Automata Theory: Assignment 2

Due date: January 17 (Thursday)

Problem 1 (3 points)
Consider the following sets of integer numbers:

$S_1 = \{1, 2, 3\}$
$S_2 = \{i : \; i \text{ is odd}\}$
$S_3 = \{i : \; i \text{ is divisible by } 3\}$

For each set below, specify its elements and determine whether it is finite or infinite:

$S_4 = 2^{S_1}$
$S_5 = 2^{S_1} \cap 2^{S_2}$
$S_6 = S_3 - S_2$

Problem 2 (3 points)
Prove that, if $S_1 \subseteq S_2$, then $2^{S_1} \subseteq 2^{S_2}$.

Problem 3 (4 points)
Prove the following equality by induction:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n \cdot (n+1) \cdot (2 \cdot n+1)}{6}.$$