These reference rules depart from the lecture notes in two minor points: (1) typing of cells is simplified by reducing it to the typing of processes that would write such a cell, and (2) in the dynamics we do not use persistent cells, but explicitly carry ephemeral cells from the left-hand side to the right-hand side of each rule that reads from memory.

Abstract Syntax

$$
\begin{array}{llll}
\text{Types} & \tau & ::= & \tau_1 \to \tau_2 \mid \&_{i \in I}(i : \tau_i) \mid \tau_1 \times \tau_2 \mid 1 \mid \sum_{i \in I}(i : \tau_i) \mid \rho\alpha.\, \tau \\
\text{Contexts} & \Gamma & ::= & \cdot \mid \Gamma, x : \tau \qquad \text{(all variables distinct)} \\
\text{Processes} & P & ::= & x \leftarrow P \,;\, Q \qquad \text{allocate/spawn} \\
 & & \mid & x^w \leftarrow y^R \qquad \text{copy} \\
 & & \mid & x^W.V \mid \mathsf{case}\ x^R\ K \qquad (1, \times, +, \rho) \\
 & & \mid & x^R.V \mid \mathsf{case}\ x^W\ K \qquad (\to, \&)
\end{array}
$$

$$
\begin{array}{llll}
\text{Small values} & V & ::= & \langle\,\rangle \mid \langle a_1, a_2 \rangle \mid i \cdot a \mid \mathsf{fold}\ a \\
\text{Continuations} & K & ::= & (\langle\,\rangle \Rightarrow P) \mid (\langle x_1, x_2 \rangle \Rightarrow P) \mid (i \cdot x_i \Rightarrow P_i)_{i \in I} \mid (\mathsf{fold}\ x \Rightarrow P) \\
\text{Cell contents} & W & ::= & V \mid K \\
\text{Configurations} & \mathcal{C} & ::= & \cdot \mid \mathcal{C}_1, \mathcal{C}_2 \mid \mathsf{proc}\ d\ P \mid \mathsf{cell}\ c\ W
\end{array}
$$

Judgments

$$
\begin{array}{ll}
\Gamma \vdash P :: (z : \sigma) & \text{process } P \text{ reads from } \Gamma \text{ and writes to } z : \sigma \\
\Gamma \vdash \mathcal{C} :: \Delta & \text{configuration } \mathcal{C} \text{ reads from } \Gamma \text{ and writes to } \Delta \\
\Gamma \vdash \mathcal{C}\ \textit{final} & \text{configuration } \mathcal{C} \text{ is final (consists only of cells)}
\end{array}
$$

Theorems

Preservation. If $\Gamma \vdash \mathcal{C} :: \Delta$ and $\mathcal{C} \mapsto \mathcal{C}'$ then $\Gamma \vdash \mathcal{C}' :: \Delta'$ for some $\Delta' \supseteq \Delta$.

Progress. If $\cdot \vdash \mathcal{C} :: \Delta$ then either $\mathcal{C} \mapsto \mathcal{C}'$ for some $\mathcal{C}'$ or $\mathcal{C}$ *final*.

Statics, Allocate and Copy

$$
\frac{\Gamma \vdash P :: (x : \tau) \quad \Gamma, x : \tau \vdash Q :: (z : \sigma)}{\Gamma \vdash (x \leftarrow P \,;\, Q) :: (z : \sigma)}\ \mathsf{tp/alloc}
\qquad
\frac{y : \tau \in \Gamma}{\Gamma \vdash x \leftarrow y :: (x : \tau)}\ \mathsf{tp/copy}
$$

Statics, Positive Types

$$\frac{}{\Gamma \vdash x^W.\langle\,\rangle :: (x : 1)}\ \text{w/unit} \qquad \frac{x : 1 \in \Gamma \quad \Gamma \vdash P :: (z : \sigma)}{\Gamma \vdash \mathsf{case}\ x^R\ (\langle\,\rangle \Rightarrow P) :: (z : \sigma)}\ \text{r/unit}$$

$$\frac{y : \tau \in \Gamma \qquad z : \sigma \in \Gamma}{\Gamma \vdash x^W.\langle y, z\rangle :: (x : \tau \times \sigma)}\ \text{w/pair} \qquad \frac{x : \tau_1 \times \tau_2 \in \Gamma \quad \Gamma, x_1 : \tau_1, x_2 : \tau_2 \vdash P :: (z : \sigma)}{\Gamma \vdash \mathsf{case}\ x^R\ (\langle x_1, x_2\rangle \Rightarrow P) : (z : \sigma)}\ \text{r/pair}$$

$$\frac{(j \in I) \quad y : \tau_j \in \Gamma}{\Gamma \vdash x^W.(j \cdot y) :: (x : \sum_{i \in I}(i : \tau_i))}\ \text{w/tag} \qquad \frac{x : \sum_{i \in I}(i : \tau_i) \in \Gamma \quad \Gamma, y : \tau_i \vdash P_i :: (z : \sigma)\ (\forall i \in I)}{\Gamma \vdash \mathsf{case}\ x^R\ (i \cdot y \Rightarrow P_i)_{i \in I} :: (z : \sigma)}\ \text{r/tag}$$

$$\frac{y : [\rho\alpha.\,\tau/\alpha]\tau \in \Gamma}{\Gamma \vdash x^W.(\mathsf{fold}\ y) :: (x : \rho\alpha.\,\tau)}\ \text{w/fold} \qquad \frac{x : \rho\alpha.\,\tau \in \Gamma \quad \Gamma, y : [\rho\alpha.\,\tau/\alpha]\tau \vdash P :: (z : \sigma)}{\Gamma \vdash \mathsf{case}\ x^R\ (\mathsf{fold}\ y \Rightarrow P) :: (z : \sigma)}\ \text{r/fold}$$

Statics, Negative Types

$$\frac{\Gamma, y : \tau \vdash P :: (z : \sigma)}{\Gamma \vdash \mathsf{case}\ x^W\ (\langle y, z\rangle \Rightarrow P) :: (x : \tau \to \sigma)}\ \text{w/fun} \qquad \frac{x : \tau \to \sigma \in \Gamma \quad y : \tau \in \Gamma}{\Gamma \vdash x^R.\langle y, z\rangle :: (z : \sigma)}\ \text{r/fun}$$

$$\frac{\Gamma \vdash P_i :: (z_i : \tau_i) \quad (\text{for all } i \in I)}{\Gamma \vdash \mathsf{case}\ x^W\ (i \cdot z_i \Rightarrow P_i)_{i \in I} :: (x : \&_{i \in I}(i : \tau_i))}\ \text{w/record} \qquad \frac{x : \&_{i \in I}(i : \tau_i) \in \Gamma \quad j \in I}{\Gamma \vdash x^R.(j \cdot z) :: (z : \tau_j)}\ \text{r/record}$$

Statics, Configurations

$$\frac{\Gamma \vdash P :: (c : \tau)}{\Gamma \vdash \mathsf{proc}\ c\ P :: (\Gamma, c : \tau)}\ \text{tp/proc}$$

$$\frac{\Gamma \vdash c^W.V :: (c : \tau)}{\Gamma \vdash \mathsf{cell}\ c\ V :: (\Gamma, c : \tau)}\ \text{tp/cell/val} \qquad \frac{\Gamma \vdash \mathsf{case}\ c^W\ K :: (c : \tau)}{\Gamma \vdash \mathsf{cell}\ c\ K :: (\Gamma, c : \tau)}\ \text{tp/cell/cont}$$

$$\frac{}{\Gamma \vdash (\cdot) :: \Gamma}\ \text{tp/empty} \qquad \frac{\Gamma \vdash \mathcal{C}_1 :: \Gamma_1 \quad \Gamma_1 \vdash \mathcal{C}_2 :: \Gamma_2}{\Gamma \vdash (\mathcal{C}_1, \mathcal{C}_2) :: \Gamma_2}\ \text{tp/join}$$

$$\frac{}{\Gamma \vdash (\cdot)\ \mathit{final}}\ \text{fin/empty} \qquad \frac{\Gamma \vdash \mathcal{C}\ \mathit{final}}{\Gamma \vdash (\mathcal{C}, \mathsf{cell}\ c\ W)\ \mathit{final}}\ \text{fin/cell}$$

Dynamics

$$
\begin{array}{llcll}
 & \mathsf{proc}\ d\ (x \leftarrow P \ ;\ Q) & \mapsto & \mathsf{proc}\ c\ ([c/x]P), \mathsf{proc}\ d\ ([c/x]Q) & \text{(alloc/spawn, } c \text{ fresh)} \\
\mathsf{cell}\ c\ W, & \mathsf{proc}\ d\ (d^W \leftarrow c^R) & \mapsto & \mathsf{cell}\ c\ W, \mathsf{cell}\ d\ W & \text{(copy)} \\
\\
 & \mathsf{proc}\ d\ (d^W.V) & \mapsto & \mathsf{cell}\ d\ V & \text{(write: } \times, 1, +, \rho) \\
\mathsf{cell}\ c\ V, & \mathsf{proc}\ d\ (\mathsf{case}\ c^R\ K) & \mapsto & \mathsf{cell}\ c\ V, \mathsf{proc}\ d\ (V \triangleright K) & \text{(read: } \times, 1, +, \rho) \\
\\
 & \mathsf{proc}\ d\ (\mathsf{case}\ d^W\ K) & \mapsto & \mathsf{cell}\ d\ K & \text{(write: } \rightarrow, \&) \\
\mathsf{cell}\ c\ K, & \mathsf{proc}\ d\ (c^R.V) & \mapsto & \mathsf{cell}\ c\ K, \mathsf{proc}\ d\ (V \triangleright K) & \text{(read: } \rightarrow, \&)
\end{array}
$$

$$
\begin{array}{rclcl}
\langle\,\rangle & \triangleright & (\langle\,\rangle \Rightarrow P) & = & P \\
\langle c_1, c_2 \rangle & \triangleright & (\langle x_1, x_2 \rangle \Rightarrow P) & = & [c_1/x_1, c_2/x_2]P \\
j \cdot c & \triangleright & (i \cdot x_i \Rightarrow P_i)_{i \in I} & = & [c/x_j]P_j \\
\mathsf{fold}\ c & \triangleright & (\mathsf{fold}\ x \Rightarrow P) & = & [c/x]P
\end{array}
$$