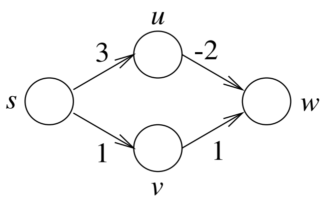
Analysis of Algorithms: Solutions 8

```
                X
                X
                X
                X
                X
                X           X
                X           X
                X           X
                X           X
                X           X
number of       X           X
homeworks       X           X
                X           X
                X           X
                X           X
                X           X
             X  X           X
             X  X           X
       X     X  X        X  X
    X  X  X  X  X     X  X  X
    --------------------------
    2  3  4  5  6  7  8  9  10
              grades
```

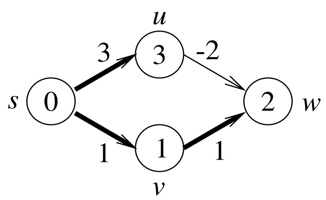
Problem 1
Using Figure 25.5 in the textbook as a model, illustrate the steps of Dijkstra's algorithm on the graph of Figure 25.2, with vertex y as the source.

The order of processing the vertices is as follows: y, s, u, v, x.

Problem 2
Give an example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers.



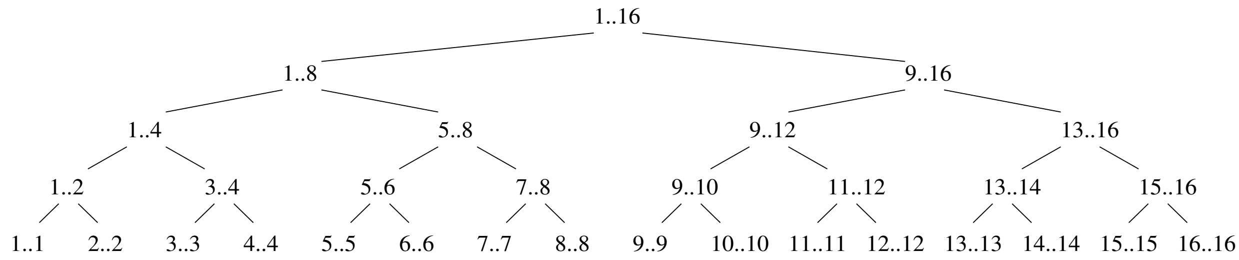
(A)



(B)

If we apply Dijkstra's algorithm to the graph in Figure A, with vertex s as the source, then it processes the vertices in the following order: s, v, w, u. Thus, it constructs the "shortest-paths" tree shown in Figure B, where the path from s to w is *not* shortest.

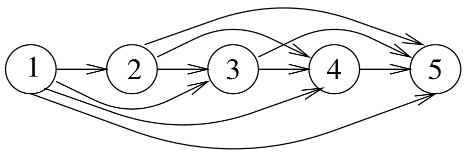
Problem 3
Using Figure 16.2 in the textbook as a model, draw the recursion tree for the MERGE-SORT procedure on a sixteen-element array. Explain why dynamic programming is ineffective for speeding up MERGE-SORT.



The MERGE-SORT procedure does *not* have overlapping subproblems, that is, all nodes of the recursion tree are distinct. We cannot re-use the results of recursive calls; hence, dynamic programming does not improve the efficiency.

Problem 4
What is the maximal possible number of edges in a directed acyclic graph with V vertices?

The maximal number of edges is $\frac{V\cdot(V-1)}{2}$. To construct a graph with that many edges, we enumerate its vertices from 1 to V, and put an edge from every vertex to every higher-number vertex. That is, the graph includes an edge (i, j) if and only if $i < j$. For example, if $V = 5$, then the graph is as follows:



To prove that this number is maximal, we observe that, for every two vertices i and j, an acyclic graph may include either the edge (i, j) or (j, i), but not both. Thus, the total number of edges is no greater than the number of vertex pairs, which is $\frac{V\cdot(V-1)}{2}$.