Algorithms: Solutions 5

Problem 1
Write an algorithm that randomly re-orders an array $A[1..n]$.

SHUFFLE(A, n)
for $i \leftarrow n$ downto 2
  do $j \leftarrow$ RANDOM$(1, i)$
    exchange $A[i] \leftrightarrow A[j]$

Problem 2
Consider a programming language that allows only integer numbers and supports addition, subtraction, and multiplication. Write an algorithm DIVIDE(n, m) that computes $\lfloor n/m \rfloor$, where n and m are positive integers.

Simple algorithm
The following computation takes $\Theta(\lceil n/m \rceil)$ time and $\Theta(1)$ space.

SLOW-DIVIDE(n, m)
$ratio \leftarrow 1$
while $ratio \cdot m \leq n$
  do $ratio \leftarrow ratio + 1$
return $ratio - 1$

Fast algorithm
The following recursive algorithm runs is $\Theta(\lg\lceil n/m \rceil)$ time. The space complexity is not constant; the algorithm requires $\Theta(\lg\lceil n/m \rceil)$ memory for the stack of recursive calls.

FAST-DIVIDE(n, m)
if $n < m$
  then return 0
$ratio \leftarrow 2 \cdot$ FAST-DIVIDE$(n, 2 \cdot m)$
if $n - ratio \cdot m < m$
  then return $ratio$
  else return $ratio + 1$