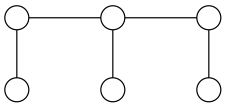
Automata Theory: Solutions 1

```
                            X
                            X
                            X
                            X
                            X  X
                            X  X
                            X  X
                      X     X  X
number of             X  X  X  X
homeworks       X     X  X  X  X
                X     X  X  X  X
                X     X  X  X  X
       X     X  X     X  X  X  X
       --------------------------
       2  3  4  5  6  7  8  9  10
                 grades
```

The histogram shows the distribution of grades.

Problem 1
Draw an example of an undirected connected acyclic graph with six vertices.



If an undirected acyclic graph has six vertices, it has five edges.

Problem 2
Prove that, if $S_1 \subseteq S_2$ and $S_3 \subseteq S_4$, then $(S_1 \cap S_3) \subseteq (S_2 \cap S_4)$.

By the definition of subsets, we need to show that every element x of $S_1 \cap S_3$ is also an element of $S_2 \cap S_4$. The proof is as follows:

$$\begin{aligned} x \in S_1 \cap S_3 &\Rightarrow x \in S_1 \text{ and } x \in S_3 \quad \text{(by definition of intersection)} \\ &\Rightarrow x \in S_2 \text{ and } x \in S_4 \quad \text{(because } S_1 \subseteq S_2 \text{ and } S_3 \subseteq S_4\text{)} \\ &\Rightarrow x \in S_2 \cap S_4 \quad \text{(by definition of set intersection)} \end{aligned}$$

Problem 3
Prove that the following results hold for every natural number n.

(a) $1+3+5+7+...+(2\cdot n-1)=n^2$

We use a proof by induction. Clearly, the equality holds for $n=1$; we now show that, if it holds for n, then it also holds for $n+1$:

$$\begin{aligned} 1+3+...+(2\cdot n-1)+(2\cdot(n+1)-1) &= (1+3+...+(2\cdot n-1))+(2\cdot n+1) \\ &= n^2+2\cdot n+1 \\ &= (n+1)^2 \end{aligned}$$

(b) $\frac{n}{n+1}<\frac{n+1}{n+2}$

We simplify the inequality as follows:

$$\begin{aligned} \frac{n}{n+1}<\frac{n+1}{n+2} &\Leftrightarrow n\cdot(n+2)<(n+1)^2 \\ &\Leftrightarrow n^2+2\cdot n<n^2+2\cdot n+1 \\ &\Leftrightarrow 0<1 \end{aligned}$$