Analysis of Algorithms: Solutions 6

```
                                X
                                X
                                X
                                X
                       X     X  X
                       X     X  X
                       X     X  X
number of              X     X  X
homeworks              X     X  X
                    X  X     X  X
                    X  X  X  X  X
              X  X  X  X  X  X  X
        X  X  X  X  X  X  X  X  X
        --------------------------
        2  3  4  5  6  7  8  9  10
                  grades
```

Problem 1
Write a *nonrecursive* version of FIND-SET with path compression, for disjoint-set forests.

FIND-SET(x)
$z \leftarrow x$
while $z \neq parent[z]$ ▷ find the root
    do $z \leftarrow parent[z]$
while $x \neq z$ ▷ set the parent pointers to the root z
    do $y \leftarrow parent[x]$
        $parent[x] \leftarrow z$
        $x \leftarrow y$
return z

Problem 2
Write efficient algorithms for converting (a) an adjacency-list representation of a graph into an adjacency matrix and (b) an adjacency matrix into adjacency lists.

We denote the adjacency list of a vertex u by *Adj-List*$[u]$, and the adjacency-matrix element for vertices u and v by *Adj-Matrix*$[u, v]$. The time complexity of both algorithms is $\Theta(V^2)$.

(a) Converting adjacency lists into a matrix.

```
LISTS-TO-MATRIX(G)        ▷ G is represented by adjacency lists
for each u ∈ V[G]
    do for each v ∈ V[G]
            do Adj-Matrix[u, v] ← 0
for each u ∈ V[G]
    do for each v ∈ Adj-List[u]
            do Adj-Matrix[u, v] ← 1
```

(b) Converting an adjacency matrix into lists.

```
MATRIX-TO-LISTS(G)        ▷ G is represented by an adjacency matrix
for each u ∈ V[G]
    do initialize an empty list Adj-List[u]
for each u ∈ V[G]
    do for each v ∈ V[G]
            do if Adj-Matrix[u, v] = 1
                    then add v to Adj-List[u]
```

Problem 3
Using Figure 23.3 in the textbook as a model, illustrate the steps of breadth-first search on the directed graph of Figure 23.2(a), with vertex 3 as the source.

The order of painting the vertices is as follows:

gray 3	black 5
gray 5	black 6
gray 6	gray 2
black 3	black 4
gray 4	black 2