Algorithms: Solutions 8

```
                             X
                             X
                             X
                             X
                             X
                             X
                             X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
number of        X           X
homeworks        X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
                 X           X
        X     X  X  X        X
        X     X  X  X        X
        -----------------------
        3  4  5  6  7  8  9  10
               grades
```

Problem 1
The depth-first search algorithm may be used to identify the connected components of an undirected graph. Write a modified version of DFS for this task.

DFS-COMPONENTS(G)
$k \leftarrow 0$ $\quad \triangleright$ component counter
for each $u \in V[G]$
 do $component[u] \leftarrow 0$
for each $u \in V[G]$
 do if $component[u] = 0$
  then $k \leftarrow k + 1$
   DFS-VISIT(u, k)
return k

DFS-VISIT(u, k)
$component[u] \leftarrow k$ $\quad \triangleright$ u has just been discovered
for each $v \in Adj[u]$
 do if $component[v] = 0$
  then DFS-VISIT(v, k)

Problem 2
Suppose that G is an undirected graph, and you need to check whether G has cycles. Design an algorithm that returns TRUE if G is acyclic, and FALSE if G has cycles.

The key observation is that an acyclic undirected graph has at most $V-1$ edges. To determine whether a graph G is acyclic, we first count its edges. If the edge counter reaches V, we immediately return FALSE, without counting the rest of edges. On the other hand, if the number of edges is less than V, we apply DFS to search for cycles. In either case, the overall running time is $O(V)$.