Analysis of Algorithms: Assignment 10

Due date: April 19 (Thursday)

Problem 1 (5 points)
Using Figure 16.2 in the textbook as a model, draw the recursion tree for the MERGE-SORT procedure on a sixteen-element array. Explain why dynamic programming is ineffective for speeding up MERGE-SORT.

Problem 2 (5 points)
Suppose that you drive along some road, and you need to reach its end. Initially, you have a full tank, which holds enough gas to cover a certain distance d.

The road has n gas stations, where you can refill your tank. The distances between gas stations are represented by an array $A[1..n]$, where $A[1]$ is the distance from the start to the first gas station, $A[2]$ is the distance from the first to the second station, $A[3]$ is that from the second to the third station, and so on. The last gas station is exactly at the end of the road. You wish to make as few stops as possible along the way.

Give an algorithm CHOOSE-STOPS(d, A, n) that identifies all places where you have to refuel, and returns the set of selected gas stations. You may assume that, for each i, $A[i] \leq d$.

Problem 3 (bonus)
This problem is optional; if you solve it, you will get one bonus point toward your final grade.

We have seen in class that a directed graph with V vertices may have at most V^2 edges. Now suppose that we consider only *acyclic* graphs with V vertices. What is the maximal possible number of edges in a directed acyclic graph? Give a formula for the maximal number of edges, in terms of the number of vertices V, and explain why it is correct.