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Generalized blockmodeling of two-mode network data

Patrick Doreian [a,*], Vladimir Batagelj [b], Anuška Ferligoj [b]

[a] *Department of Sociology, University of Pittsburgh, 2G245 WWPH, Pittsburgh, PA 15260, USA*
[b] *University of Ljubljana, Ljubljana, Slovenia*

Abstract

We extend the direct approach for blockmodeling one-mode data to two-mode data. The key idea in this development is that the rows and columns are partitioned simultaneously but in different ways. Many (but not all) of the generalized block types can be mobilized in blockmodeling two-mode network data. These methods were applied to some 'voting' data from the 2000–2001 term of the Supreme Court and to the classic Deep South data on women attending events. The obtained partitions are easy to interpret and compelling. The insight that rows and columns can be partitioned in different ways can be applied also to one-mode data. This is illustrated by a partition of a journal-to-journal citation network where journals are viewed simultaneously as both producers and consumers of scientific knowledge.
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1. Introduction

Blockmodeling tools were developed to partition network actors into clusters, called positions, and, at the same time, to partition the set of ties into blocks that are defined by the positions (see Lorrain and White (1971), Breiger et al. (1975), Burt (1976) for the foundational statements). For these authors, and those using their methods, the foundation for the partitioning was structural equivalence. White and Reitz (1983) generalized structural equivalence to regular equivalence as another principle for blockmodeling networks. For all of these authors, the use of blockmodeling tools was inductive in the sense of specifying an equivalence type and searching for partitions that approximated those equivalence types.[1]

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* Corresponding author.
E-mail address: pitpat@pitt.edu (P. Doreian).

[1] An exception is Heil and White (1976) but their algorithm did not enjoy widespread usage for blockmodeling.

The procedures were indirect in the sense of converting network data into a (dis)similarity matrix and using some clustering algorithm. Batagelj et al. (1992a,b) suggested an alternative strategy where the partitioning was done by using the network data directly. In essence, their approach was built upon the recognition that both structural and regular equivalence define certain block types if a partition of actors and ties is *exact* and consistent with the type of equivalence. For structural equivalence, the ideal blocks are null and complete (Batagelj et al., 1992a), and for regular equivalence, the ideal block types are null and regular (Batagelj et al., 1992b). Subsequently, blockmodeling was generalized to permit many new types of blocks (see Batagelj (1997) and Doreian et al. (1994)). The notion of constructing blockmodeling in terms of a larger set of block types, together with the use of optimization methods mobilized within a direct approach is called *generalized blockmodeling* (Doreian et al., 2005). Hitherto, these methods have been applied only to one-mode network data. Here, we consider another extension of blockmodeling by including two-mode network data.

2. Two-mode network data

Wasserman and Faust (1994, Chapter 8) provide a discussion of affiliation networks as two-mode data. In essence, two-mode data are defined for two sets of social units and contain measurements of a relation from the units in one set to units in the other set. Pairs of network actor types and relations include: people attending events, organizations employing people, justices on a court rendering decisions, and nations belonging to alliances. The most used example of a two-mode network is the Deep South data, also known as the 'Southern Women' data, collected by Davis et al. (1941) for a set of women attending social events over a 9-month period. We consider these data shortly as our second example.

3. Approaches to two-mode network data

In a two-mode network, $\mathbf{N} = (\mathcal{U}_1, \mathcal{U}_2, R, w)$, one set of social units is denoted by $\mathcal{U}_1 = \{u_1, u_2, \ldots, u_{n1}\}$ and the second set of units is denoted by $\mathcal{U}_2 = \{v_1, v_2, \ldots, v_{n2}\}$. By definition, $\mathcal{U}_1 \cap \mathcal{U}_2 = \emptyset$. The social relation $R \subseteq \mathcal{U}_1 \times \mathcal{U}_2$ is defined as one between the units in these two sets and is represented by the set of lines (which can be arcs or edges) with initial vertices in the set $\mathcal{U}_1$ and terminal vertices in the set $\mathcal{U}_2$. The mapping $w : R \to \mathbb{R}$ is a weight. Two examples of weighted two-mode networks are (persons ($\mathcal{U}_1$) goods/services ($\mathcal{U}_2$), consumed (R), frequency (w)) and (countries ($\mathcal{U}_1$), countries ($\mathcal{U}_2$), imported from (R), value (w)). If no weight is defined we can assume a constant weight $w(u, v) = 1$ for all uRv.

Many network analytic tools have been used to study the structure contained in two-mode data with distinct ways of representing this type of data. We focus here on the approaches that are primarily algebraic. One takes the form of using a *rectangular $n_1 \times n_2$* matrix $\mathbf{A} = [a_{uv}]$ to represent the data for the two sets of units, $\mathcal{U}_1$ and $\mathcal{U}_2$.

$$a_{uv} = \begin{cases} w(u, v), & uRv \\ 0, & \text{otherwise} \end{cases}$$

This (rectangular) matrix can be binary or valued. For two of the examples that follow, **A** has a binary form. The third example that we consider involves valued data.

A two-mode network can be viewed also as an ordinary (one-mode) network on the vertex set $\mathcal{U}_1 \cup \mathcal{U}_2$, divided into two disjoint sets $\mathcal{U}_1$ and $\mathcal{U}_2$, where the arcs can only go from $\mathcal{U}_1$ to $\mathcal{U}_2$—it is a bipartite directed graph. Borgatti and Everett (1997: 248) provide a pictorial representation of a bipartite graph for the Southern Women data set and advocate the inclusion of analyses of the bipartite matrix, in this form, in the analysis of two-mode data. Freeman (2003) also displays the Southern Women data in this fashion.

One algebraic approach to two-mode data takes the form of exploring the 'duality' of two-mode data (Breiger, 1974) by constructing two valued one-mode networks. The matrix, $\mathbf{A}_1 = \mathbf{A}\mathbf{A}^T$, where T denotes transposition, is in one-mode form for the actors in $\mathcal{U}_1$ and $\mathbf{A}_2 = \mathbf{A}^T\mathbf{A}$ is in one-mode form for the actors in $\mathcal{U}_2$. Analyses then are conducted by using these two one-mode representations.

Atkin (1974) introduced Q-analysis as a way of delineating the structure of the dual simplicial complexes of $\mathbf{A}_1$ and $\mathbf{A}_2$. Doreian (1979) analyzed the structure of the Southern Women data using these tools. Freeman (1980) provides another example of using Q-analysis for network data and Seidman (1981) provides a related discussion in terms of hypergraphs. Galois lattices provide another algebraic approach to two-mode data, one taken by Freeman and White (1994) who also applied these methods to the Southern Women data. Once $\mathbf{A}_1$ and $\mathbf{A}_2$ have been constructed as one-mode matrices, all of the conventional techniques for analyzing one-mode data can be used.

4. Blockmodels for two-mode network data

Thinking of applying generalized blockmodeling tools to two-mode data implies making some adjustments to this set of techniques, as well as the thinking behind them. Because the data come in the form of rectangular arrays the language of diagonal and off-diagonal blocks is no longer applicable. Blockmodels applied to (the usual) 'square' network data, require that the rows and columns are partitioned simultaneously in exactly the same way. It makes no sense, conceptually and technically, to partition rows and columns of a rectangular array in the same fashion. In this paper, the rows and columns of the rectangular array *are* partitioned at *the same time* but they are partitioned differently.

We do not regard the use of partitioning tools for the above matrix representation of the bipartite graph (using **A**, $\mathbf{A}^T$ and two null matrices) as appropriate. The reason is simple: it is possible to have *mixtures* of the two *distinct types* of actors in the same cluster. Indeed, exactly this happened with Borgatti and Everett (1997) partition of the bipartite network in this form for the Southern Women data. We propose the following procedure to *ensure* that actors of different types are *never* clustered together: the two-mode data are treated as two-mode data.

We establish a simple notation given $\mathcal{U}_1 = \{u_1, u_2, \ldots, u_{n1}\}$ and $\mathcal{U}_2 = \{v_1, v_2, \ldots, v_{n2}\}$. Let k_1 be the number of clusters for actors in $\mathcal{U}_1$ and let k_2 the number of clusters of actors in $\mathcal{U}_2$ Clearly, $1 \leq k_1 \leq n_1$ and $1 \leq k_2 \leq n_2$. While it is possible to consider up to n_1 and n_2 for the number of clusters of the two sets of actors, the point of the blockmodeling

effort is to use fewer clusters than actors. We will label a partition with k_1 clusters of the actors in ($\mathcal{U}_1$) and k_2 clusters of the actors in ($\mathcal{U}_2$) as a (k_1, k_2)-partition of the two-mode array.

5. A formalization of blockmodeling two-mode data

The theoretical background for two-mode blockmodeling comes from Batagelj et al. (1992a,b); Doreian et al. (1994); Batagelj et al. (1998): we view the blockmodeling of two-mode data as a simple extension of one-mode blockmodeling.

The main difference is that in blockmodeling of a two-mode network $\mathbf{N} = (\mathcal{U}_1, \mathcal{U}_2, R, w)$ we are trying to identify a two-clustering $\mathbf{C} = (\mathbf{C}_1, \mathbf{C}_2)-$ where $\mathbf{C}_1$ is a partition of $\mathcal{U}_1$ and $\mathbf{C}_2$ is a partition of $\mathcal{U}_2$—such that they induce blocks of the selected types. We denote the set of all feasible two-clusterings with Φ. The block $R(C_u, C_v)$ is defined by $R \subseteq \mathcal{U}_1 \times \mathcal{U}_2$ restricted to $C_u \subseteq \mathcal{U}_1$ and $C_v \subseteq \mathcal{U}_2$.

The two-mode generalized problem can be formulated as an optimization problem (Φ, P, min):

Determine the two-clustering $\mathbf{C}^* = (\mathbf{C}_1^*, \mathbf{C}_2^*) \in \Phi$ for which

$$P(\mathbf{C}^*) = \min_{\mathbf{C} \in \Phi} P(\mathbf{C})$$

where Φ is the set of feasible two-clusterings and P is the criterion function.

The criterion function $P(\mathbf{C})$ is obtained in the same way as in the ordinary case:

$$P(\mathbf{C}) = P(\mathbf{C}_1, \mathbf{C}_2) = \sum_{C_u \in \mathbf{C}_1, C_v \in \mathbf{C}_2} \min_{B \in \mathcal{B}(C_u, C_v)} \delta(R(C_u, C_v), B)$$

where $\mathcal{B}(C_u, C_v)$, $C_u \subseteq \mathcal{U}_1$ and $C_v \subseteq \mathcal{U}_2$ denote the set of all ideal blocks corresponding to block $R(C_u, C_v)$, and the term $\delta(R(C_u, C_v), B)$ measures the difference (number of inconsistencies) between the block $R(C_u, C_v)$ and the ideal block B. The value of δ is constructed on the basis of the characterizations of the types of blocks in such as fashion that makes δ compatible with the selected type of equivalence (see Batagelj et al. (1992a) and Batagelj (1997)). The criterion function $P(\mathbf{C}_1, \mathbf{C}_2)$ is sensitive iff $P(\mathbf{C}_1, \mathbf{C}_2) = 0 \Leftrightarrow$ the blockmodel determined by $(\mathbf{C}_1, \mathbf{C}_2)$ is exact.

The resulting optimization problem can be solved by a local optimization procedure as fol-
lows:
    Determine the initial clustering $\mathbf{C}$
    repeat
    when in the neighborhood of the current clustering $\mathbf{C}$
    if there exists a clustering $\mathbf{C}'$ such that $P(\mathbf{C}') < P(\mathbf{C})$
    then move to clustering $\mathbf{C}'$

Usually, the *neighborhood* is determined by two transformations: *moving* a unit from one cluster to another cluster; *interchanging* of two units between different clusters. The procedure is repeated for many initial clusterings and the (those) partition(s) with the smallest value of the criterion function are selected. Once the partitions $(\mathbf{C}_1, \mathbf{C}_2)$ and types of blocks are determined, we can also compute the values of connections by using averaging rules.

6. Three empirical examples

We consider three examples. One features some of the some decisions handed down by the US Supreme Court in their 2000–2001 term. The second comes in the form of the Deep South data discussed above and the third is a journal-to-journal citation network.

6.1. Supreme Court Voting

These data come from the simple (preliminary) study by Doreian and Fujimoto (2003) of the Supreme Court Justices and their 'votes' on a set of 26 "important decisions". These data are presented in Table 1. The nine Justices (in the order in which they joined the Supreme Court) are Rehnquist (1972), Stevens (1975), O'Connor (1981), Scalia (1982), Kennedy (1988), Souter (1990), Thomas (1991), Ginsburg (1993) and Breyer (1994). The distribution of votes are organized in terms of the substantive content of the selection of decisions as organized by Greenhouse (2001).

Each row of Table 1 represents a decision handed down by the Supreme Court. On the left is a descriptive label for that decision. A much fuller description of these case can be found in Greenhouse (2001) and in Doreian and Fujimoto (2003). The substantive details of these cases, while stark, are of secondary concern here. The columns correspond to the nine Justices where the Justices are represented by a label: Breyer (Br), Ginsburg (Gi), Souter (So), Stevens (St), O'Connor (OC), Kennedy (Ke), Rehnquist (Re), Scalia (Sc) and Thomas (Th). In these data, a '+' in the column of a Justice represents voting in the majority for that issue and a '−' represents voting in the minority for that decision. The decisions range from the unanimous decision[2] in the case involving the Clean Air Act to many 5-4 decisions. The latter suggest an ideologically divided court.

In the usual representation of such data, $\mathbf{A}$ is written with the actors (in this case the justices) in the rows and the events (in this case the decisions) in the columns. Table 1 has been written as the transposed form for formatting reasons. In terms of the analysis that follows, rows of $\mathbf{A}$ represent the justices and the columns their decisions.

Doreian and Fujimoto viewed these data as 'signed' two-mode data. They constructed a matrix, $\mathbf{A}_p$, for majority voting where 1 represented a majority vote and 0 a vote that was not for the majority (i.e. either a vote against the decision or an abstention). Similarly, they constructed a matrix $\mathbf{A}_n$ for the negative votes for a decision, where 1 represented a vote in the minority and 0 represented either an abstention or a vote with the majority. Their analyses involved (for a focus on the justices) $\mathbf{A}_p\mathbf{A}_p^{\mathrm{T}}$ for majority voting, $\mathbf{A}_n\mathbf{A}_n^{\mathrm{T}}$ for minority voting and $\mathbf{A}_t$ where $\mathbf{A}_t = \mathbf{A}_p\mathbf{A}_p^{\mathrm{T}} + \mathbf{A}_n\mathbf{A}_n^{\mathrm{T}}$ for all voting. Their analyses were designed to explore the differing patterns of joint majority voting, joint minority voting and joint overall voting patterns. They used a combination of tools including Q-analysis and partitions based on structural equivalence for the generated one-mode matrices. Some nuanced interpretations follow as a result of looking at the differences between majority and minority voting patterns. As a result of these different analyses, they reached a partition—having four clusters of justices and eight clusters of decisions—with 13 inconsistencies with an ideal partition

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[2] There is one case that was unanimous except for Breyer who abstained because his brother had ruled on the case at a lower court level.

Table 1
Supreme Court Voting for 26 important decisions

Issue	Br	Gi	So	St	OC	Ke	Re	Sc	Th
Presidential election	−	−	−	−	+	+	+	+	+
Criminal law cases									
Illegal Search 1	+	+	+	+	+	+	−	−	−
Illegal Search 2	+	+	+	+	+	+	−	−	−
Illegal Search 3	+	+	+	−	−	−	−	+	+
Seat Belts	−	−	+	−	−	+	+	+	+
Stay of execution	+	+	+	+	+	+	−	−	−
Federal authority cases									
Federalism	−	−	−	−	+	+	+	+	+
Clean Air Act	+	+	+	+	+	+	+	+	+
Clean water	−	−	−	−	+	+	+	+	+
Cannabis for health	0	+	+	+	+	+	+	+	+
United Foods	−	−	+	+	−	+	+	+	+
New York Times copyrights	−	+	+	−	+	+	+	+	+
Civil rights cases									
Voting Rights	+	+	+	+	+	−	−	−	−
Title VI disabilities	−	−	−	−	+	+	+	+	+
PGA vs. handicapped player	+	+	+	+	+	+	+	−	−
Immigration law cases									
Immigration Jurisdiction	+	+	+	+	−	+	−	−	−
Deporting Criminal Aliens	+	+	+	+	+	−	−	−	−
Detaining Criminal Aliens	+	+	+	+	−	+	−	−	−
Citizenship	−	−	−	+	−	+	+	+	+
Speech and press cases									
Legal Aid for the Poor	+	+	+	+	−	+	−	−	−
Privacy	+	+	+	+	+	+	−	−	−
Free Speech	+	−	−	−	+	+	+	+	+
Campaign Finance	+	+	+	+	+	−	−	−	−
Tobacco Ads	−	−	−	−	+	+	+	+	+
Labor and property rights cases									
Labor rights	−	−	−	−	+	+	+	+	+
Property rights	−	−	−	−	+	+	+	+	+

based on structural equivalence.[3] Here, we focus on the two-mode data as given in Table 1 and use a direct partition of the two-mode data with four clusters of the justices and seven clusters of the decisions:[4] i.e. a (4, 7)-partition in the notation of Section 3. The clustering of the justices is identical with the partition reported by Doreian and Fujimoto. The difference between to two procedures comes with the partition of the decisions.

Fig. 1 shows the *fitted* blockmodel for structural equivalence where four clusters were specified for the justices and seven clusters were specified for the decisions.

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[3] The value of the criterion function for their reported partition was computed for the partition they reported and was not used as a part of their partitioning methods.

[4] In selecting this particular partition we were helped by the partition reported by Doreian and Fujimoto (2003).
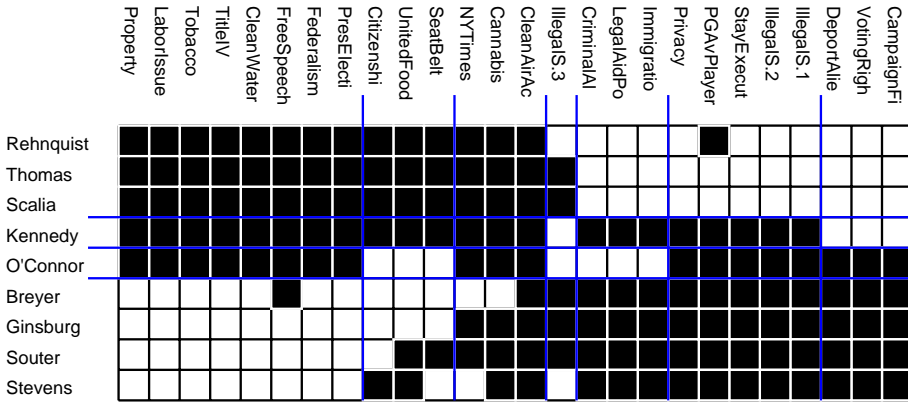
Fig. 1. A (4, 7)-partition of the Supreme Court Voting.

For the justices, two of the four clusters are {Breyer, Ginsburg, Souter, Stevens}, and {Rehnquist, Scalia, Thomas} that can be viewed, respectively, as the liberal and conservative wings of the Supreme Court (Doreian and Fujimoto, 2003). The two singletons, O'Connor and Kennedy can be interpreted in several ways. In one interpretation, they can be seen as justices that form a bridge between the two wings. Alternatively, they can be viewed as being a part of the conservative wing that joins with the liberal wing on *some* of the issues that come before the court. Kennedy is the justice who most often is a part of the majority (22 times in the 26 decisions considered here). O'Connor is in the majority for 19 of the decisions.[5]

For the set of 26 decisions, the first cluster on the left (on top) in Fig. 1 are decisions where the conservative core plus O'Connor and Kennedy won with a narrow 5-4 vote plus the Free Speech case where Breyer joins the majority. His vote on this issue is an inconsistency with a perfect structural equivalence partition. The second cluster of cases (on the left) has the Seat Belts, United Foods and the Citizenship cases. For these decisions, the conservative core plus Kennedy are joined by justices from the liberal wing. Justices Souter and Stevens each joins them twice. These four votes from the liberal wing are all inconsistent with the ideal null block specified under structural equivalence for these three cases. They are the black squares in the block defined by these decisions and the liberal wing cluster. Justice O'Connor's votes are all consistent with a structural equivalence partition for these three cases because she was a singleton and voted in the minority for each of these cases. The third cluster from the left has the cases concerning the Clean Air Act, Cannabis and the New York Times cases. The conservative core of Rehnquist, Thomas and Scalia are joined by both Kennedy and O'Connor together with at least one member of the liberal wing of the court. The inconsistencies with structural equivalence are the white squares in the block defined by these cases and the liberal wing.

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[5] These two justices are differentiated by Doreian and Fujimoto (2003) in terms of minority voting, A_n, where O'Connor has a bridging role not shared by Kennedy.
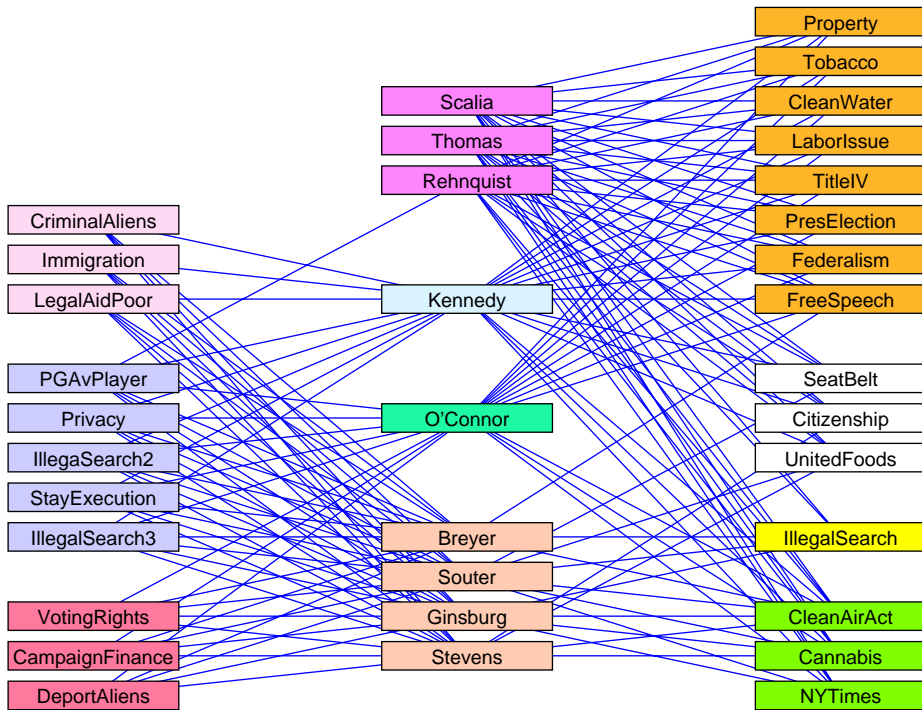
Fig. 2. The Bipartite Supreme Court Network with the (4, 7)-partition.

The remaining clusters of cases are those where the three justices of the conservative core are not a part of the majority (except for Rehnquist with the PGA versus a handicapped player). The singleton, Illegal Search 3, is a most improbable decision where Scalia and Thomas are joined by three members of the liberal wing in a yet another 5-4 decision. The white squares for Rehnquist and Stevens are the inconsistencies with structural equivalence. Next comes three decisions—Legal Aid for the Poor, Detaining Criminal Aliens and Immigration Jurisdiction—where Kennedy joins the liberal wing to produce more 5-4 decisions. There are no inconsistencies with structural equivalence. Next comes five decisions where the liberal wing is joined by both Kennedy and O'Connor.[6] Rehnquist's vote is inconsistent with structural equivalence. Finally, there are three decisions where the liberal wing is joined by O'Connor. These are the Campaign Finance, Voting Rights, and the Deporting Criminal Aliens cases. There are no inconsistencies with structural equivalence.

Fig. 2 provides a pictorial representation of the partitioned two-mode Supreme Court data as a bipartite graph. The clusters that are produced are homogenous—there is no mixing of the two types of social actors in them. Finally, together, the matrix display in Fig. 1 and the picture of the bipartite graph in Fig. 2 provide clean displays of the partitioned two-mode data.

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[6] As noted above, the case of the PGA versus a handicapped player has Rehnquist in the majority.

Table 2
Sorted participation matrix

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	1	0	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0

Having a partition based on structural equivalence, without really pre-specifying a model, that is sufficient to partition the Supreme Court two-mode data set is appealing. However, in general, using just these two types of blocks (null and complete) that could appear anywhere in a blockmodel will not be sufficient to blockmodel all two-mode data structures. We turn now to consider other types of blockmodels and strategies for modeling two-mode data.

6.2. The southern women event participation data

Our second example is the Davis et al. (1941) Southern Women participating in social events data set. They are shown in Table 2 where a 1 indicates attendance at an event and 0 indicates not attending an event. See also Homans (1950) for a discussion of these data.

As Freeman (2003) notes, this data set "... reappears whenever any network analyst wants to explore the utility of some new tool for analyzing data." We join the long line of analysts who have mined these data. Our assessment of the utility of using blockmodeling tools for two-mode data has been made much easier by Freeman's (2003) meta-analysis of 21 analyses of these data. His analysis provides a consensual (induced) criterion regarding the 'true structure' in these data. It is also a point of departure for other partitioning strategies.

Freeman is clear that the 'true structure' of the Southern Women data on the participation in social events is one where there are two subgroups. One is composed of {Evelyn, Laura, Theresa, Brenda, Charlotte, Frances, Eleanor, Pearl, Ruth} and the other has {Verne,
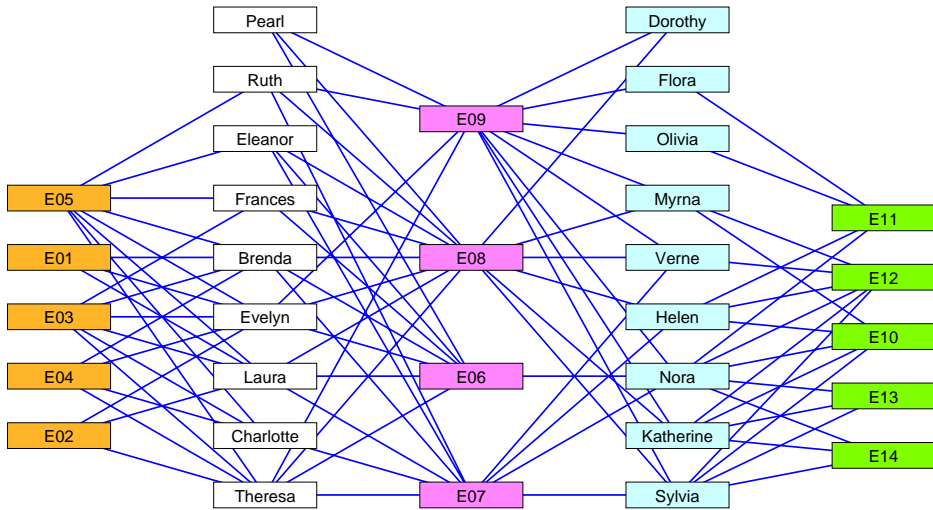
Fig. 3. Southern Women Bipartite Network: Version 1.

Myra, Katherine, Sylvia, Nora, Helen, Dorothy, Olivia, Flora} as its members. These two subgroups of women are labeled as 'Group A' and 'Group B'. Freeman is silent about the corresponding partition of the set of events. We label the events E_j for $1 \leq j \leq 14$ where the labeling of the events is consistent with the partitions of those events that we present. We have divided them into three clusters that almost have the form: events E_1–E_5 were events attended only by women of Group A; events E_6–E_9 were events attended by both groups of women and events E_{10}–E_{14} were attended only by the women in Group B. Pearl is the exception in Group A because she attends only E_6, E_8 and E_9 and Dorothy, by attending only E_8 and E_9, is the exception in Group B. This partition is labeled 'Version 1' and is shown as Table 2. Consistent with that description, Fig. 3 shows a picture of the bipartite graph with these two clusters of women and the three clusters of events.

Can the (2, 3)-partition shown in Table 2 be recovered as a unique (or near unique) solution by means of (generalized) blockmodeling tools for two-mode data? If only the block types are specified and there are no further specifications, then the answer is no. It becomes necessary to *pre-specify* (Batagelj et al., 1998) the location of specific block types and it is possible to impose different penalties on some inconsistencies relative to others. Moving in this direction is aided by the summary of the event participation as shown in Table 3.

Table 3 makes it clear that there are events attended only by women from Group A and a set of events attended only by women from Group B. The blockmodel suggested by the counts in Table 3 is:

com	com	nul
nul	com	com

Table 3
Counts of participation in events

Actor	E_1 - E_5	E_6 - E_9	E_{10} - E_{14}
Evelyn	5	3	0
Laura	4	3	0
Theresa	4	4	0
Brenda	4	3	0
Charlotte	3	1	0
Frances	2	2	0
Eleanor	1	3	0
Pearl	0	3	0
Ruth	1	4	0
Verne	0	3	1
Myra	0	2	2
Katherine	0	2	4
Sylvia	0	3	4
Nora	0	3	5
Helen	0	2	3
Dorothy	0	2	0
Olivia	0	1	1
Flora	0	1	1

With structural equivalence in mind, the specification can be 'strengthened' in the sense of pre-specifying a model in terms of block types and their location within the image. In this case, if there are events attended only be women of Group A and events attended only by women of Group B then there are clear null blocks. It seems sensible to use this information about the structure of the data in Table 2. So, in addition to specifying that only null and complete blocks are allowed, we can go further and specify *where* in a blockmodel these null blocks should be *located* and specify that inconsistencies in the form of 1's in these null blocks are to be penalized. That is, 1's in these null blocks are more consequential than 0's in the otherwise complete blocks. The penalty we imposed for 1's in null blocks was 100 with the expectation that null blocks would be identified as null but with the consequence of increasing the number of 0's in blocks whose corresponding ideal block is complete.

Fitting this blockmodel type, with the penalty, does yield a unique blockmodel with the specified form and 64 inconsistencies with the corresponding ideal blockmodel. The partition for the women is the 'best' partition emerging from Freeman's meta-analysis. The partition of the events is problematic where the partition is $\{E_1, E_2, E_4\}$, $\{E_3, E_5–E_9\}$ and $\{E_{10}–E_{14}\}$. We note that all of the structural equivalence partitions—where only block types were specified—has optimized criterion functions of 53.

By making sure that the null blocks were null, the 'total' number of inconsistencies was raised in the form of 0's in complete blocks. This is not surprising because seeking partitions with additional *constraints* (in the pre-specification), in general, leads to partitions with larger values for the criterion function. There is a price to pay if constraints are placed on solutions. In this case, there are null blocks that *must be preserved* and preserving them for all fitted blockmodels increases the inconsistency count from 53 to 64. It is a price we are willing to pay and insist that it is a price we often must pay to delineate meaningful

Table 4
Sorted participation matrix

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	1	0	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0

partitions if *we know more* about the data than the presence of blocks consistent with an equivalence type.

The partition shown in Fig. 3 and Table 2 is appealing and reproduces the 'truth' identified in Freeman's (2003) meta-analysis of previous analyses of the Davis data. Yet there is a problem in the pattern of ties for Pearl and Dorothy. They attend only events *common* to Group A and Group B. This suggests the partition shown in Table 4 where Pearl and Dorothy form their own group. This suggests a blockmodel with a pre-specified structure of:

com	com	nul
nul	com	com
nul	com	nul

When this pre-specified model is fitted, a unique solution is returned for the blockmodeling problem for these data and it is exactly the partition shown in Table 4. This blockmodel is shown in Fig. 4.

The partition shown in Table 4 can be reached in at least two other ways. Doreian et al. (2005) show that both of the following specifications also produce exactly the same partitions of women and events:

reg	reg	nul
nul	reg	reg
nul	rre	nul

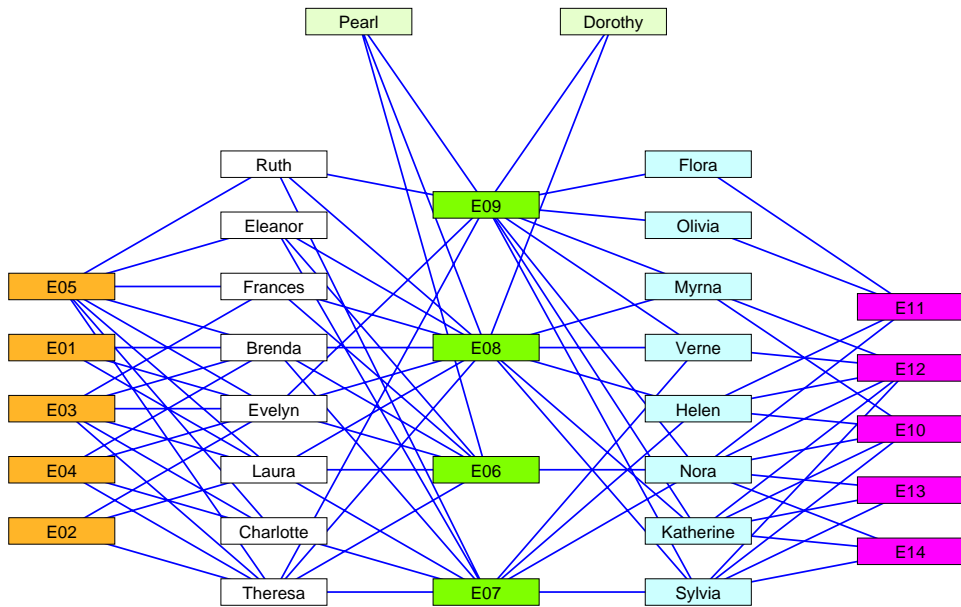|		
rdo	rdo	nul
nul	rdo	rdo
nul	cdo	nul

Fig. 4. Southern Women Bipartite Network: Version 2.

An examination of Table 4 makes it clear that these are both appropriate pre-specified blockmodels for these data. For both, the same heavy penalty for 1's in null blocks was imposed. Of course, the measures of inconsistency change with the change in the definition of block types. See Doreian et al. (2005) for a discussion of having multiple blockmodel types for complementary interpretations of a body of data and a discussion of why inconsistencies for different block types and blockmodels cannot be compared directly.

We turn now to consider delineating the subgroup structure of Group A and Group B. However, we do so with a certain amount of trepidation because there may not be enough information to determine the internal structure of Group A and Group B in an unequivocal fashion. Certainly, we do not determine this internal structure in a unique fashion. Given the robustness of the partition of events into three clusters, we first consider (5, 3)-partitions. We know already that Dorothy and Pearl belong to neither Group A nor Group B. As a result, we need to specify a cluster for them. If Group A and Group B are each to be split into two subgroups, then k_1 is 5. All our attempts to get subgroups of A and B with predicates of complete, regular, row-regular and null blocks were not successful. Either there are multiple equally well-fitting partitions or the unique partitions, when they are returned, are less than satisfying. The unique partitions most often break Group A into a singleton (usually Evelyn) and the rest of A and to break Group B into a singleton (usually Nora) and the rest of B. It appears that the use of complete, regular and row-regular blocks precludes the identification of reasonable subgroups. When complete blocks are used, there is a tendency to return singletons as one of the subgroups. This suggests weakening such

Table 5
(5, 3)-Partition for subgroups using density blocks

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	1	0	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0

a specification to a regular block. The key problem is that the step away from complete blocks to regular blocks permits a large number of candidate regular blocks of greatly varying densities. This leads to multiple equally well-fitting partitions.

If there are subgroups of A and B, then their attendance patterns must differ if they are to be differentiated. Core women attend more group events than the other women. This suggests blocks of differential densities. To accommodate this notion, we pre-specify the following ideal blockmodel where 'den' indicated that a density is to be specified:

den	reg	nul
rre	reg	nul
nul	rre	nul
nul	reg	den
nul	rre	rre

The density has to specified as high and we selected a density of 0.8 for the blocks specified as 'den'. Under this specification, 'den' indicated that there are women who attend heavily a subset of events for their group. The remaining women of a group attend only some of their group's events. For these women we specify a row-regular block. For common events, we use either regular or row-regular blocks. The third row of the blockmodel is specified for Dorothy and Pearl. When this pre-specified blockmodel is fitted with 50,000 repetitions and the heavy penalty (100) for null blocks, we obtained the (seemingly) unique partition shown in Table 5.

Table 6
A second (5, 3)-partition for subgroups using density blocks

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	1	0	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0

Unfortunately for a simple narrative, a second run of 100,000 repetitions yielded the partition shown in Table 6 which means that we cannot establish a unique partition with the above pre-specified model. The partition in Table 5 is quite appealing. Evelyn, Laura Theresa and Brenda all have heavy participation in events E_1–E_5. The remaining women {Charlotte, Frances, Eleanor, Ruth} of Group A have a much lower participation in events E_1–E_5. The partition shown in Table 6 has appeal also insofar as {Evelyn, Laura, Brenda} still have a heavy participation in events E_1–E_5. Theresa also has a heavy participation in these events, she does not attend E_1 and is located in the second cluster of women in Group A in Table 6. This ambiguity is problematic even though the difference seems minor. If anything the partition shown in Table 5 is preferable as it provides a sharper delineation between the two subgroups of Group A. When we turn to Group B, in Table 5, a similar problem occurs. In the partition shown, it seems that Katherine and Nora form the core of Group B with their heavy attendance of events E_{10}–E_{14}, the events attended by the women of Group B only. However, a visual inspection shows that the partition with Katherine and Sylvia interchanged also satisfies this pre-specified model and there is a lingering doubt that Katherine ought to be located in the first subgroup of B. In short, while we have the broad outline of the subgroup structure of the two groups of women, we do not have a unique partition with this pre-specified blockmodel.

An alternative strategy for considering subgroups of Group A and Group B is to include a finer grained partition of the events. As a part of this alternative, it seems reasonable to focus on null blocks within the events for Group A and within the events for Group B. An

Table 7
(5, 5)-Partition for subgroups using more event clusters

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	1	0	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0

inspection of the data suggested the following blockmodel:

reg	com	reg	nul	nul
nul	com	reg	nul	nul
nul	nul	rre	nul	nul
nul	nul	reg	reg	com
nul	nul	rre	reg	nul

When this blockmodel is fitted over 50,000 repetitions, the blockmodel shown in Table 7 results. The partition is exact with zero inconsistencies and seems unique. Event E_5 is a singleton and all of the women of Group A attend it. For events E_1–E_4, Eleanor and Frances are detached from the rest of Group A by attending none of these events while the remaining members of Group A attend at least one of them. For Group B, Katherine, Sylvia and Nora all attend E_{13} and E_{14} while the other members of Group B do not attend them. This fitted blockmodel resolves the ambiguity concerning the subgroup structure of Group B.

Each of the three partitions (in Tables 5–7) delineates slightly different features of the internal structure of the two larger groups of women in the Deep South data. We could rest at this point but there is the sense that these partitions are rather 'forced'. The pre-specified models were obtained after a close inspection of the data and it is no surprise that the models fit exactly. The trouble is that they are not unique and the blockmodels might be 'over-fitted' through looking closely at the details of the potential blocks. So, we step back and think of

Table 8
Structural equivalence (5, 5)-partition for subgroups

Actor	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}
Evelyn	1	1	1	1	1	1	0	1	1	0	0	0	0	0
Laura	1	1	0	1	1	1	1	1	0	0	0	0	0	0
Theresa	0	1	1	1	1	1	1	1	1	0	0	0	0	0
Brenda	1	0	1	1	1	1	1	1	0	0	0	0	0	0
Charlotte	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Frances	0	0	0	1	1	1	0	1	0	0	0	0	0	0
Eleanor	0	0	0	0	1	1	1	1	0	0	0	0	0	0
Ruth	0	0	0	0	1	0	1	1	1	0	0	0	0	0
Pearl	0	0	0	0	0	1	0	1	1	0	0	0	0	0
Dorothy	0	0	0	0	0	0	0	1	1	0	0	0	0	0
Katherine	0	0	0	0	0	0	0	1	1	1	0	1	1	1
Sylvia	0	0	0	0	0	0	1	1	1	1	0	1	1	1
Nora	0	0	0	0	0	1	1	0	1	1	1	1	1	1
Verne	0	0	0	0	0	0	1	1	1	0	0	1	0	0
Myra	0	0	0	0	0	0	0	1	1	1	0	1	0	0
Helen	0	0	0	0	0	0	1	1	0	1	1	1	0	0
Olivia	0	0	0	0	0	0	0	0	1	0	1	0	0	0
Flora	0	0	0	0	0	0	0	0	1	0	1	0	0	0

one further pre-specified model:

com	com	com	nul	nul
nul	com	com	nul	nul
nul	nul	com	nul	nul
nul	nul	com	com	com
nul	nul	com	com	nul

Again, we fit it with the same heavy penalty on the null blocks. The unique partition (over 100,000 repetitions) is shown in Table 8. There are 44 inconsistencies with structural equivalence with the pre-specified model. Group A is split into {Evelyn, Laura, Theresa, Brenda, Charlotte} and {Frances, Eleanor, Ruth} while Group B is split into {Katherine, Sylvia, Nora} and {Verne, Myra, Helen, Olivia, Flora}. The partition of events in Table 8 differs slightly from the partition of events in Table 7 with {E_1, E_2, E_4} and {E_3, E_5} as the partition of the first five events. The only difference in the partition of the women in Group A between the partitions shown in Table 8 and Table 7 concerns the location of Frances. The null block defined by {Frances, Eleanor, Ruth} and {E_1, E_2, E_4} has considerable appeal in delineating the subgroup structure of Group A. The partition of the women of Group B is identical in the two tables. If anything, we give a slight preference to the partition shown in Table 8 because the specification of null blocks and non-null blocks requires less 'over-fitting' compared to specifying a complex pattern involving complete, regular and row-regular block types. Moreover, it appears to be a unique partition given the pre-specified model.

Table 9
Examples of blocks with types of connections

	Y				
	1	1	1	1	1
X	1	1	1	1	1
	1	1	1	1	1
	1	1	1	1	1

complete

	Y				
	0	1	0	0	0
X	1	1	1	1	1
	0	0	0	0	0
	0	0	0	1	0

row-dominant

	Y				
	0	0	1	0	0
X	0	0	1	1	0
	1	1	1	0	0
	0	0	1	0	1

col-dominant

	Y				
	0	1	0	0	0
X	1	0	1	1	0
	0	0	1	0	1
	1	1	0	0	0

regular

	Y				
	0	1	0	0	0
X	0	1	1	0	0
	1	0	1	0	0
	0	1	0	0	1

row-regular

	Y				
	0	1	0	1	0
X	1	0	1	0	0
	1	1	0	1	1
	0	0	0	0	0

col-regular

	Y				
	0	0	0	0	0
X	0	0	0	0	0
	0	0	0	0	0
	0	0	0	0	0

null

	Y				
	0	0	0	1	0
X	0	0	1	0	0
	1	0	0	0	0
	0	0	0	1	0

row-functional

	Y			
	1	0	0	0
	0	1	0	0
X	0	0	1	0
	0	0	0	0
	0	0	0	1

col-functional

7. Thinking of one-mode data as if they were two-mode data

Recognizing that the row and column partitions may be different makes it possible to view one-mode network data in a different fashion *by thinking of them as two-mode data*. People in an organization can seek advice from each other and form advice networks. Because seeking advice and providing advice are directional activities, the advice networks that they generate will be asymmetric.[7] This suggests that partitioning the rows and columns in different ways will be useful. For social service agencies that refer clients, the act of referral is asymmetric. As a result, these inter-agency one-mode networks can be examined fruitfully as if they were two-mode networks. A final example comes in the form of journal-to-journal networks for academic disciplines. Blockmodeling has been used to partition such networks as one-mode networks in terms of structural equivalence. While the partitions had value, there is a sense in which a partitioning strategy for one-mode data is overly restrictive. If there are 'consumer' journals (as the first set of actors) and 'producer' journals (as the second set of actors), it makes sense to think of partitioning the rows and columns of such citation networks in different ways.

The following discussion requires the use of some of the expanded set of block types presented by Doreian et al. (1994, 2005). Table 9 shows these block types and, from them, we

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[7] If some pairs of actors seek and provide advice to each other, then the advice network will be non-symmetric.

concentrate on expanding the set of null, complete, and regular blocks with the row-regular, column-regular and row-function blocks. See Doreian et al. (1994, 2005) for a pictorial representation of these block types.

7.1. Journal-to-journal citation networks

Journal networks are defined with journals as the vertices and have valued elements. These elements are counts of the number of times articles on one journal cite articles in another journal. The diagonal elements of these (one-mode) journal networks are counts of self-citation for the journals in the network. These (valued) networks have been studied as one-mode networks and partitioned in terms of structural equivalence (Doreian, 1985, 1988). In these partitioning efforts, the rows and columns are clustered in the *same* way and the diagonal elements are ignored. Such networks have also been studied as one-mode core–periphery structures (Borgatti and Everett, 1999). At face value, these one-mode networks are *better viewed* as two-mode networks. When this is done, journals are seen as both 'consuming journals' (in the rows as *citing* journals) and 'producing journals' (in the columns as *cited* journals). And, consistent with the approach taken in this paper, the rows and columns can be clustered in *different* ways. Additionally, the self-citation entries (diagonal elements) can be included as an integral part of the data even though the 'diagonal' elements of the *image* have no meaning.

Viewing citation networks in this fashion allows us to examine the different roles that journals can perform as producers and consumers of knowledge. This can be approached as a simple form of 'hypothesis testing'. If there is no difference between these roles, then the rows and columns, as an empirical matter, will be clustered in the same way. To the extent that rows and columns are clustered differently, we have evidence that journals function differently with regard to the production and consumption of disciplinary knowledge.

In approaching journal networks in this way, we had only some intuitions at the outset. We thought that there would be a core consumer position and a core producer position, consistent with the idea of distinguishing cores from non-cores. Further, to reflect 'coreness', the block corresponding to the two (producer and consumer) cores should be *complete. Every* core consumer journal ought to cite *every* core producer journal and *every* core producer journal ought to be cited by *every* core consumer journal. Second, there will be non-core consumers and non-core producers. Some will vary in the extent to which they approximate a core and others will not be core journals. We speculate that a second level of consumer journals will occupy a position where the block defined by them and the core producer journals is weaker than a complete block. We specify that the (1, 2) block is regular. But differently, for such a set of near-core consumer journals, each will cite at least one core producer journal and every core producer journal will be cited by at least one the second level of near-core consumer journals. In a similar fashion, we think there will be a second level of producer journals whose block with the core consumer journals would be regular—i.e. the (2, 1) block is a regular block. Third, weaker consumer and producer positions will have blocks, involving the core producer and core consumer journals, that would be row-regular and column-regular. Fourth, there will be null blocks. It seems reasonable to expect 'holes' in the structure if journals occupy different niches. There will be sets of consumer journals

Table 10
Journals in social work citation network

Title	Label	Id
Administration in Social Work	ASW	m
British Journal of Social Work	BJSW	n
Child Abuse and Neglect	CAN	j
Child Care Quarterly	CCQ	o
Child Welfare	CW	a
Children and Youth Services Review	CYSR	b
Clinical Social Work Journal	CSWJ	k
Family Relations	FR	l
Journal of Gerontological Social Work	JGSW	p
Journal of Social Policy	JSP	q
Journal of Social Work Education	JSWE	e
Public Welfare	PW	r
Social Casework	SCW	c
Social Services Review	SSR	f
Social Work	SW	d
Social Work with Groups	SWG	g
Social Work in Health Care	SWHC	h
Social Work Research and Abstracts	SWRA	i

that never cite some of the producer journals and there will sets of producer journals that are not cited by sets of consumer journals. Finally, we need to allow for certain 'almost null' blocks: some 'diagonal' elements might cause some problems by appearing in erstwhile null blocks. If there are consumer journals that cite some of the core journals and then only themselves, these self-citation counts will appear in null blocks. This suggested the use of the row-function block type to allow a single non-zero element to appear in each row of such a null block. The specified model is:

com	reg	cre	rre
reg	reg	nul	nul
reg	nul	rfn	nul
rre	rre	nul	rfn
cre	nul	nul	rfn

With the above arguments in mind, we re-analyze the social work journal network reported by Baker (1992) and analyzed by Borgatti and Everett (1999). The included journals are listed in Table 10. In addition to thinking in terms of blockmodeling these data as two-mode data, there is one important difference between out approach and that of Borgatti and Everett. The original data are not symmetric and, in contrast to Borgatti and Everett, we do not symmetrize them for a simple reason. When the data are symmetrized there can be no distinction between production of, and consumption of, knowledge. A two-mode analysis of such a symmetrized citation network would yield the same partition of the rows and columns. This is especially true when a symmetrized element is constructed from a non-zero and zero citation rate for a pair of journals.

Table 11
Journal citation matrix with the (5, 4)-partition

	Id	a	c	f	d	m	e	g	h	i	j	k	l	n	o	b	p	q	r
CW	a	187	32	10	58	0	11	0	0	0	7	0	0	0	0	6	0	0	7
CYSR	b	70	8	14	28	0	0	0	0	5	12	0	0	0	5	26	0	0	6
SCW	c	17	149	36	124	8	21	8	6	18	6	8	6	0	0	0	6	0	0
SW	d	52	58	53	356	15	33	15	43	8	0	0	9	0	0	0	0	0	19
JSWE	e	0	18	16	58	9	104	0	7	16	0	0	0	0	0	0	0	0	0
SSR	f	17	30	105	106	7	9	0	0	25	0	0	0	0	0	0	0	0	0
SWG	g	0	9	7	40	0	9	41	9	0	0	0	0	0	0	0	0	0	0
SWHC	h	0	20	0	26	0	0	0	86	0	0	0	0	0	0	0	0	0	0
SWRA	i	8	8	39	44	0	24	0	0	40	0	0	0	0	0	0	0	0	0
CAN	j	9	6	0	8	0	0	0	0	0	109	0	0	0	0	0	0	0	0
CSWJ	k	0	47	20	45	0	0	0	0	0	0	40	0	0	0	0	0	0	0
FR	l	0	18	0	9	0	0	0	0	0	0	0	205	0	0	0	0	0	0
ASW	m	0	0	21	73	70	18	0	0	7	0	0	0	0	0	0	0	0	13
BJSW	n	0	0	0	19	0	13	0	0	0	0	0	0	95	0	0	0	0	0
CCQ	o	12	0	0	0	0	0	0	0	0	0	0	0	0	92	0	0	0	0
JGSW	p	0	16	0	18	0	0	0	0	0	0	0	0	0	0	0	9	0	0
JSP	q	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	35	0
PW	r	0	0	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0	9

We leave out two of the journals included by Baker. One is the Indian Journal of Social Work (IJS) really does not belong to the network.[8] It is linked weakly with only one other journal (Table 10). Also eliminated was Administration in Mental Health because it was linked weakly to only one other journal. This leaves the 18 journals whose citation volumes are shown in Table 11.

Table 11 has been organized so that rows columns that have been permuted to reflect a partitioned structure with five consumer positions and four producer positions. The producer positions (where PJ_i is used to label producer clusters) are:

PJ_1	{CW, SCW, SSR, SW}
PJ_2	{ASW, JSWE, SWG, SWHC, SWRA}
PJ_3	{CAN, CSWJ, FR}
PJ_4	{BJSW, CCQ, CYSR, JGSW, JSP, PW}

and the consumer positions (where CJ_j is used to label the consumer positions) are:

CJ_1	{CW, CYSR, SCW, SW}
CJ_2	{JSWE, SSR, SWO, SWHC, SWRA}
CJ_3	{CAN, CSWJ, FR}
CJ_4	{ASW, BJSW}
CJ_5	{CCQ, JGSW, JSP, PW}

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[8] Consulting members of a social work faculty about these data, all indicated that US, regardless of its merits, is not visible in the US and is seldom or never consulted.

It is clear that the consuming positions differ from the producing positions. There is a single inconsistency between the fitted blockmodel and the corresponding ideal blockmodel. This is the bolded element (7) in the block defined by the fifth row position and the third column position.

Three journals (CW, SCW, SW) belong to both of the core positions—they are core producers in PJ_1 and are core consumers in CJ_1. Social Service Review (SSR) is in PJ_1 but not CJ_1 while Children and Youth Services Review (CYSR) is CJ_1 but is not CP_1. SSR does appear in the second consumer position. Four journals (JSWE, SWG, SWHC, SWRA) are common to the second producer position (PJ_2) and the second consumer position (CJ_2). Together, these most prominent journals vary in their roles as producers and consumers of social work knowledge. We note that the block defined by CJ_2 and PJ_2 is regular. This is also the case for the block defined by CJ_1 and PJ_2. The third producer position (PJ_3) and the third consumer position (CJ_3) share the same set of journals, namely CAN, CSWJ, and FR. The block defined by CJ_3 and PC_1 regular as is the block defined by CJ_1 and PC_3 (although we only specified it as row-regular). The block defined by CJ_3 and PJ_3 is a row-functional block. Apart from these block types, the blocks associated with these positions are null blocks. The block defined by CP_4 and CJ_1 is row-regular while the block defined by CP_1 and CJ_5 is column-regular. Apart from these, the blocks for the last consumer and producer positions are null or row-functional. The one inconsistency for the fitted blockmodel is the tie from PW to CAN. The bipartite graph (for incidence of ties but not their values) is shown in Fig. 5 where the journals in each of the producer and consumer positions are grouped together.

There is an interesting contrast with the analysis of Borgatti and Everett (1999) who found a single core in a core–periphery structure. The core they established was {SSR, SCW, SW}. These three journals are in our producer core (PJ_1) but only SCW and SW are in the consumer core (CJ_1). In our analysis, the producer core includes also CW while the core consumer includes CW and CYSR. Analyzing these data with a two-mode blockmodeling approach provides a more differentiated view of the core.

8. Summary and discussion

While (generalized) blockmodeling has been used primarily as a powerful way of modeling one-mode network data, we have extended (generalized) blockmodeling to examine two-mode data. To do this, it is necessary to think in terms of partitioning the rows and columns *separately* as a part of the overall partitioning procedure. Two examples, the Supreme Court data and the Southern Women data, were two-mode data sets. For the former we used structural equivalence in an unrestricted fashion and, for the latter, we used the same equivalence idea but added to it the use of a pre-specified blockmodel.

The partitions returned for both of these data sets are straightforward and compelling. To the extent that the 'true' partition stemming from Freeman's meta-analysis of the many prior analyses of the Southern Women data provides a benchmark, the two-mode block-modeling approach discussed here does delineate the true structure of the subgroup formation for these data, consistent with the analysis of Freeman and White (1994). Additionally,
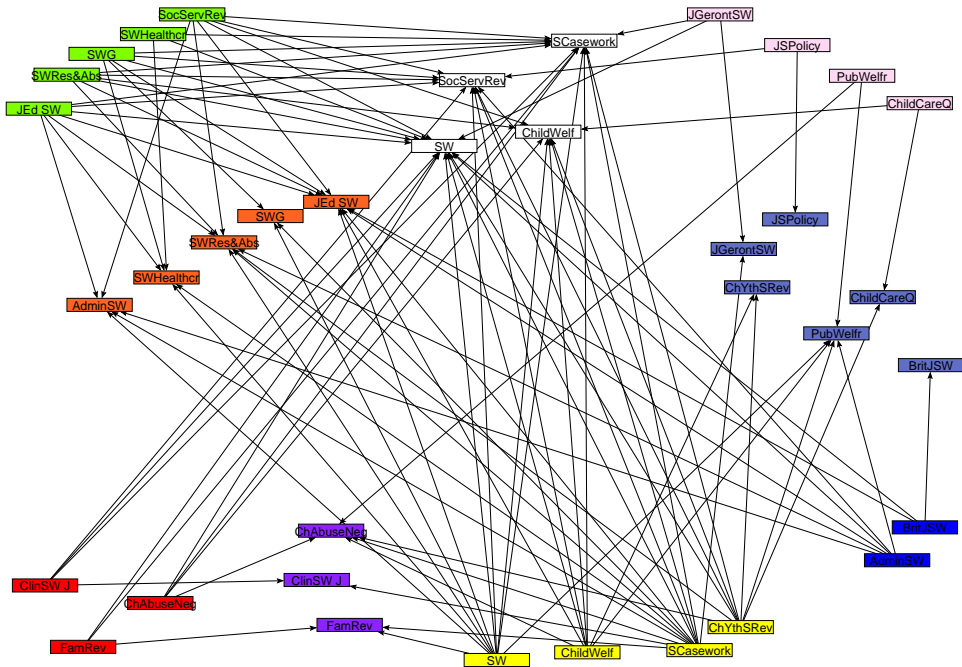
Fig. 5. Social Work Journal Bipartite (5, 4) Network.

the corresponding partition of the events is obtained. In the same fashion, the joint partition of justices and decisions of the Supreme Court data provides considerable insight into the political stance of the justices and the events (decisions) that reveal their differences.

The idea of partitioning the rows and columns in different ways is imposed by the nature of two-mode data. There is no other way to proceed. However, the idea of partitioning rows and columns in distinct ways need not be restricted to two-mode data and can be used for one-mode data. For symmetric one-mode data structures, there is no reason to use two-mode data analytic methods. However, for asymmetric and non-symmetric one-mode networks, there is much to be gained by viewing them through the two-mode lens. The third example, in the form of a journal-to-journal citation network, shows the utility of treating one-mode data as though they were two-mode data. Journals differ in terms of their producer and consumer positions and roles in the communication of knowledge.

We offer one final caution because we have found that fitting 'two-mode blockmodels' is difficult. On the positive side, considerable subtleties in the partition structures can be delineated through pre-specifying a complex pattern of block types. The potential downside is that a lot of 'forcing' or 'over-fitting' can occur. We attach particular importance for obtaining unique partitions given a pre-specification even if this may create an overly stringent criterion for accepting the adequacy of a fitted blockmodel. The liberal use of

regular, row-regular and column-regular block types runs the risk of leading to multiple equally well-fitting partitions. No doubt, having a larger set of block types that are attuned to particular substantive contexts will help resolve such ambiguities in fitting two-mode blockmodels.

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