Analysis of Algorithms: Solutions 2

```
                              X
                              X
                          X   X   X
                          X   X   X
                          X   X   X
                          X   X   X
                          X   X   X
number of                 X   X   X
homeworks             X   X   X   X
                      X   X   X   X
                      X   X   X   X
                      X   X   X   X
              X       X   X   X   X
          X   X   X   X   X   X   X
          -------------------------
          4   5   6   7   8   9   10
                  grades
```

The histogram shows the distribution of grades, from 0 to 10.

Problem 1
Suppose that $A[1..n]$ is an integer array, and some integers occur several times in this array. Write an algorithm for counting integers that occur several times; note that each of them must be counted only once.

For each element $A[j]$, we increment the counter if we find a duplicate in the $A[(j+1)..n]$ segment and no duplicate in $A[1..(j-1)]$. The DUPLICATES function first checks for duplicates of $A[j]$ in $A[1..(j-1)]$, and then looks for duplicates in $A[(j+1)..n]$.

COUNT-DUPLICATES(A, n)	*cost*	*times*
counter $\leftarrow 0$	c_1	1
for $j \leftarrow 1$ **to** n	c_2	$n+1$
do if DUPLICATES(A, j, n)	see below	
then *counter* $\leftarrow$ *counter* $+1$	c_3	$\leq n$
return *counter*	c_4	1

DUPLICATES(A, j, n)	*cost*	*times*
for $i \leftarrow 1$ **to** $j-1$	c_5	$1+2+3+...+n = \frac{n(n+1)}{2}$
do if $A[i] = A[j]$	c_6	$0+1+2+...+(n-1) = \frac{n(n-1)}{2}$
then return FALSE	c_7	$\leq 0+1+2+...+(n-1) = \frac{n(n-1)}{2}$
for $i \leftarrow j+1$ **to** n	c_8	$\leq n+(n-1)+(n-2)+...+1 = \frac{n(n+1)}{2}$
do if $A[i] = A[j]$	c_9	$\leq (n-1)+(n-2)+(n-3)+...+0 = \frac{n(n-1)}{2}$
then return TRUE	c_{10}	$\leq (n-1)+(n-2)+(n-3)+...+0 = \frac{n(n-1)}{2}$
return FALSE	c_{11}	$\leq n$

Problem 2
Estimate the worst-case running time of your algorithm.

$$\begin{aligned}
T(n) &\leq c_1 + c_2(n+1) + c_3 n + c_4 + c_5\frac{n(n+1)}{2} + c_6\frac{n(n-1)}{2} + c_7\frac{n(n-1)}{2} \\
&\quad + c_8\frac{n(n+1)}{2} + c_9\frac{n(n-1)}{2} + c_{10}\frac{n(n-1)}{2} + c_{11}n \\
&= \left(\frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2} + \frac{c_8}{2} + \frac{c_9}{2} + \frac{c_{10}}{2}\right) n^2 \\
&\quad + \left(c_2 + c_3 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2} + \frac{c_8}{2} - \frac{c_9}{2} - \frac{c_{10}}{2} + c_{11}\right) n \\
&\quad + \left(c_1 + c_2 + c_4\right) \\
&= \Theta(n^2)
\end{aligned}$$