Analysis of Algorithms: Results of Exam 1

```
 number      X
of exams     X           X     X                 X     X
             X     X     X     X     X     X     X     X     X
       X     X     X     X     X     X     X     X     X     X
       X     X     X     X     X     X     X     X     X     X     X     X
     -----------------------------------------------------------------------
     0-39 40-44 45-49 50-54 55-59 60-64 65-69 70-74 75-79 80-84 85-89 90-100
```

The histogram shows the distribution of grades, *not* including the bonus.

Problem 10

A *ternary heap* is like a binary heap, but instead of two children, nodes have three children.

(a) How would you represent a ternary heap by an array? What are the expressions for determining the parent and children of a given element?

The following expressions determine the parent and children of element i:

$$\begin{aligned}
\textsc{Parent}(i) &= \left\lfloor \frac{i+1}{3} \right\rfloor \\
\textsc{Left-Child}(i) &= 3i - 1 \\
\textsc{Middle-Child}(i) &= 3i \\
\textsc{Right-Child}(i) &= 3i + 1
\end{aligned}$$

(b) What is the height of a ternary heap of n elements in terms of n?

The height h of a heap is *approximately* equal to $\log_3 n$. The exact height is

$$h = \lceil \log_3(2n+1) - 1 \rceil$$