Automata Theory: Solutions 1

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  number of                                 X
  homeworks                     X           X
                                X     X     X
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  X                 X     X     X     X     X
  X                 X     X     X     X     X
------------------------------------------------
2.5-3 3.5-4 4.5-5 5.5-6 6.5-7 7.5-8 8.5-9 9.5-10
                      grades
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This histogram shows the distribution of grades (from 0 to 10) for the homeworks submitted on time; it does not include the late submissions.

Problem 1
Consider the following sets of integer numbers:

$S_1 = \{4, 5, 6\}$
$S_2 = \{i : \ i \text{ is even}\}$
$S_3 = \{i : \ i \text{ is divisible by } 3\}$

For each set below, specify its elements and determine whether it is finite or infinite:

$S_4 = S_1 \times S_1 = \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$
$S_5 = 2^{S_1} = \{\emptyset, \{4\}, \{5\}, \{6\}, \{4,5\}, \{4,6\}, \{5,6\}, \{4,5,6\}\}$
$S_6 = S_1 \cap S_2 = \{4, 6\}$
$S_7 = S_2 \cap S_3 = \{i : \ i \text{ is divisible by } 6\}$

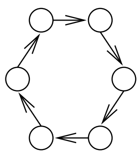
The sets S_4, S_5, and S_6 are finite, whereas S_7 is infinite.

Problem 2
Show that, if $S_1 \subseteq S_2$, then $\overline{S_2} \subseteq \overline{S_1}$.

By the definition of subsets, we need to show that every element x of $\overline{S_2}$ is also an element of $\overline{S_1}$. The proof is as follows:

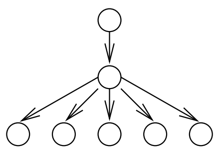
$$\begin{aligned} x \in \overline{S_2} \quad &\Rightarrow \quad x \notin S_2 \quad \text{(by definition of } \overline{S_2}\text{)} \\ &\Rightarrow \quad x \notin S_1 \quad \text{(because } S_1 \subseteq S_2\text{)} \\ &\Rightarrow \quad x \in \overline{S_1} \quad \text{(by definition of } \overline{S_1}\text{)} \end{aligned}$$

Problem 3
(a) Draw an example of a graph that has six vertices and six edges. Mark all simple cycles in your graph.



This graph has one simple cycle; other examples may have a different number of cycles.

(b) Draw an example of a tree that has seven vertices, five of which are leaves. How many edges are in your tree?



A tree with n vertices always has $n-1$ edges; in particular, a seven-vertex tree has six edges.