Constructive Logic (15-317), Fall 2017
Assignment 2: Proofs as Programs, Harmony

Due Tuesday, September 19, 2017

You will be using Tutch for the majority of this assignment, but there is also a written portion. This assignment is due at the beginning of class on the above date and must be submitted electronically via autolab. Submit your homework as a **tar** archive containing seven files: **hw2.pdf** (your written solutions) and **hw2_task1{a, b, c}.tut** and **hw2_task2{a, b, c}.tut** (your Tutch solutions).

1 Tutch Proofs

Tutch allows you to annotate your proof with proof terms by declaring it with `annotated proof`. An annotated proof is just like a regular Tutch proof, but each line, normally `A`, is annotated with the term that justifies it, making it `M : A`.

```
annotated proof andComm : A & B => B & A =
begin
[ u : A & B;
  snd u : B;
  fst u : A;
  (snd u, fst u) : B & A];
fn u => (snd u, fst u) : A & B => B & A
end;
```

It is also possible to simply give a program. To give a program as a proof in Tutch, declare it with `term` rather than `proof`:

```
term andComm : A & B => B & A =
  fn u => (snd u, fst u);
```

The syntax for terms is, coincidentally, precisely the syntax that we have used for annotating proof terms in lecture.

For more examples, see Chapter 4 of the Tutch User's Guide. The proof terms are very similar to the ones given in lecture and are summarized in Section A.2.1 of the Guide.

Task 1 (6 points). Give annotated proofs for the following theorems using Tutch.

   a. `annotated proof oroverand : (A | (B & C)) => ((A | B) & (A | C));`

   b. `annotated proof choose : ((A => B) | (A => C)) => A => (B | C);`

   c. `annotated proof split : ((A | B) => C) => (A => C) & (B => C);`

Task 2 (6 points). Give proof terms for the following theorems using Tutch.

   a. `term smap : A => (A => B) => B;`

   b. `term exception : (A | B) => ~B => A;`

   c. `term split : (A | B => C) => (A => C) & (B => C);`

On a machine with Tutch installed, you can check your progress against a particular requirements file by running

   `$ tutch -r ./hw2_task1a.req hw2_task1a.tut`

Substituting 1a for the appropriate task number and letter to denote the problem.

2 Harmony

Task 3 (10 points). Consider a connective $\bowtie$ defined by the following rules:

$$\overline{A\ true}\ u$$
$$\vdots$$
$$\frac{A\ true \quad B\ true}{A \bowtie B\ true} \bowtie I^u \qquad \frac{A \bowtie B\ true}{B\ true} \bowtie E$$

1. Is this connective locally sound? If so, show the reduction; if not, explain (informally) why no such reduction exists.

2. Is this connective locally complete? If so, give an appropriate local expansion; otherwise, explain (informally) why no such expansion exists.

Task 4 (10 points). Consider a connective $\odot$ with the following elimination rules:

$$\overline{A\ true}\ u \qquad \overline{B\ true}\ v$$
$$\vdots$$
$$\frac{A \odot B\ true \qquad C\ true}{C\ true} \odot E^{u,v}$$

2

(Normally we take the verificationist perspective that introduction rules come first, but this time we'll go in the opposite direction.)

1. Come up with a set of zero or more introduction rules for this connective.

2. Show that the connective is locally sound and complete for your choice of introduction rules.

3. Is it possible to come up with a notational definition $A \odot B \triangleq$ _____ so that both your defined introduction rule(s) as well as the elimination rule given above are merely derived rules? You needn't prove that this fact, merely state yes or no. However, partial credit may be awarded for partially correct arguments.

3 All the things you can do with a $\diamond$

Consider the $\diamond$ connective.

$$\dfrac{\dfrac{\overline{A\ true}\ ^u}{\vdots}\ }{\dfrac{B\ true}{\diamond(A,B,C)\ true}}\ \diamond I_1^u \qquad \dfrac{\dfrac{\overline{A\ true}\ ^u}{\vdots}\ }{\dfrac{C\ true}{\diamond(A,B,C)\ true}}\ \diamond I_2^u \qquad \dfrac{\diamond(A,B,C)\ true \quad A\ true \quad \dfrac{\overline{B\ true}\ ^u}{\vdots}\ D\ true \quad \dfrac{\overline{C\ true}\ ^v}{\vdots}\ D\ true}{D\ true}\ \diamond E^{u,v}$$

Task 5 (4 points). Give a proof term assignment for the rules.

Task 6 (4 points). Show all the local reduction(s) and expansion(s) for these rules (proving local soundness and completeness) in proof term notation. Be sure to indicate which are reductions and which are expansions.

Submitting your solutions

Please generate a tarball containing your solution files by running

```
$ tar cf hw2.tar hw2.pdf hw2_task1a.tut hw2_task1b.tut \
        hw2_task1c.tut hw2_task2a.tut hw2_task2b.tut hw2_task2c.tut
```

and submit the resulting `hw2.tar` file to Autolab.