Automata Theory: Assignment 2

Due date: September 9 (Thursday)

Problem 1 (3 points)
Prove the following equality by induction:

$$1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + ... + n)^2.$$

Problem 2 (3 points)
Consider the following two languages on the alphabet $\Sigma = \{a, b\}$:

$$L_1 = \{a^n : \ n \geq 1\}$$
$$L_2 = \{b^n : \ n \geq 1\}$$

Describe the languages below, using either the set notation or precise definitions in English:

$$L_3 = L_1^*$$
$$L_4 = \overline{L_1}$$
$$L_5 = L_1 \cup L_2$$
$$L_6 = L_1 L_2$$
$$L_7 = ({L_1}^2)({L_2}^2)({L_1}^2)$$
$$L_8 = (L_1 \cup L_2)^*$$
$$L_9 = (L_1 L_2)^*$$

Problem 3 (4 points)
Consider the alphabet $\Sigma = \{a, b\}$. Is there any language L on this alphabet for which $(\overline{L})^* = \overline{L^*}$? If yes, give an example of such a language; if no, explain why.