# Analysis of Algorithms: Assignment 3

**Due date: January 28 (Thursday)**

**Problem 1** (5 points)
A *d-ary heap* is like a binary heap, but instead of 2 children, nodes have $d$ children.

**(a)** How would you represent a $d$-ary heap in an array? What is the height of a $d$-ary heap of $n$ elements in terms of $n$ and $d$?

**(b)** Give an efficient implementation of HEAP-EXTRACT-MAX for a $d$-ary heap.

**(c)** Give an efficient implementation of a HEAP-INCREASE-KEY$(A, i, k)$ algorithm, which sets $A[i] \leftarrow \max(A[i], k)$ and updates the heap structure appropriately. Give its time complexity, in terms of $d$ and $n$, and briefly explain your answer.

**Problem 2** (5 points)
Consider the following sorting algorithm:

```
STOOGE-SORT(A, i, j)
if A[i] > A[j]
    then exchange A[i] ↔ A[j]
if i + 1 ≥ j
    then return
k ← ⌊(j − i + 1)/3⌋
STOOGE-SORT(A, i, j − k)      ▷ First two-thirds.
STOOGE-SORT(A, i + k, j)      ▷ Last two-thirds.
STOOGE-SORT(A, i, j − k)      ▷ First two-thirds again.
```

**(a)** Argue that STOOGE-SORT$(A, 1, n)$ correctly sorts the input array $A[1..n]$.

**(b)** Give the recurrence for the worst-case running time of STOOGE-SORT and a tight asymptotic ($\Theta$-notation) bound on the worst-case running time.

**(c)** Compare the worst-case running time of STOOGE-SORT with that of insertion sort, merge-sort, heap-sort, and quick-sort. Is it a good algorithm?

**Problem 3** (bonus)
*This problem is optional, and it does not affect your grade for the homework; however, if you solve it, then you get 2 bonus points toward your* **final grade** *for the course. You cannot submit this bonus problem after the deadline.*

We consider an integer array $A[1..n]$ and define a segment sum from $p$ to $r$, where $1 \le p \le r \le n$, as follows:

$$sum(p, r) = \sum_{p \le i \le r} A[i].$$

That is, it is the sum of all array elements in the segment $A[p..r]$. Note that the total number of distinct segments is $\frac{n(n+1)}{2}$. Write a *linear-time* (that is, $\Theta(n)$) algorithm that determines the maximum over all segment sums.