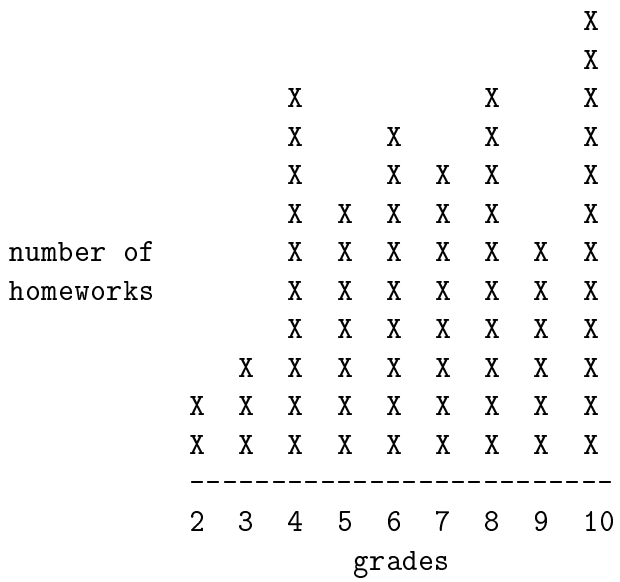
Automata Theory: Solutions 6



Problem 1
Give a context-free grammar for each of the following languages:

(a) $\{a^n b^{2n} : \ n \geq 0\}$
(b) $\{a^n b^m c^{n+m} : \ n, m \geq 0\}$
(c) $\{a^n b^m : \ 2n \leq m \leq 3n\}$

(a) $S \to aSbb \mid \lambda$

(b) $S \to aSc \mid A$
$A \to bAc \mid \lambda$

(c) $S \to aSbb \mid aSbbb \mid \lambda$

Problem 2
Consider the following grammar:

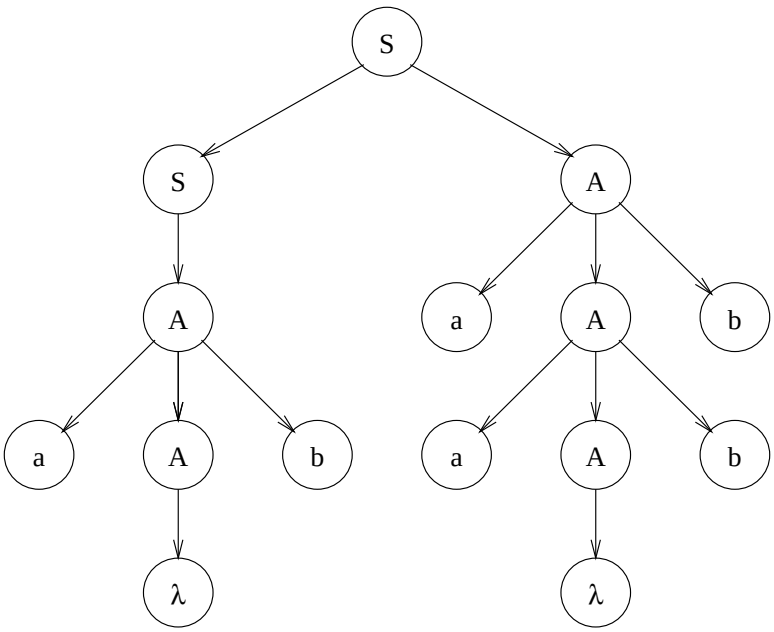
$S \to SA \mid A$
$A \to aAb \mid \lambda$

Show a left-most derivation, right-most derivation, and derivation tree for the string $abaabb$.

Left-most derivation:
$S \Rightarrow SA \Rightarrow AA \Rightarrow aAbA \Rightarrow abA \Rightarrow abaAb \Rightarrow abaaAbb \Rightarrow abaabb$

Right-most derivation:
$S \Rightarrow SA \Rightarrow SaAb \Rightarrow SaaAbb \Rightarrow Saabb \Rightarrow Aaabb \Rightarrow aAbaabb \Rightarrow abaabb$

Derivation tree:



Problem 3
Find *simple* grammars (a.k.a. s-grammars) for the following languages:

(a) $\{a^n b : \ n = 0 \text{ or } n \geq 3\}$
(b) $\{a^n b^n : \ n \geq 1\}$

(a) $S \to aA \mid b$
$A \to aB$
$B \to aC$
$C \to aC \mid b$

(b) $S \to aA$
$A \to aAB \mid b$
$B \to b$