Analysis of Algorithms: Results of Exam 1

```
                                  X
                            X     X           X     X
                            X     X           X     X
number                      X     X           X     X
of exams                    X     X     X     X     X
                            X     X     X     X     X
                X           X     X     X     X     X
                X           X     X     X     X     X     X     X
          X     X     X     X     X     X     X     X     X     X
         0-54 55-59 60-64 65-69 70-74 75-79 80-84 85-89 90-94 95-100
```

The histogram shows the distribution of grades, *not* including the bonus.

Problem 10

Suppose that you are using a programming language that allows only integer numbers and supports four operations on them: addition, subtraction, multiplication, and integer division; the running time of each operation is constant, that is, $\Theta(1)$. The result of the integer division of n over m is $\lfloor n/m \rfloor$; for example, $\lfloor 10/3 \rfloor = 3$. The language does *not* allow fractional numbers, and does *not* have operations for logarithms and exponentiation. Write an efficient algorithm $\text{Power}(n, m)$ that computes n^m, where n and m are positive integers, and give the asymptotic time complexity (Θ-notation) of your algorithm.

Simple algorithm

The following brute-force computation takes $\Theta(m)$ time:

Simple-Power(n, m)
$k \leftarrow 1$
for $i \leftarrow 1$ **to** m
    do $k \leftarrow k \cdot n$
return k

Fast algorithm

We next note that, if m is even, then $n^m = (n \cdot n)^{\lfloor m/2 \rfloor}$; similarly, if m is odd, then $n^m = n \cdot (n \cdot n)^{\lfloor m/2 \rfloor}$. These observations lead to a faster algorithm, whose complexity is $\Theta(\lg m)$:

Fast-Power(n, m)
if $m = 0$
    then return 1
if $\lfloor m/2 \rfloor \cdot 2 = m$ $\triangleright$ Is m even?
    then return Fast-Power$(n \cdot n, \lfloor m/2 \rfloor)$
    else return $n \cdot$Fast-Power$(n \cdot n, \lfloor m/2 \rfloor)$