Analysis of Algorithms: Assignment 5

Due date: February 25 (Thursday)

Problem 1 (3 points)
Suppose that we apply the CONNECTED-COMPONENTS algorithm to an undirected graph G, with vertices $G[V] = \{a, b, c, d, e, f, g, h, i, j, k\}$, and its edges $E[G]$ are processed in the following order: $(d, i), (f, k), (g, i), (b, g), (a, h), (i, j), (d, k), (b, j), (d, f), (g, j), (a, e), (i, d)$. Using Figure 22.1 in the textbook as a model, illustrate the steps of CONNECTED-COMPONENTS on this graph.

Problem 2 (5 points)
Write pseudocode for MAKE-SET, FIND-SET, and UNION, using the linked-list representation of disjoint sets. Your UNION algorithm must always append the shorter list to the longer one. For every linked list, you will need to store its size, a pointer to the first element, and a pointer to the last element.

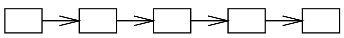
Problem 3 (2 points)
Write a *nonrecursive* version of the FIND-SET algorithm with path compression, for disjoint-set forests.

Problem 4 (bonus)
This problem is optional; if you solve it, then you get 2 bonus points toward your final grade for the course. You cannot submit this bonus problem after the deadline.

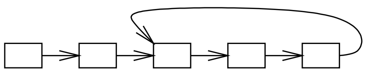
If we need to print all elements of a standard linked list (see Figure 1), then we may write a simple program that starts at the beginning of the list and follows pointers until reaching the end; however, novice programmers sometimes mistakingly create a list whose last element points into the middle of the list (see Figure 2), and then an attempt to print all elements leads to an infinite loop.

Write an algorithm CHECK-LOOP(x) that determines whether a given linked list is "looped." The algorithm's argument x is the first element of the list. If a linked list is looped (Figure 2), then CHECK-LOOP returns TRUE; if the list is not looped (Figure 1), then it returns FALSE.

Your algorithm has to run in *linear time.* Furthermore, it must run *in-place* (no extra memory) and preserve the initial contents of the list. These restrictions mean the you *cannot* mark the elements of the list that you have visited, because storing such marks would require a lot of additional memory.



1. Standard linked list.



2. Looped linked list.