)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

Smelly	bad	sellable	good
yes	1	1	$\frac{2}{3}$
no	0	0	$\frac{1}{3}$

Color	bad	sellable	good
green	$\frac{1}{2}$	0	0
red	$\frac{1}{2}$	1	$\frac{1}{3}$
white	0	0	$\frac{2}{3}$

Temp	bad	sellable	good
cold	0	0	$\frac{2}{3}$
cool	$\frac{1}{2}$	1	$\frac{1}{3}$
hot	$\frac{1}{2}$	0	0

"testing"

f_1: Smelly = yes, Color = red, Temperature = cool

$$P(Y = \text{good} \mid X) = \frac{P(X \mid Y = \text{good})\, P(Y = \text{good})}{P(X)}$$

$$P(Y = \text{good} \mid X) =$$

$$\frac{1}{P(X)} P(\text{Smelly} = \text{yes} \mid Y = \text{good})\, P(\text{Color} = \text{red} \mid Y = \text{good})\, P(\text{Temp} = \text{cool} \mid Y = \text{good})\, P(Y = \text{good})$$

$$= \frac{1}{P(X)} \frac{2}{3} \frac{1}{3} \frac{1}{3} \frac{1}{2} = \mathbf{\frac{1}{27} \frac{1}{P(X)}}$$

$$P(Y = \text{bad} \mid X) =$$

$$\frac{1}{P(X)} P(\text{Smelly} = \text{yes} \mid Y = \text{bad})\, P(\text{Color} = \text{red} \mid Y = \text{bad})\, P(\text{Temp} = \text{cool} \mid Y = \text{bad})\, P(Y = \text{bad})$$

$$= 1 \times \frac{1}{2} \frac{1}{2} \frac{1}{3} = \mathbf{\frac{1}{12} \frac{1}{P(X)}}$$

$$= \frac{1}{P(X)}$$

$$P(Y = \text{sellable} \mid X) = \mathbf{\frac{1}{P(X)} \frac{1}{6}}$$

$$P(X) = \sum_{y=1}^{k} P(X, Y = k)$$

This fish is Sellable despite being Smelly

f_2: Smelly = yes, Color = white, Temperature = cool

Recall Summary:

□	bad	sellable	good
P (Y)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{2}$

Smelly	bad	sellable	good
yes	1	1	$\frac{2}{3}$
no	0	0	$\frac{1}{3}$

Color	bad	sellable	good
green	$\frac{1}{2}$	0	0
red	$\frac{1}{2}$	1	$\frac{1}{3}$
white	0	0	$\frac{2}{3}$

Temp	bad	sellable	good
cold	0	0	$\frac{2}{3}$
cool	$\frac{1}{2}$	1	$\frac{1}{3}$
hot	$\frac{1}{2}$	0	0

$$P(Y = \text{bad}, X)$$

$$= P(\text{Smelly} = \text{yes} \mid Y = \text{bad})\, P(\text{Color} = \text{white} \mid Y = \text{bad})\, P(\text{Temp} = \text{cool} \mid Y = \text{bad})\, P(Y = \text{bad})$$

$$= 1 \times 0$$

Laplace smoothing

$$\theta \equiv P(Y = \text{bad}) = \frac{\#(Y = \text{bad}) + 1}{\#(\text{examples}) + 3} = \frac{\square}{\square}$$

$$P(\text{Color} = \text{white} \mid Y = \text{bad}) = \frac{\#(Y = \text{bad},\ \text{Color} = \text{white}) + 1}{\#(Y = \text{bad}) + \#(\text{bad fish hallucinated for all Colors})} = \frac{\square}{\square}$$

no zeros!

Gaussian Naive Bayes

Parameters to learn

$$\theta \equiv P(Y = 1) = \text{same way as for Mult. NB}$$

$$P(\text{Smelly} \mid Y = \text{bad}) \sim N(\mu_{01},\ \sigma_{01})$$

$$\mu_{01} = \frac{1}{\#(Y = \text{bad})} \sum_{bad\ \text{examples}} \text{value of Smelly}$$

What's the problem?

Smelly: Binary → {0,1}

Temperature: can get the numeric values → Gaussian makes sense since closer numbers are more "similar"

Color: green=0, red=1, white=2?

doesn't really make sense... can still model as multinomial and combine with the gaussian features! (why not? ← rhetorical)