Algorithms: Solutions 1

Problem 1

Write an algorithm that finds a given value k in a sorted array $A[1..n]$. It should return the index of the found element; if the array does not include k, it should should return 0.

BINARY-SEARCH(A, n, k)
$p \leftarrow 1$
$r \leftarrow n$
while $p < r$
  do $q = \lfloor (p + r)/2 \rfloor$
    if $k \leq A[q]$
      then $r \leftarrow q$
      else $p \leftarrow q + 1$
if $k = A[p]$
  then return p
  else return 0

Problem 2

Prove that, if n is a natural number, then $n^3 - n$ is divisible by 6.

Observe that $n^3 - n = (n - 1) \cdot n \cdot (n + 1)$, which is a product of three consecutive integers. One of these integers is divisible by 3, and at least one of them is divisible by 2, which implies that their product is divisible by 6.