Algorithms: Solutions 9

Problem 1
Suppose that we apply RB-INSERT to add a node to a red-black tree, and then immediately call RB-DELETE to remove this node. Can the resulting tree differ from the initial tree?

We use the example in Figure 13.4 (page 282) of the textbook to demonstrate that the new tree may differ from the original. The example shows the insertion of a node with value 4 into a red-black tree. If we then delete the value 4, we obtain a tree that differs from the initial tree. Note that, in this case, the deletion does not involve color changes or rotations.

Problem 2
Suppose that we augment a normal programming language with a "magic" function MAGIC-MAX(A, i, j). The arguments of this function include an array $A[1..n]$ and two indices, i and j, such that $1 \leq i \leq j \leq n$. The function sometimes returns the index of the largest element and sometimes the index of the second largest element in $A[i..j]$; it returns an answer in constant time. Your task is to use this language to develop a procedure that sorts an array of real values in linear time.

MAGIC-SORT(A, n)
for $j \leftarrow n$ **downto** 2
    do $k \leftarrow$ MAGIC-MAX$(A, 1, j)$
        exchange $A[k] \leftrightarrow A[j]$
for $i \leftarrow 2$ **to** n
    do if $A[i-1] > A[i]$
            then exchange $A[i-1] \leftrightarrow A[i]$