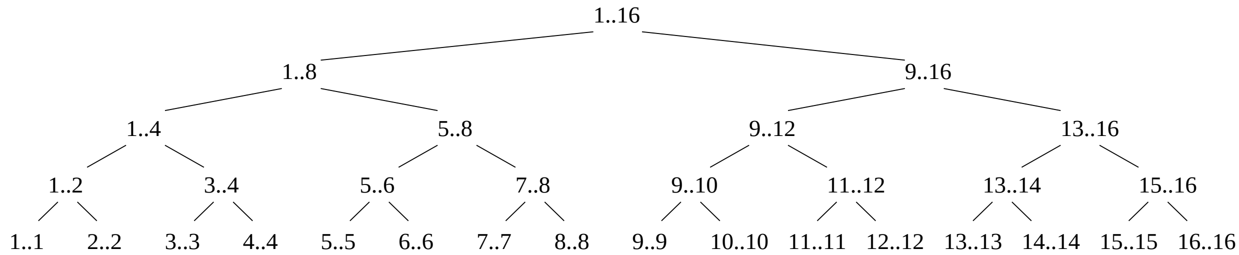
Analysis of Algorithms: Solutions 8

```
                                X
                                X
                                X
                                X
                                X
                                X
                          X     X
number of                 X     X
homeworks      X  X     X X  X  X
               X  X     X X  X  X
               X  X  X  X X  X  X
            X  X  X  X  X X  X  X  X
            -------------------------
            1  2  3  4  5  6  7  8  9
```

The histogram shows the distribution of grades for the homeworks submitted on time.

Problem 1
Using Figure 16.2 in the textbook as a model, draw the recursion tree for the MERGE-SORT procedure on a sixteen-element array. Explain why dynamic programming is ineffective for speeding up MERGE-SORT.



The MERGE-SORT procedure does *not* have overlapping subproblems, that is, all nodes of the recursion tree are distinct. We cannot re-use the results of recursive calls and, hence, dynamic programming does not improve the efficiency.

Problem 2
Determine a longest common subsequence of $\langle 1,0,0,1,0,1\rangle$ and $\langle 0,1,0,1,1,0,1,1\rangle$. Using Figure 16.3 in the book as a model, draw the table constructed by the LCS-LENGTH algorithm for these two sequences (you do *not* need to show arrows in your table).

The longest common subsequences are $\langle 0,0,1,0,1\rangle$, $\langle 1,0,1,0,1\rangle$, and $\langle 1,0,0,1,1\rangle$. The table computed by LCS-LENGTH is as follows:

```
6 | 0 1 2 3 4 4 4 5 5
5 | 0 1 2 3 3 3 4 4 4
4 | 0 1 2 2 3 3 3 4 4
3 | 0 1 1 2 2 2 3 3 3
2 | 0 1 1 2 2 2 2 2 2
1 | 0 0 1 1 1 1 1 1 1
0 | 0 0 0 0 0 0 0 0 0
    -----------------
    0 1 2 3 4 5 6 7 8
```

Problem 3
Write an algorithm INCREASING-LENGTH(A,n) that determines the *length* of a longest increasing subsequence of an array $A[n]$.

Let $d[i]$ denote the length of a longest increasing subsequence whose last element is $A[i]$. For example, suppose that $A=\langle 1,2,1,2,3\rangle$ and $i=3$; then, the longest increasing subsequence that ends at $A[3]$ is $\langle 1,1\rangle$, which implies that $d[3]=2$.

Observe that this subsequence contains at least one element and, hence, $d[i]\geq 1$. Furthermore, if $j<i$ and $A[j]\leq A[i]$, then $d[i]\geq d[j]+1$, because we may construct a $(d[j]+1)$-element increasing subsequence that ends at $A[i]$, by adding $A[i]$ to the longest increasing subsequence that ends at $A[j]$. These observations lead to the following algorithm, whose running time is $\Theta(n^2)$:

INCREASING-LENGTH(A,n)
d-max $\leftarrow 0$
for $i \leftarrow 1$ **to** n
    do $d[i] \leftarrow 1$
        for $j \leftarrow 1$ **to** $i-1$
            do if $A[i]\geq A[j]$ and $d[i]<d[j]+1$
                    then $d[i] \leftarrow d[j]+1$
    if *d-max* $< d[i]$
        then *d-max* $\leftarrow d[i]$
return *d-max*