Planning and control of dynamic contact manipulation

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Center for the Foundations of Robotics Talk
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Introduction

The manipulation problem

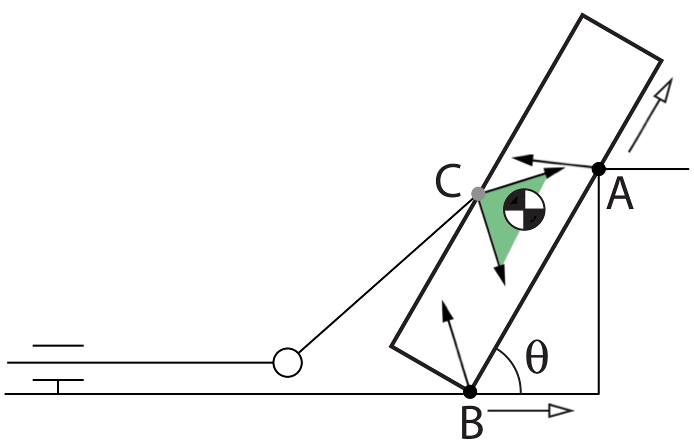
- Move object from start to goal
- Maintain desired contact conditions - rolling, sliding

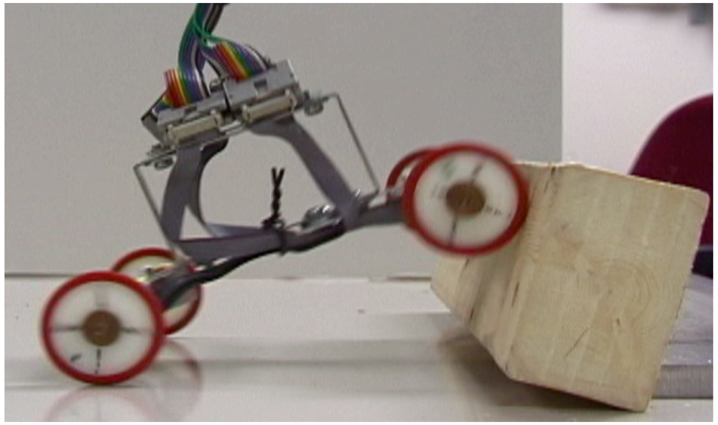
Why is it hard

- Friction is non-Newtonian
- Constraints exist in different spaces
  - *Contact force constraints*
  - *Contact velocity constraints*

Our goal

- Combine constraints
- The set of feasible control and feasible trajectories
- Feedback controllers
- Properties of the system - reachability, controllability





Outline

Motivation

Two example problems

Feedback control

Global controllability

Future work

Dynamic Contact Manipulation

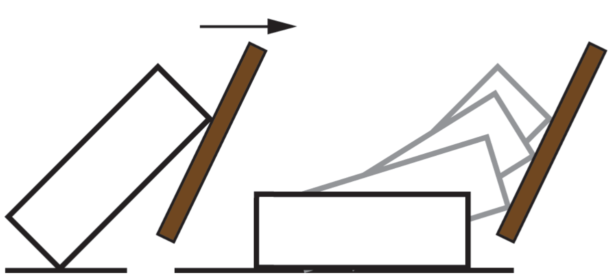
Dynamic :

- Controlling the motion of the object during its dynamic phase
- NOT : Having plans where the object moves dynamically
  - *Nodes = statically stable configurations*
  - *Links = discrete actions that cause dynamic motion*

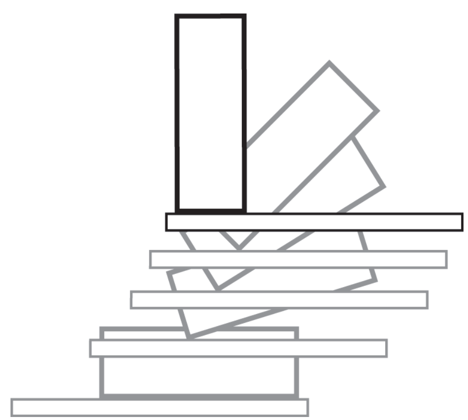
Caveats :

- Requires high manipulator speed
- Harder to act on a moving object, might require feedback
- Open-loop unstable, lose the funneling effect

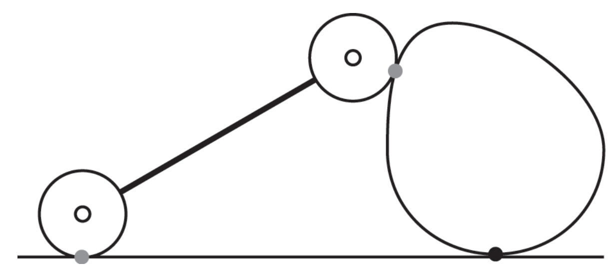
Error recovery



Richer actions



Necessity



Outline

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- Example problem #1 - The Mobipulator

Planning

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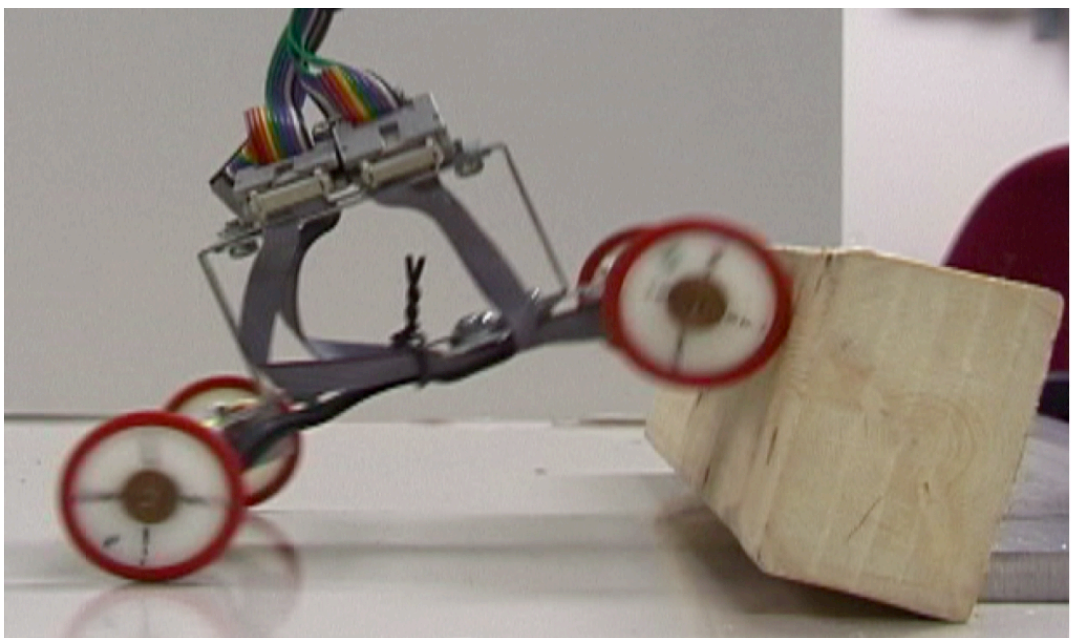
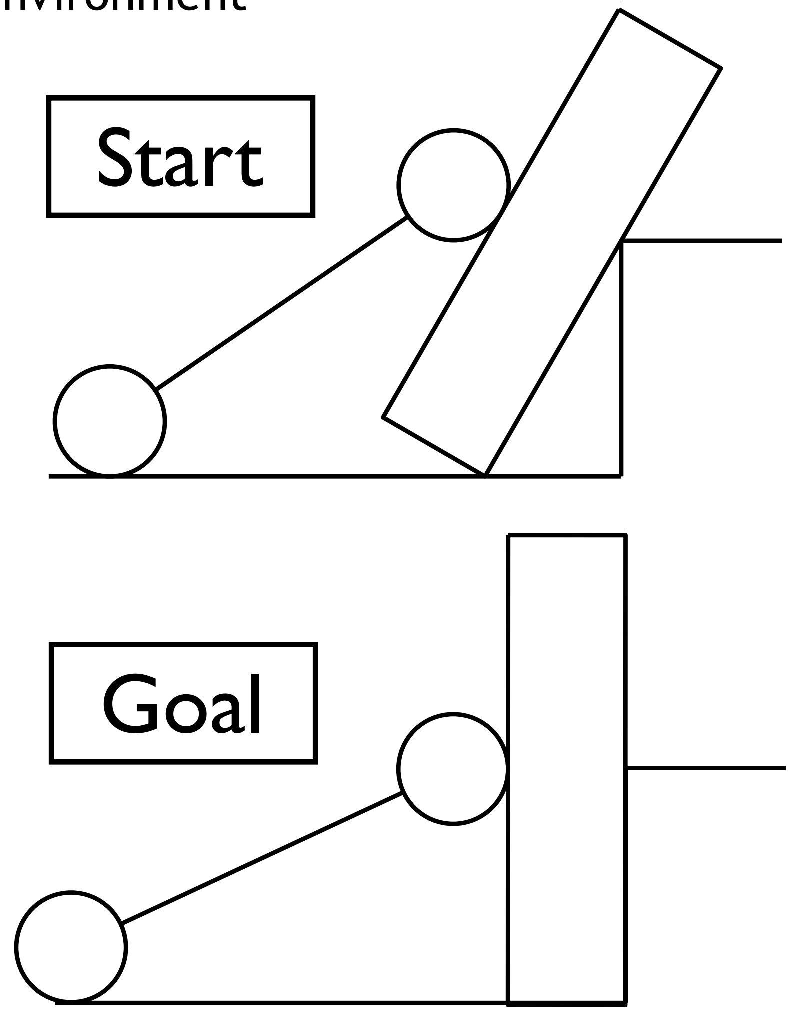
Example problem #1

Mobipulator standing a block up

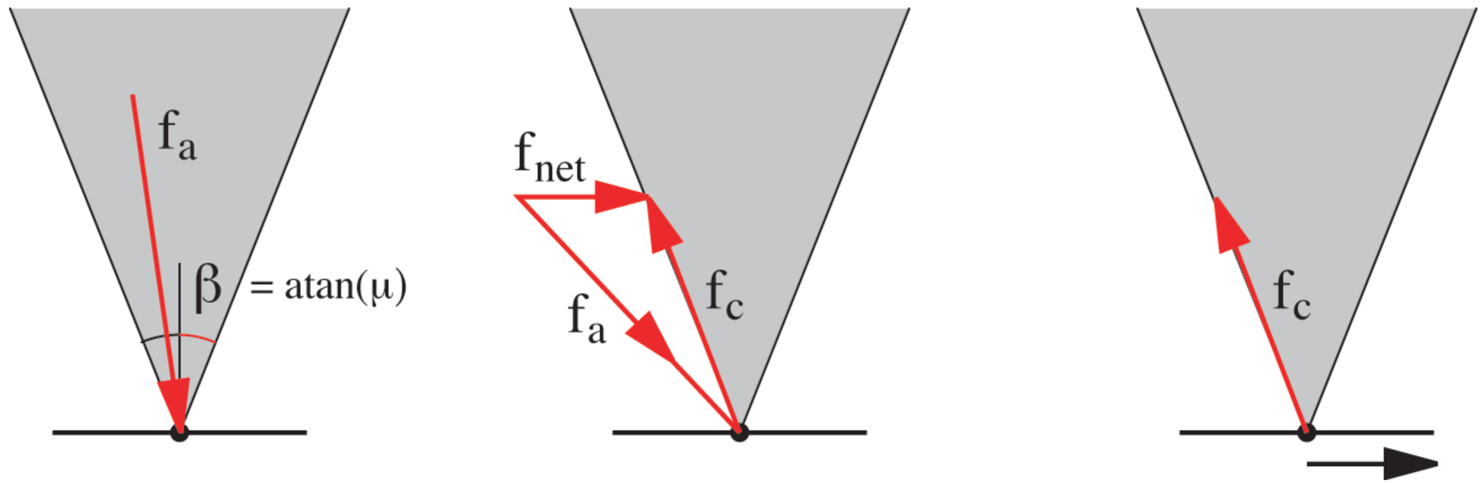
Why is it interesting?

- Contacts both with the robot and the environment
- No quasi-static solution





Friction



Fixed contact | Impending slip | Right sliding

Model : dry Coulomb friction [1872]

Non-Newtonian

- There can be no solution or multiple solutions (Painleve's paradox)

Forward dynamics problem

- Given an applied wrench, what is the motion of the object?
- Erdmann [1984] - Configuration space friction cones
- Lotstedt [1984] and others - Framed it as a linear complimentarity problem

Constraints

Constraints on the system

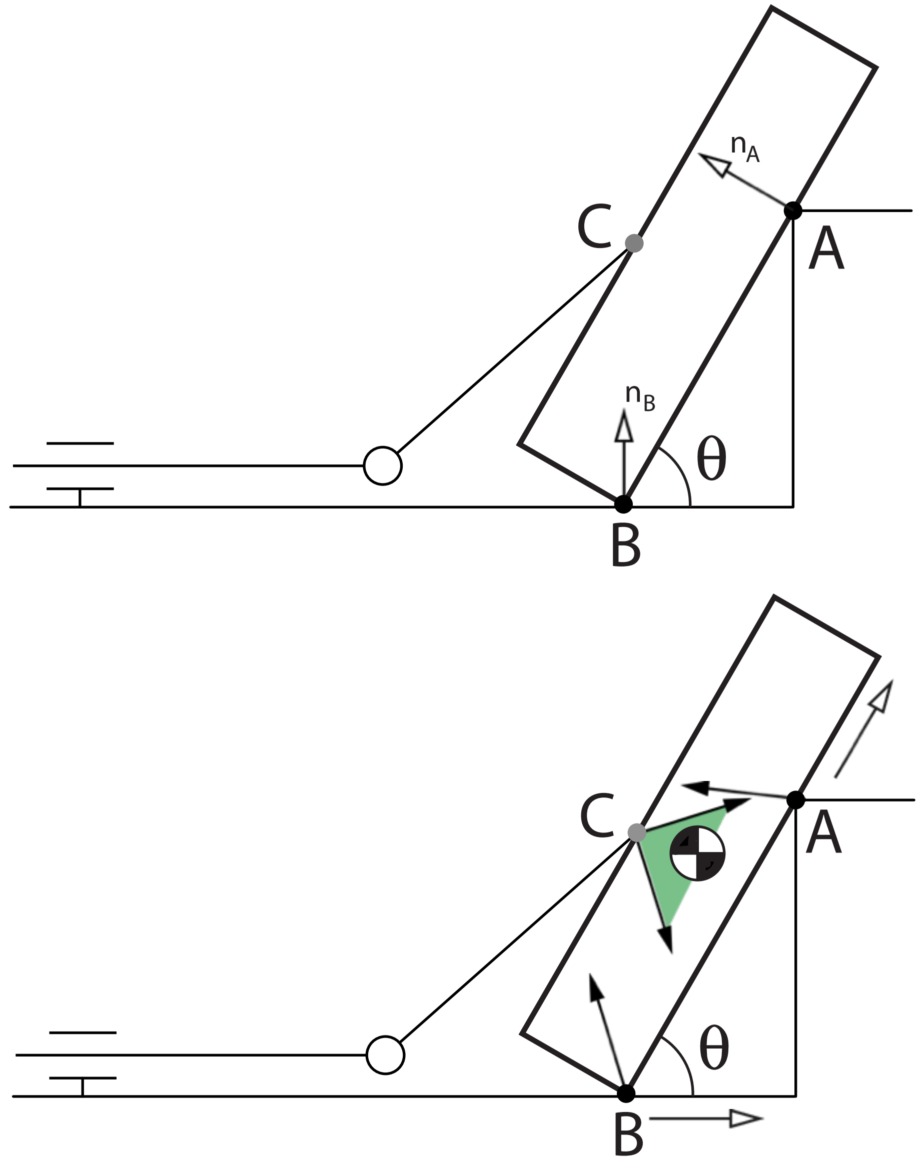
- Fixed contact at C
- Sliding contact at A and B

$$\boldsymbol{v_A} \cdot \boldsymbol{n_A} = 0$$

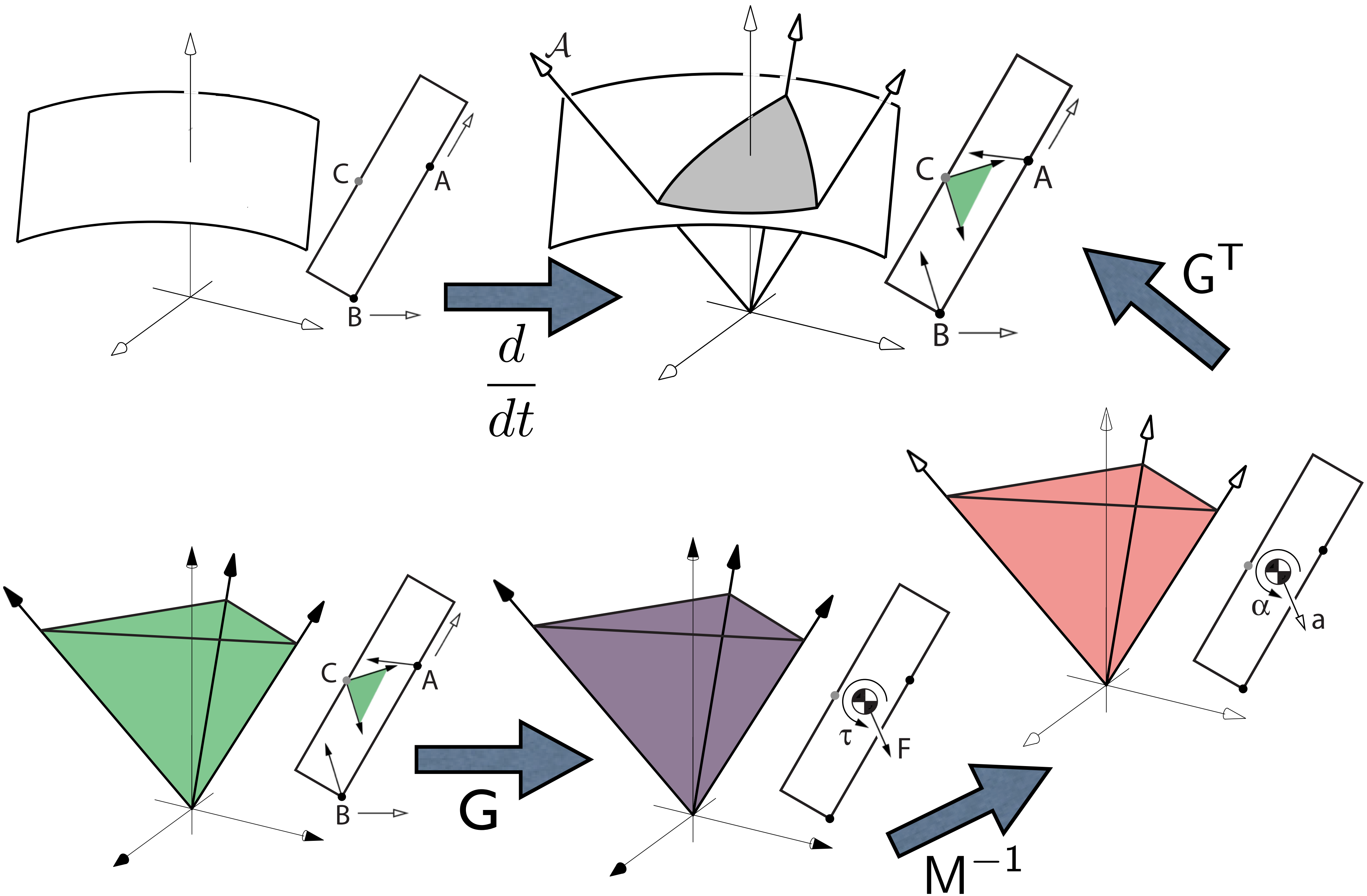
$$\boldsymbol{v_B} \cdot \boldsymbol{n_B} = 0$$

$$\boldsymbol{v_C} = \boldsymbol{v_R}$$

$$\boxed{\begin{aligned} \boldsymbol{f_C} &\in \mathcal{F}_C \\ \boldsymbol{f_A} &\in \mathcal{F}_{A,L} \\ \boldsymbol{f_B} &\in \mathcal{F}_{B,L} \end{aligned}} \qquad \dot{\theta} > 0$$



Combining constraints in contact acceleration space



Combining constraints

Given

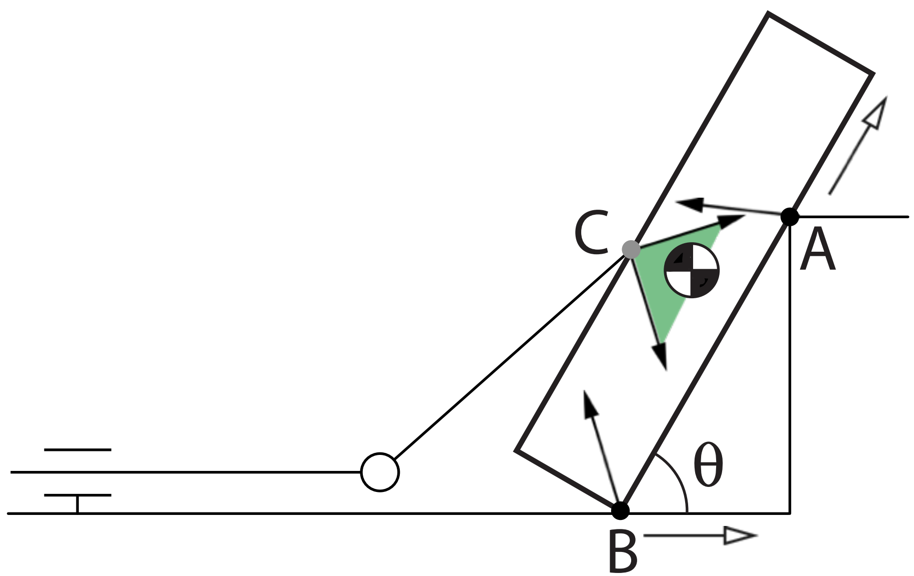
- Configuration and velocity of the system
- Desired contact conditions
  - *Force constraints*
  - *Velocity constraints*

Theorem : Allowable robot joint acceleration must satisfy

$$\boxed{\mathrm{J_c}\ddot{\boldsymbol{q}}_{\boldsymbol{r}}} + \mathrm{V}(\dot{\boldsymbol{q}}_{\boldsymbol{o}}, \dot{\boldsymbol{q}}_{\boldsymbol{r}}, \boldsymbol{n}_{\boldsymbol{o}}) \in \boxed{\mathcal{A}}$$

Robot joint acceleration

Contact space acceleration cone

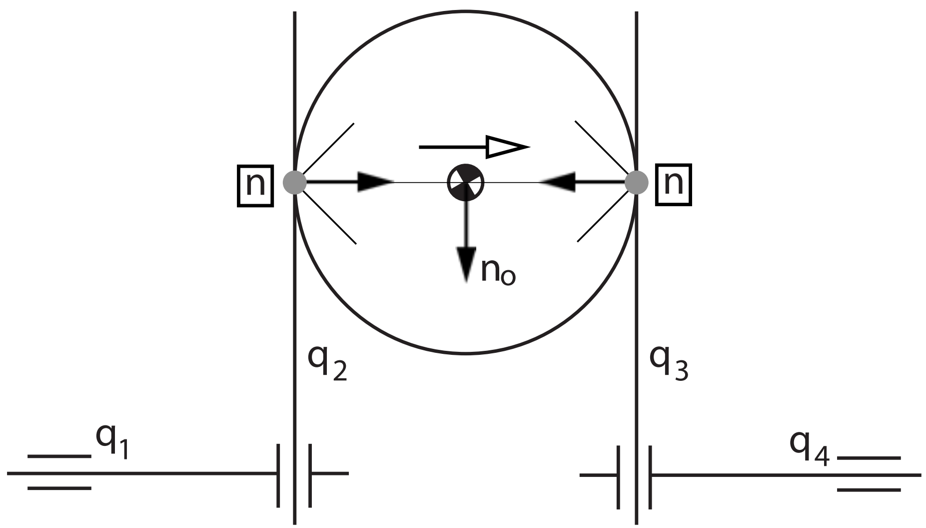


Control synthesis for dynamic contact manipulation
Srinivasa, Mason and Erdmann
IEEE ICRA 2005

However ...

The condition is necessary but not sufficient if there exist internal forces

- Internal force - a force that produces no acceleration of the object
- Sits in the nullspace of our projection map



$$\ddot{q}_2 = \ddot{q}_3 = 0$$

$$\ddot{q}_1 = \ddot{q}_4$$

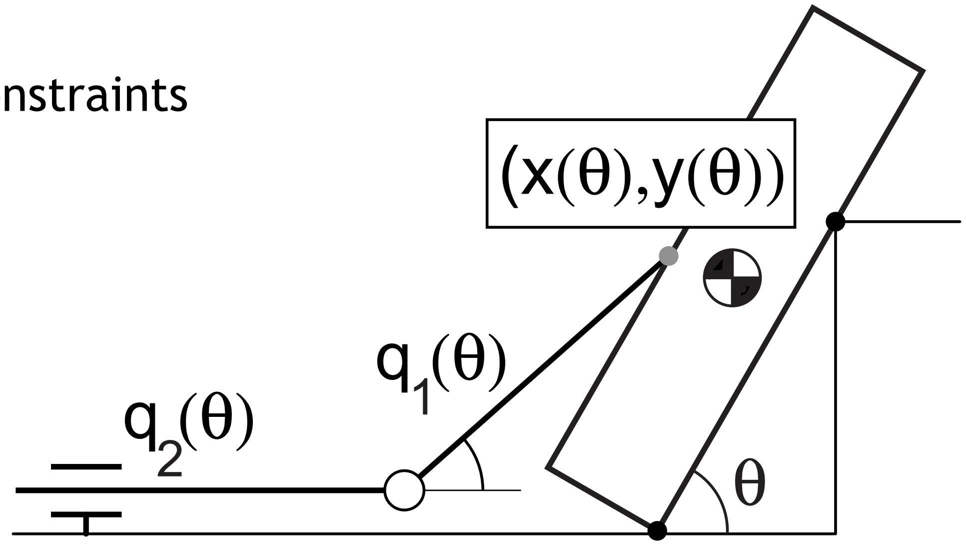
Time-scaling

Decouple trajectory generation into

- Path planning in configuration space : $q(s)$
- Time-scaling the path - planning in phase space : $(s, \dot{s})$

For our problem

- 1D configuration space with the constraints
- Use θ as time-scaling parameter
- Add manipulator joint limits



$$\mathsf{J_c}\ddot{\boldsymbol{q}}_{\boldsymbol{r}} + \mathsf{V}(\dot{\boldsymbol{q}}_{\boldsymbol{o}}, \dot{\boldsymbol{q}}_{\boldsymbol{r}}, \boldsymbol{n}_{\boldsymbol{o}}) \in \mathcal{A}$$

$$\Downarrow$$

$$\boldsymbol{u_1}(s)\ddot{s} + \boldsymbol{u_2}(s)\dot{s}^2 + \boldsymbol{u_3}(s) \in \mathcal{A}(s)$$

Minimum-time control of robotic manipulators with geometric path constraints
Shin, McKay, IEEE TRA 1985

Time-optimal control of robotic manipulators along specified paths
Bobrow, Dubowsky, Gibson
IJRR 1985

The contact acceleration constraints

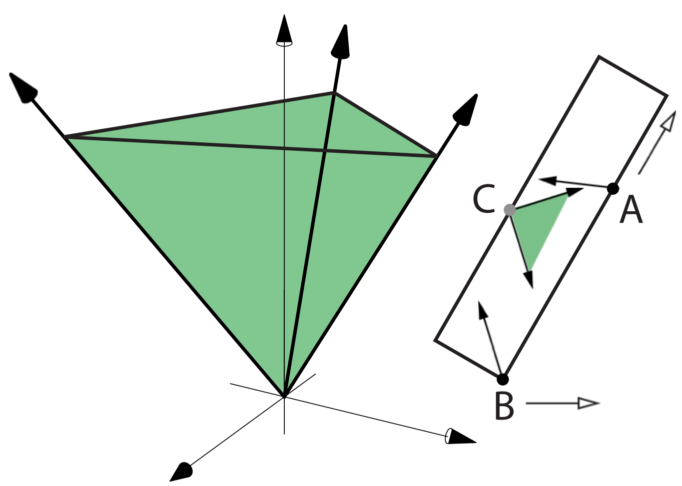
For polyhedral convex cones

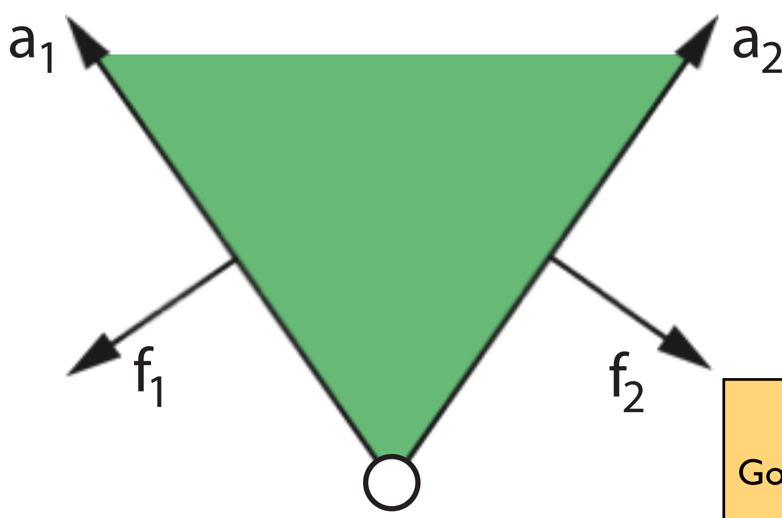
- Span representation - positive linear span of vectors
- Face normal representation - intersection of half spaces

Use face normal representation to get inequalities

$$\boldsymbol{u_1}(s)\ddot{s} + \boldsymbol{u_2}(s)\dot{s}^2 + \boldsymbol{u_3}(s) \in \mathcal{A}(s)$$

$$\mathrm{F_A}\left(\boldsymbol{u_1}(s)\ddot{s} + \boldsymbol{u_2}(s)\dot{s}^2 + \boldsymbol{u_3}(s)\right) \preceq \boldsymbol{0}$$

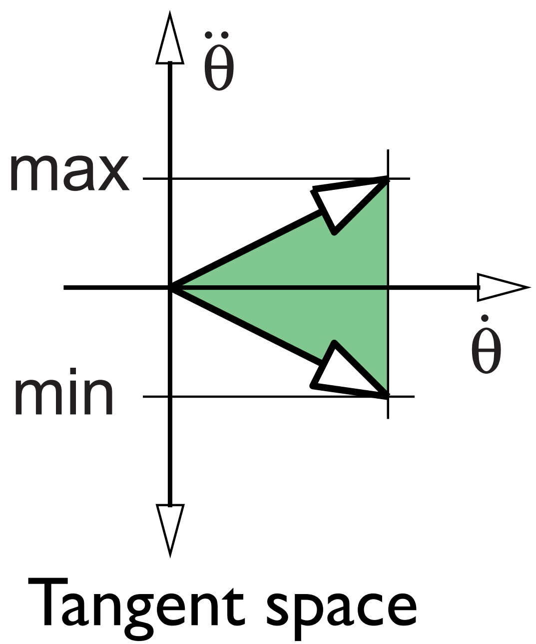


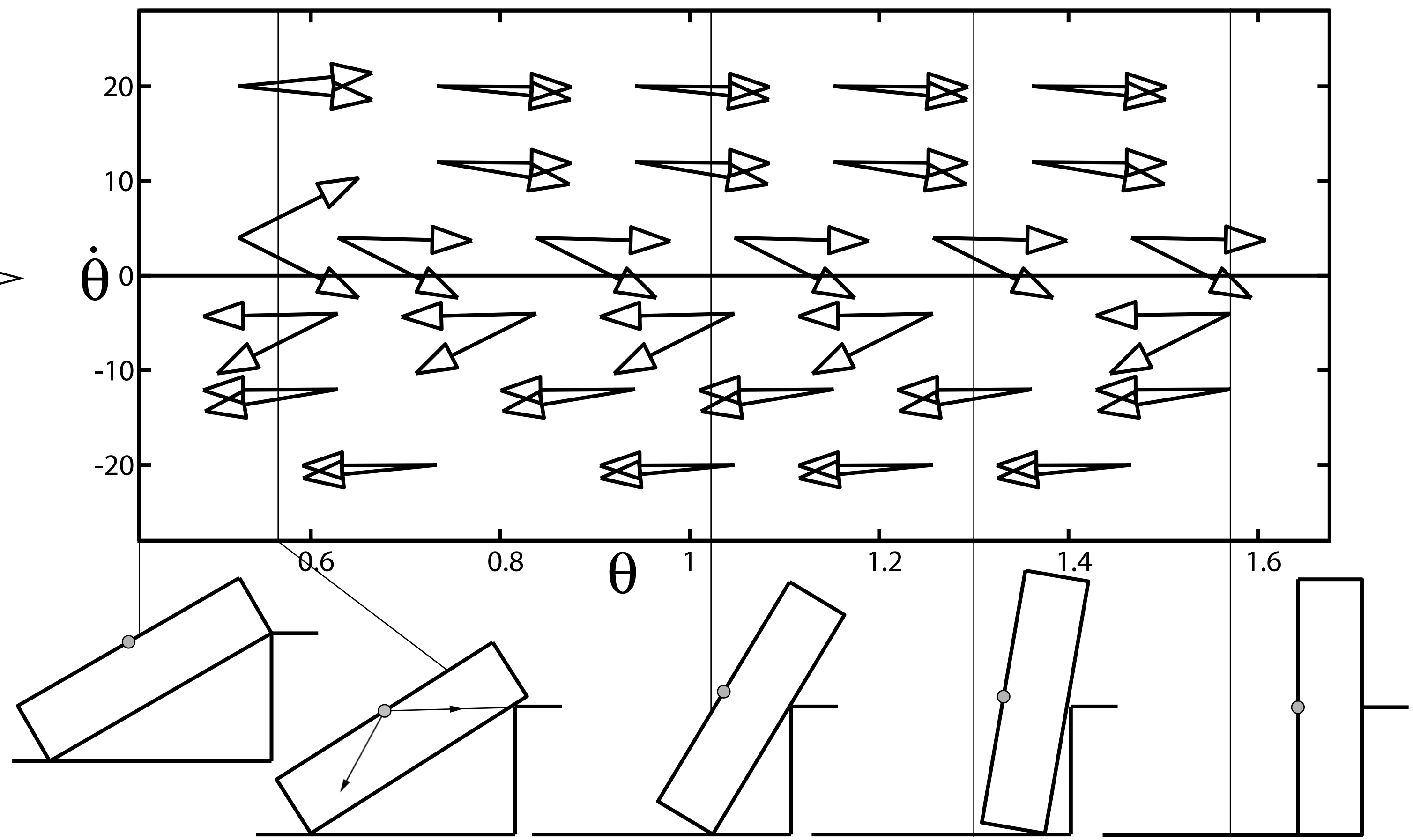


Polyhedral convex cones
Goldman, A.W. and Tucker, A.J.
In Linear inequalities and related systems 1956

The phase space constraints

$$\mathsf{F_A}\left(\boldsymbol{u_1}(s)\ddot{s} + \boldsymbol{u_2}(s)\dot{s}^2 + \boldsymbol{u_3}(s)\right) \preceq \mathbf{0}$$

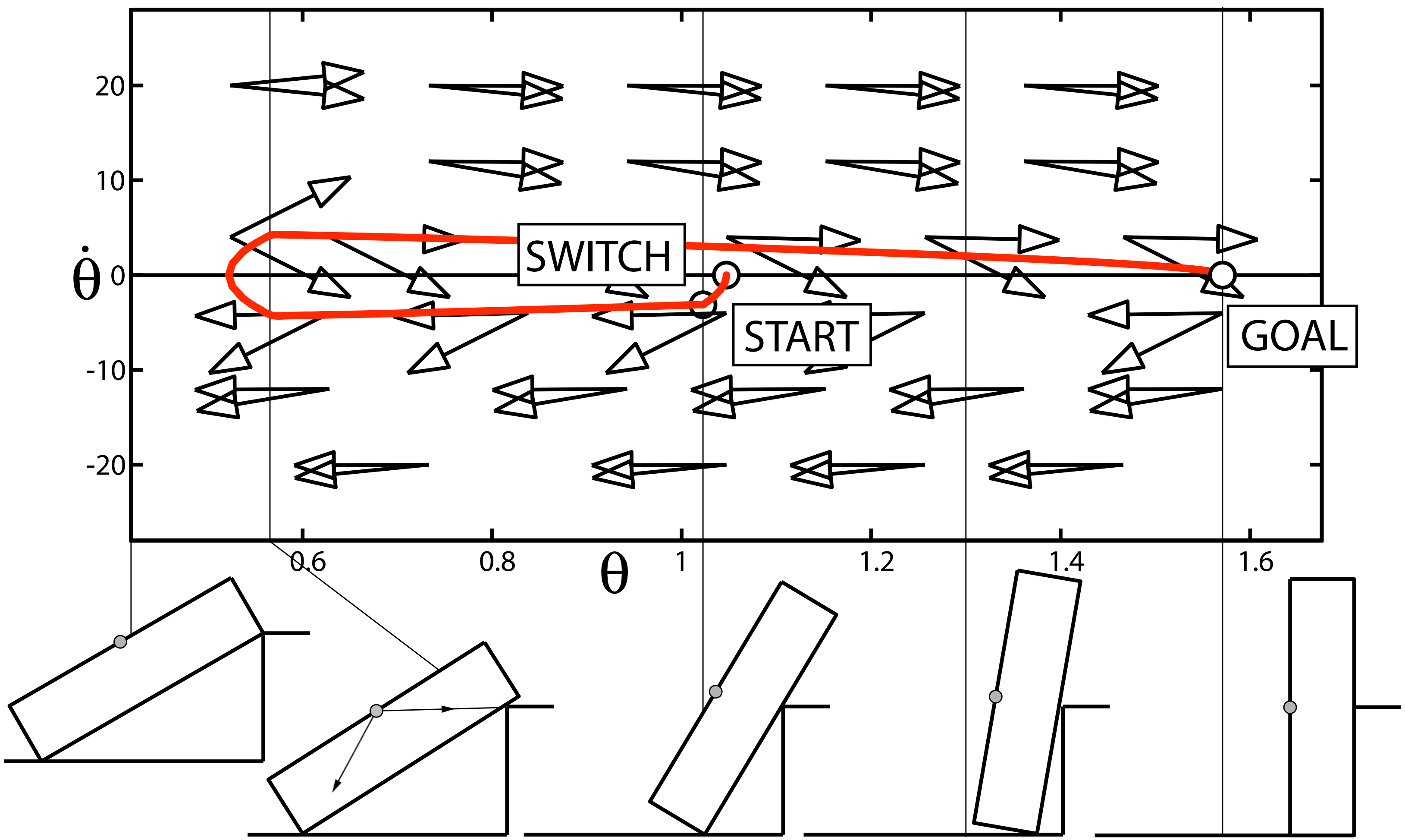




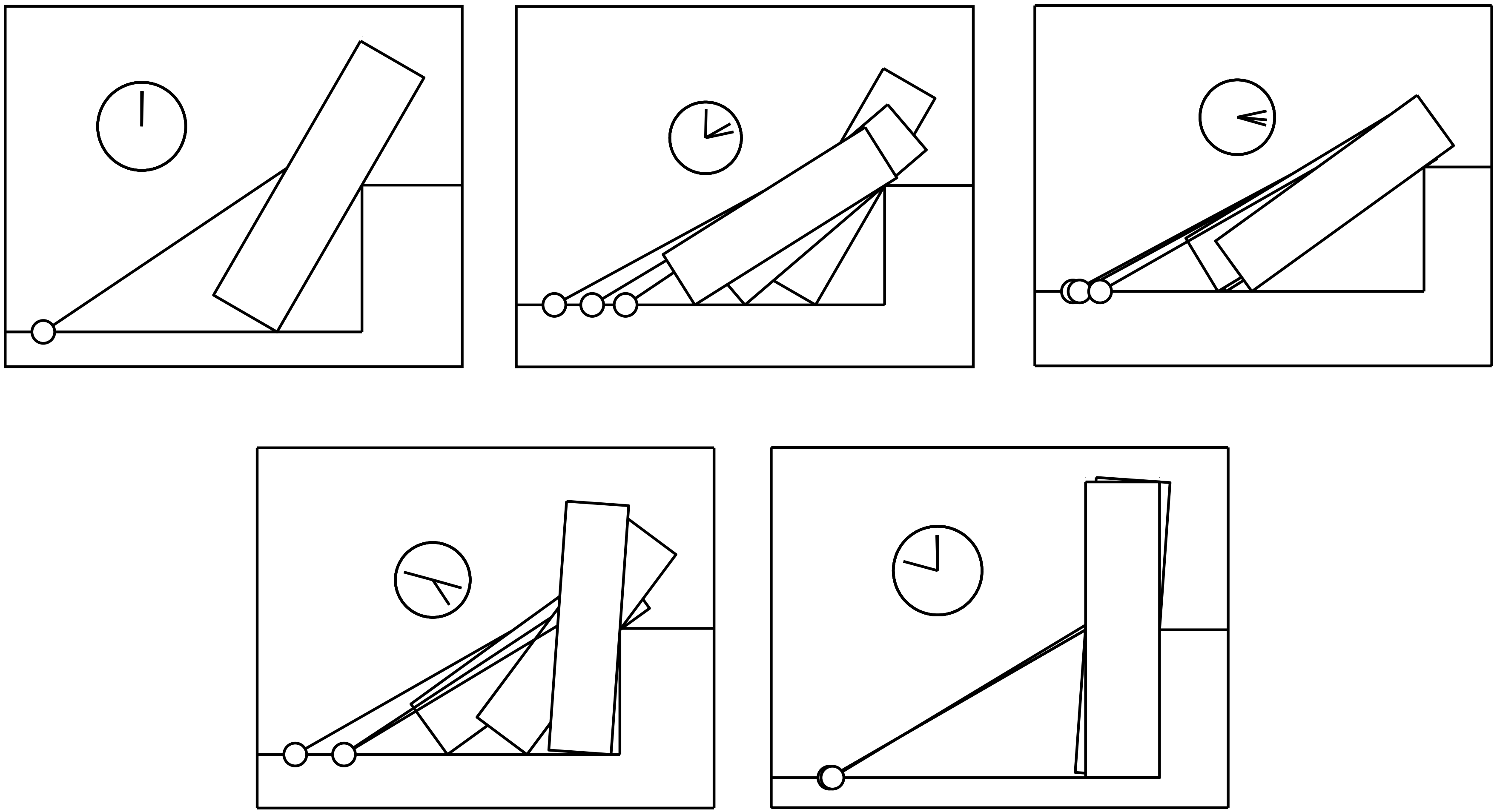
Phase space path

A simple algorithm

- Choose a vector field to follow from start
- Back project from goal
- Switch at intersection point



The final trajectory

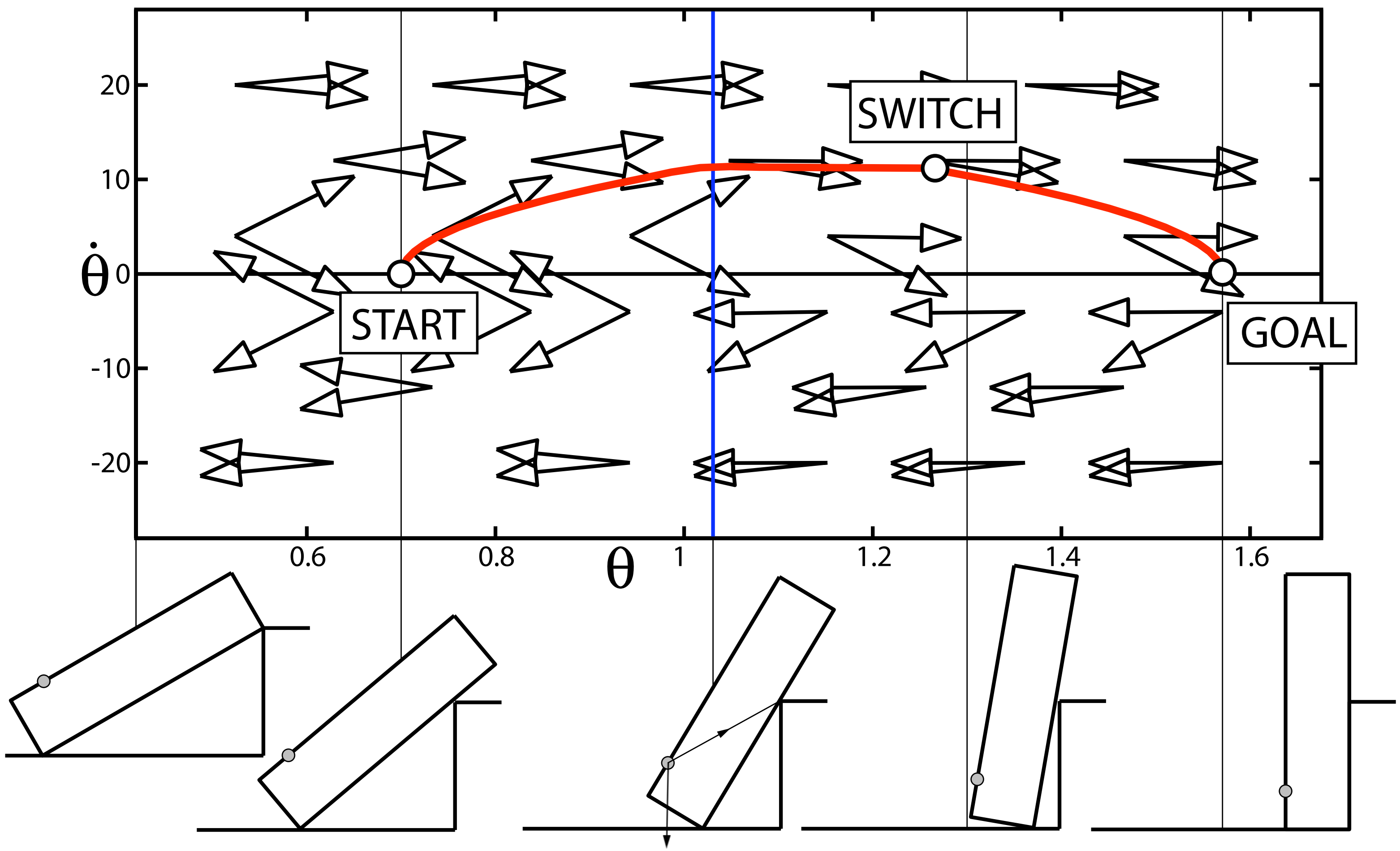


Other plans

Alternative strategy

- There are plans where rolling back is not necessary
- Release contact, move down, and push forward

There is a critical configuration at which plans change



Summary

The contact acceleration constraint

Time-scaling for motion planning

Dual representation of cones for planar problems

Dynamic trajectory generation where no quasi-static solution exists

Problems with decoupling

- What if only a small subset of paths can be successfully time-scaled?

Outline

Motivation

Two example problems

- Example problem #1 - The Mobipulator
- Example problem #2 - Block tipping

Feedback control

Global controllability

Future work

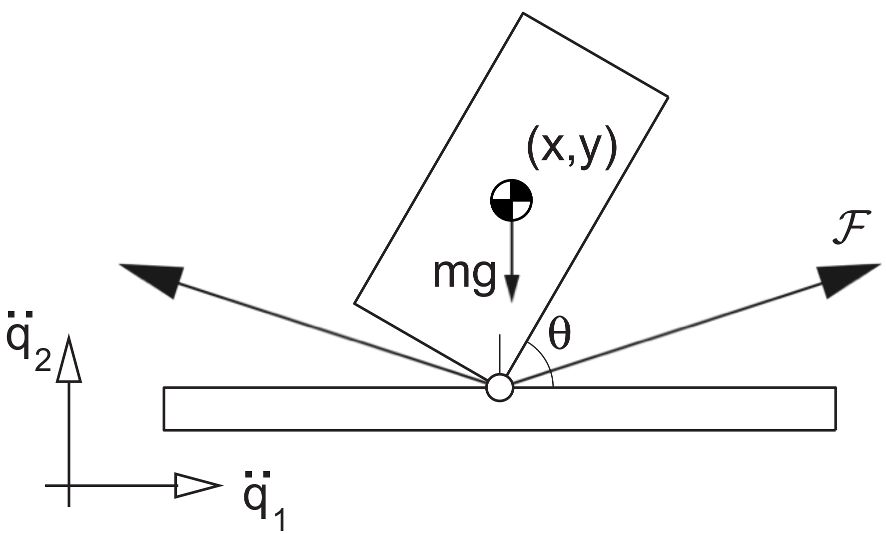
Example problem #2

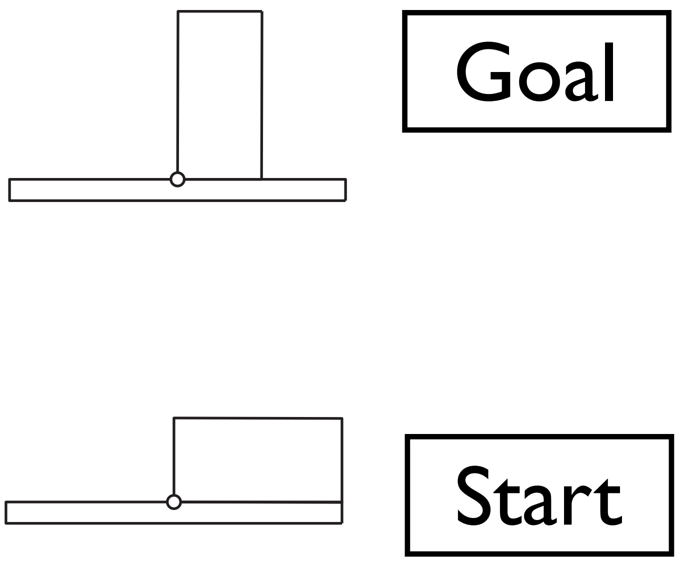
Tip a block resting on a palm

- Go from θ = 0 to 90 while maintaining rolling contact
- Control the motion of the palm in (x,y). Palm only translates.
- Acceleration of the palm bounded

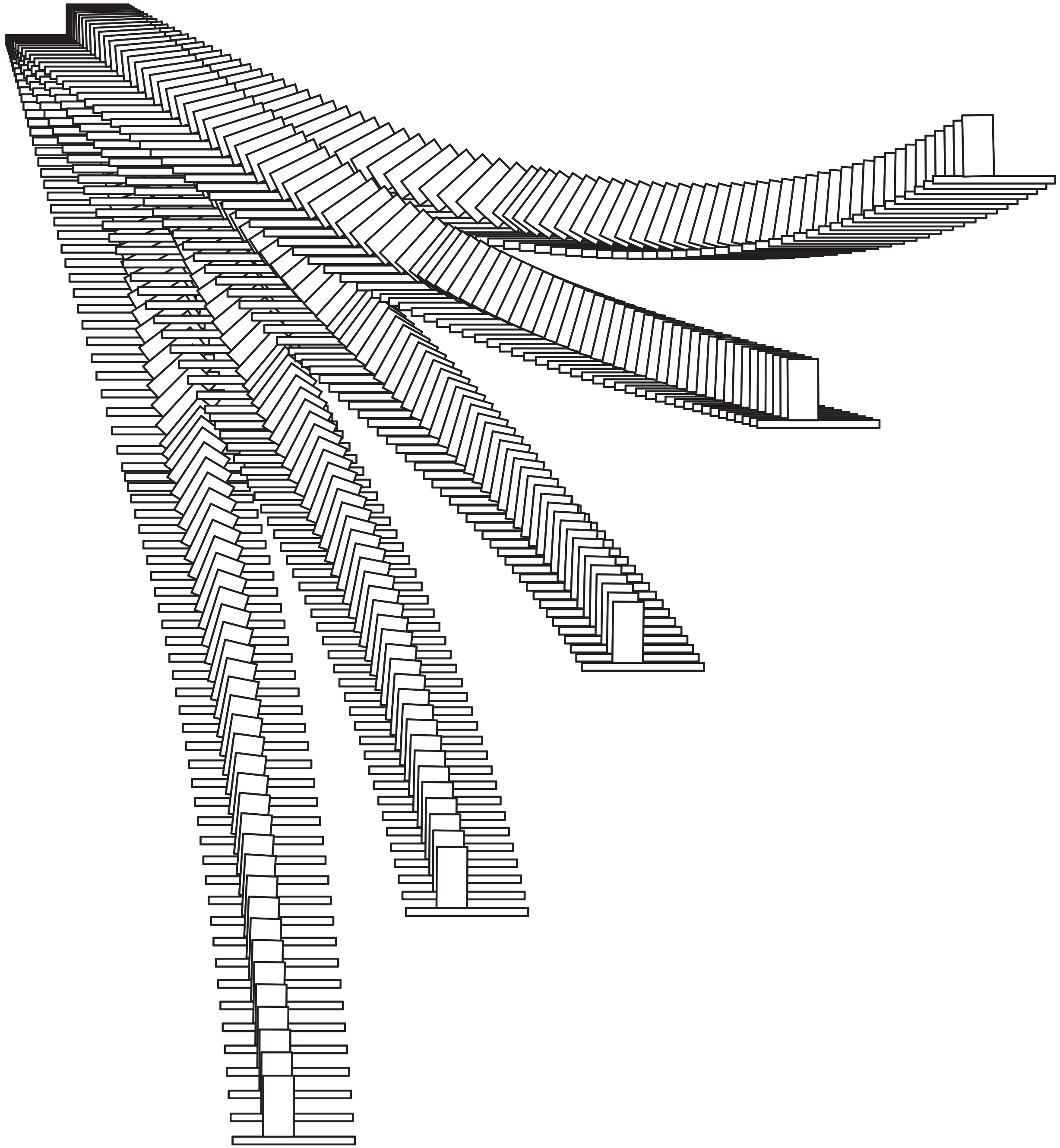
Why is it interesting

- Richer actions
- Hard to find feasible paths in (x,y, θ) that can be time-scaled
- Lynch and Mason [1999] - 1 DOF revolute arm to tip, throw and catch



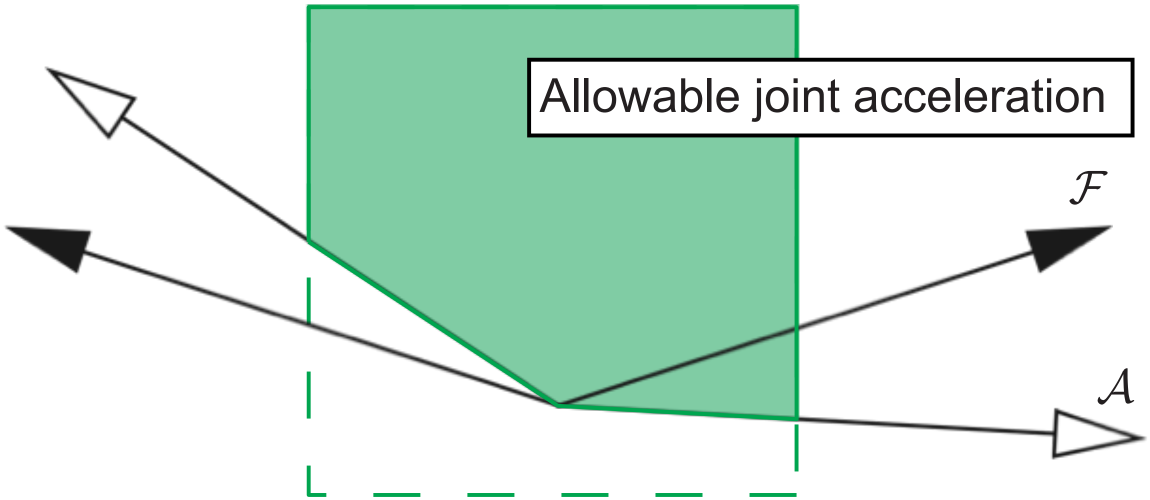


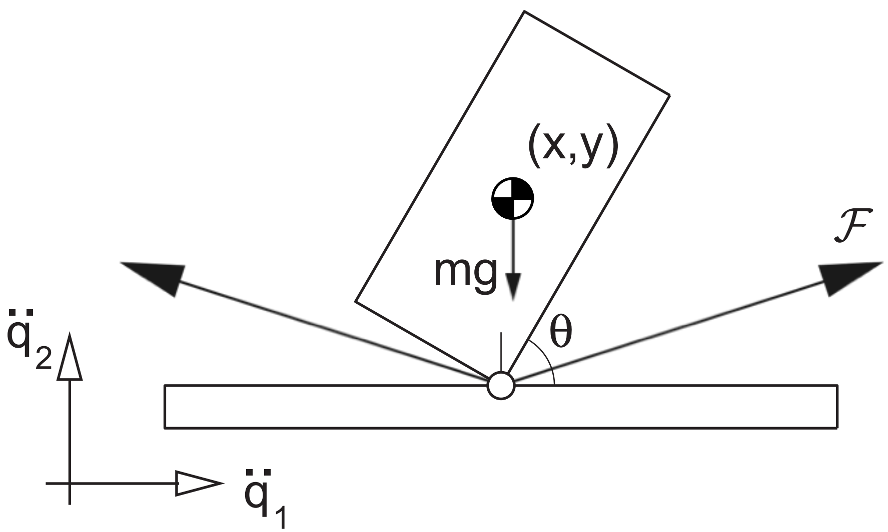
All feasible trajectories

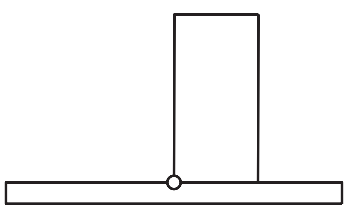


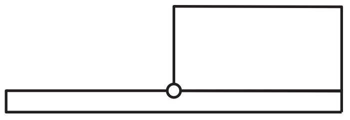
Contact space constraints

$$\mathsf{J_c}\ddot{\boldsymbol{q}}_{\boldsymbol{r}} + \mathsf{V}(\dot{\boldsymbol{q}}_{\boldsymbol{o}}, \dot{\boldsymbol{q}}_{\boldsymbol{r}}, \boldsymbol{n}_{\boldsymbol{o}}) \in \mathcal{A}$$









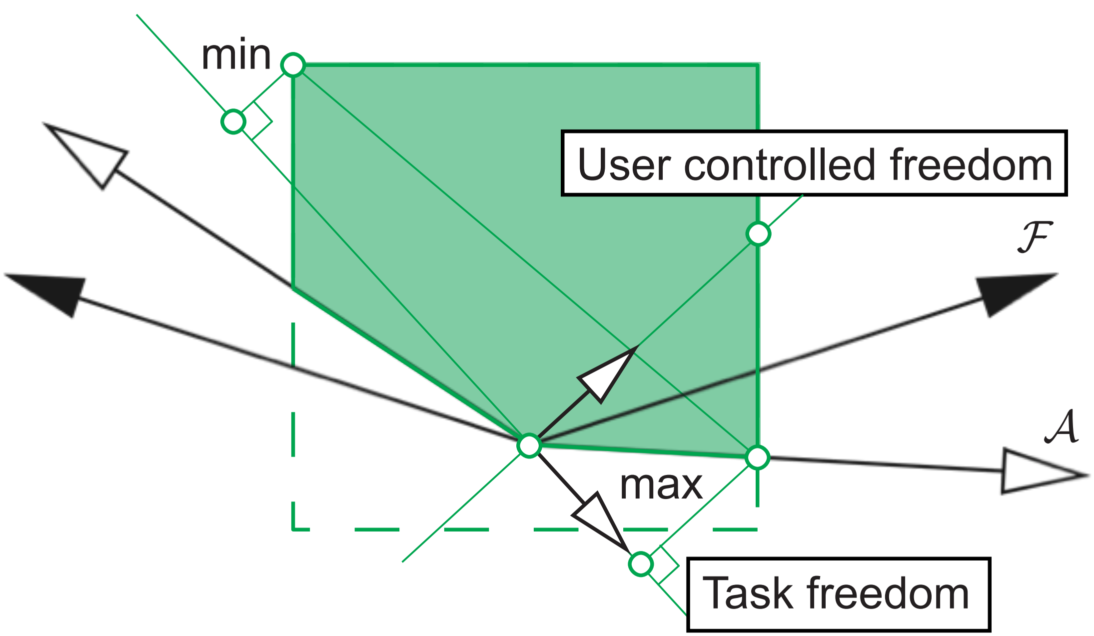
Projecting onto task freedom

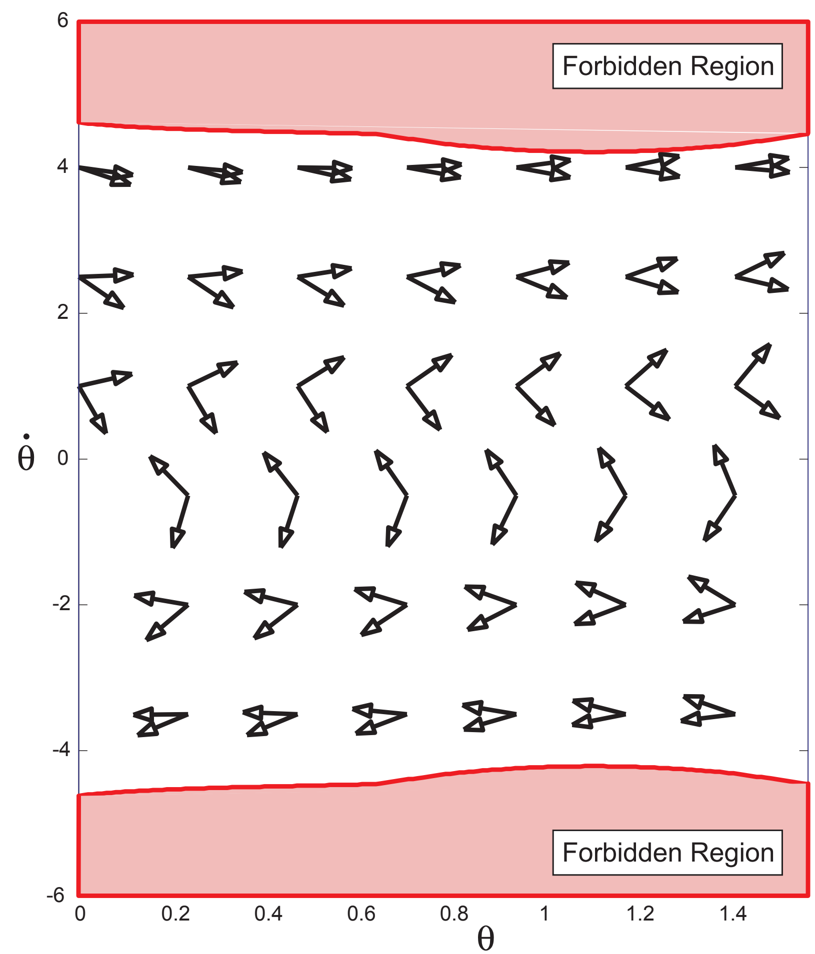
Task freedoms

- Set of state variables we care about
- Here, the orientation of the block

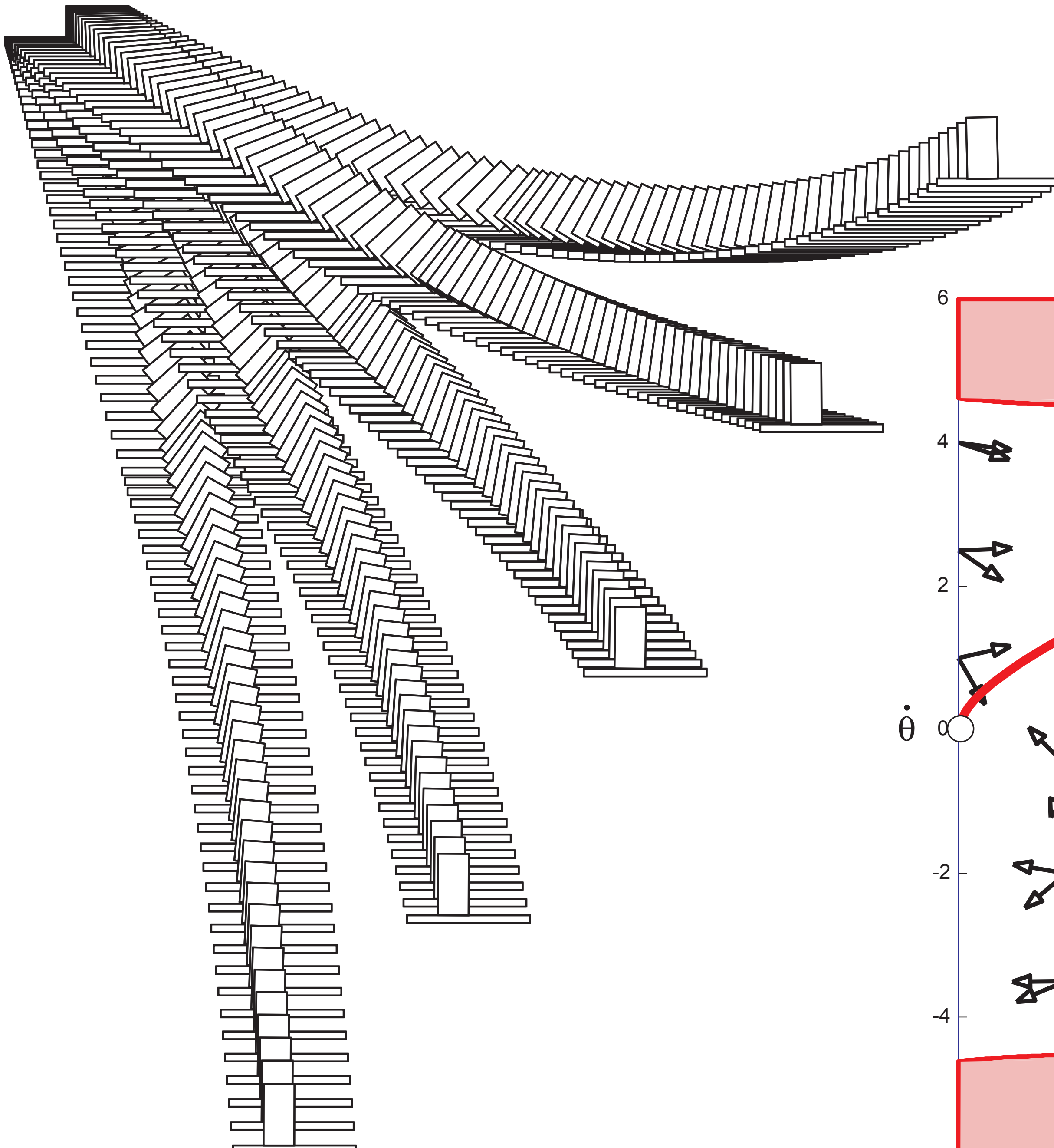
Project the dynamics onto the task freedoms

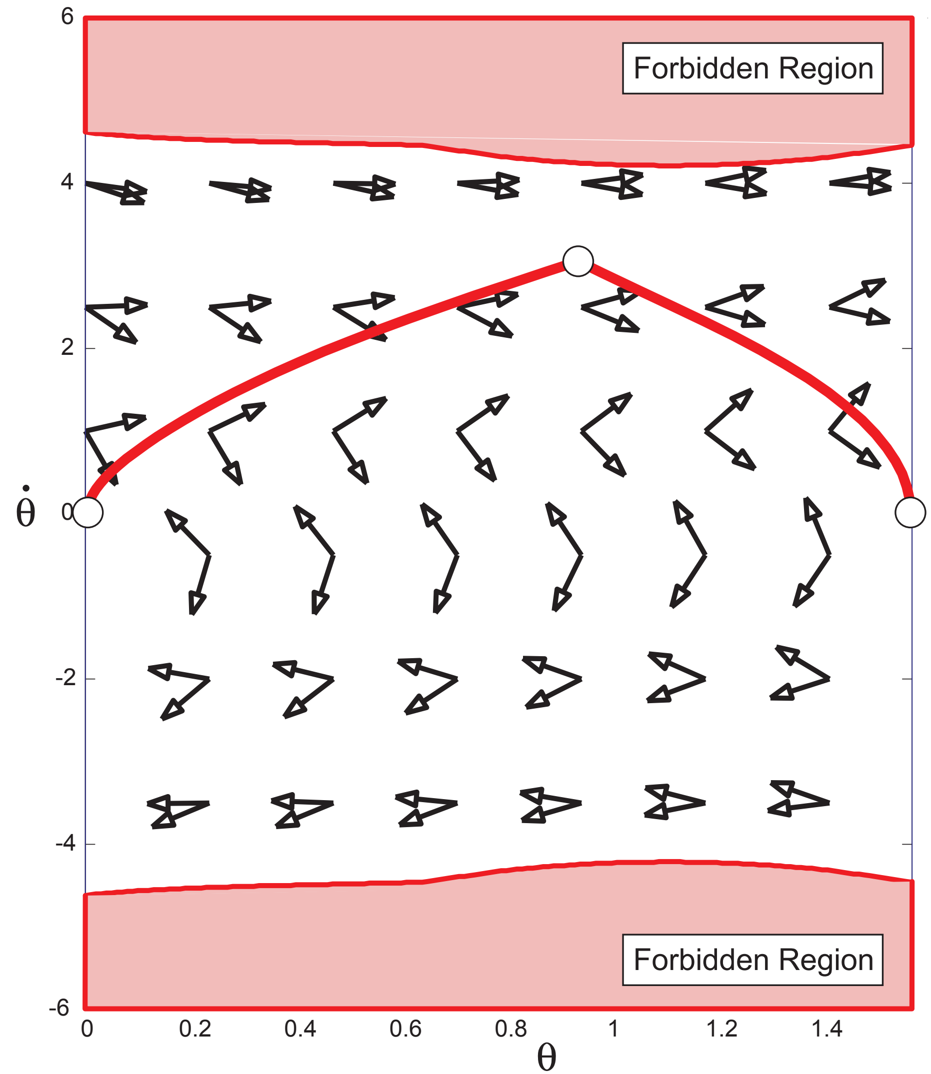
Let the user control the orthogonal space





Controllable simulations





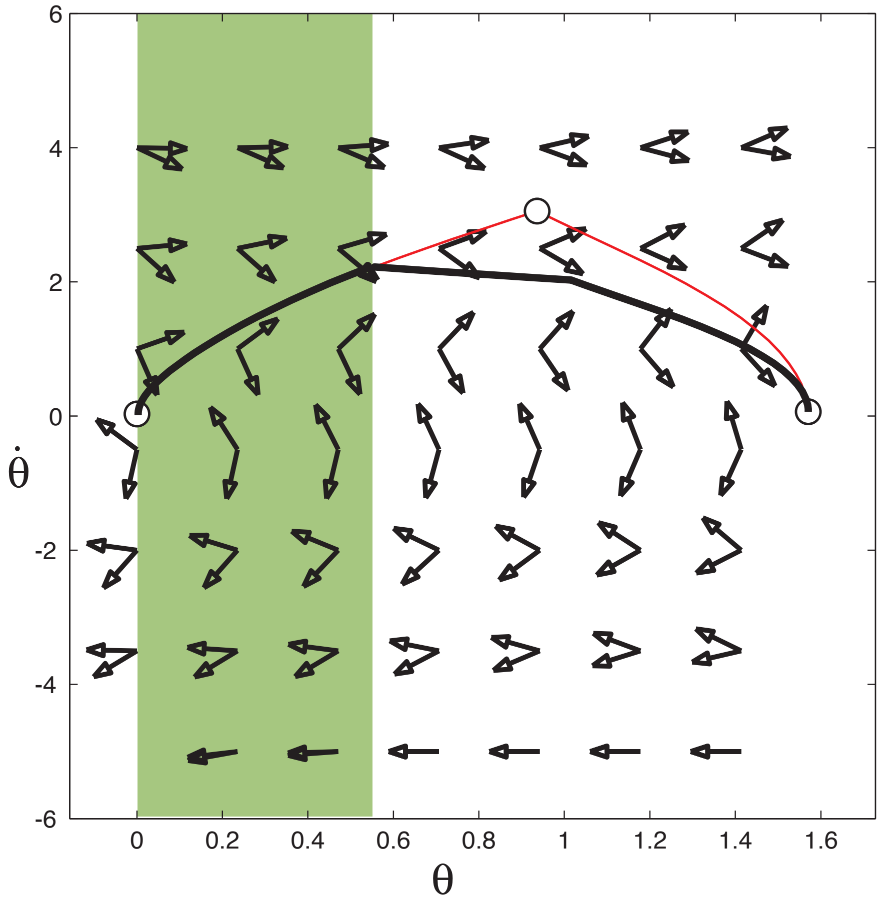
Sliding

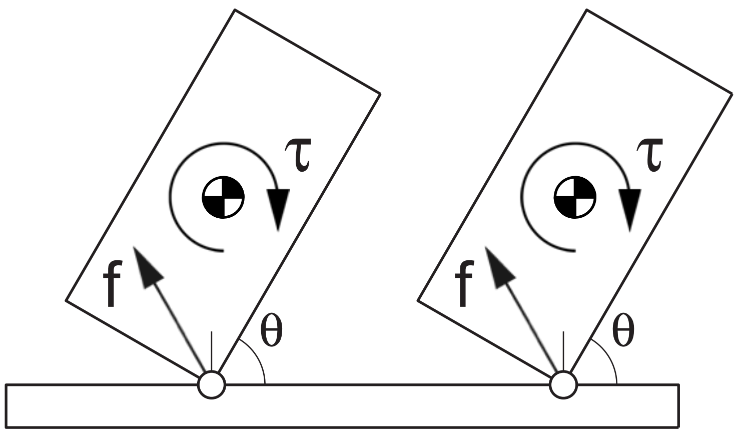
Contact is of Type B

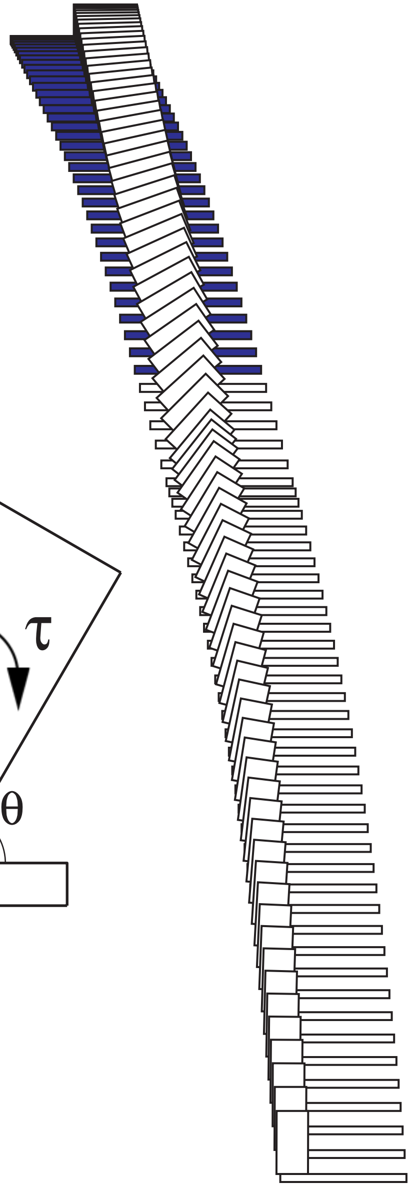
- Location of contact on palm does not change torque on object

Plan

- Move phase space acceleration to extremal
- Slide palm from relative rest to relative rest
- Continue tipping plan







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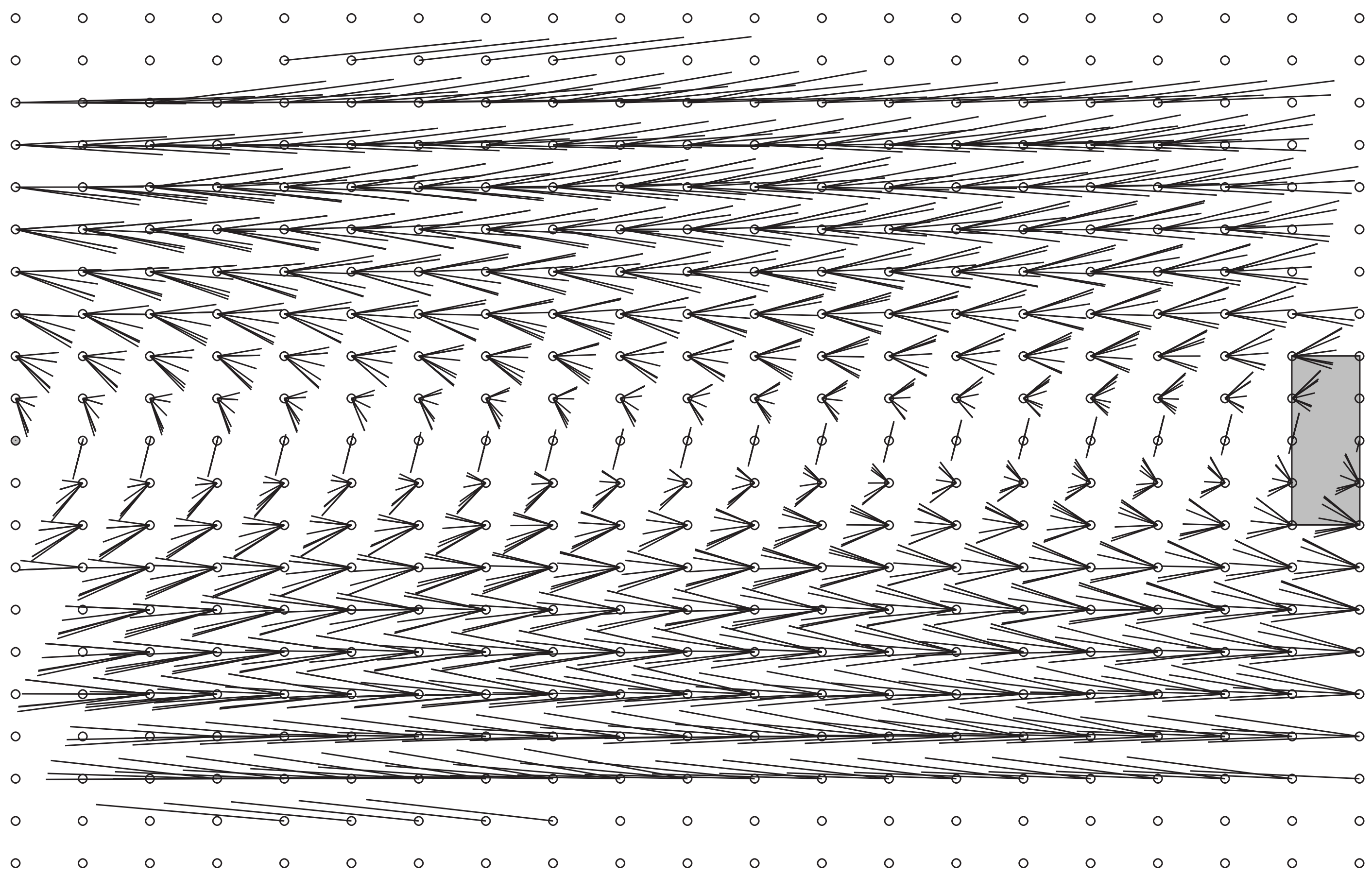
Feedback control

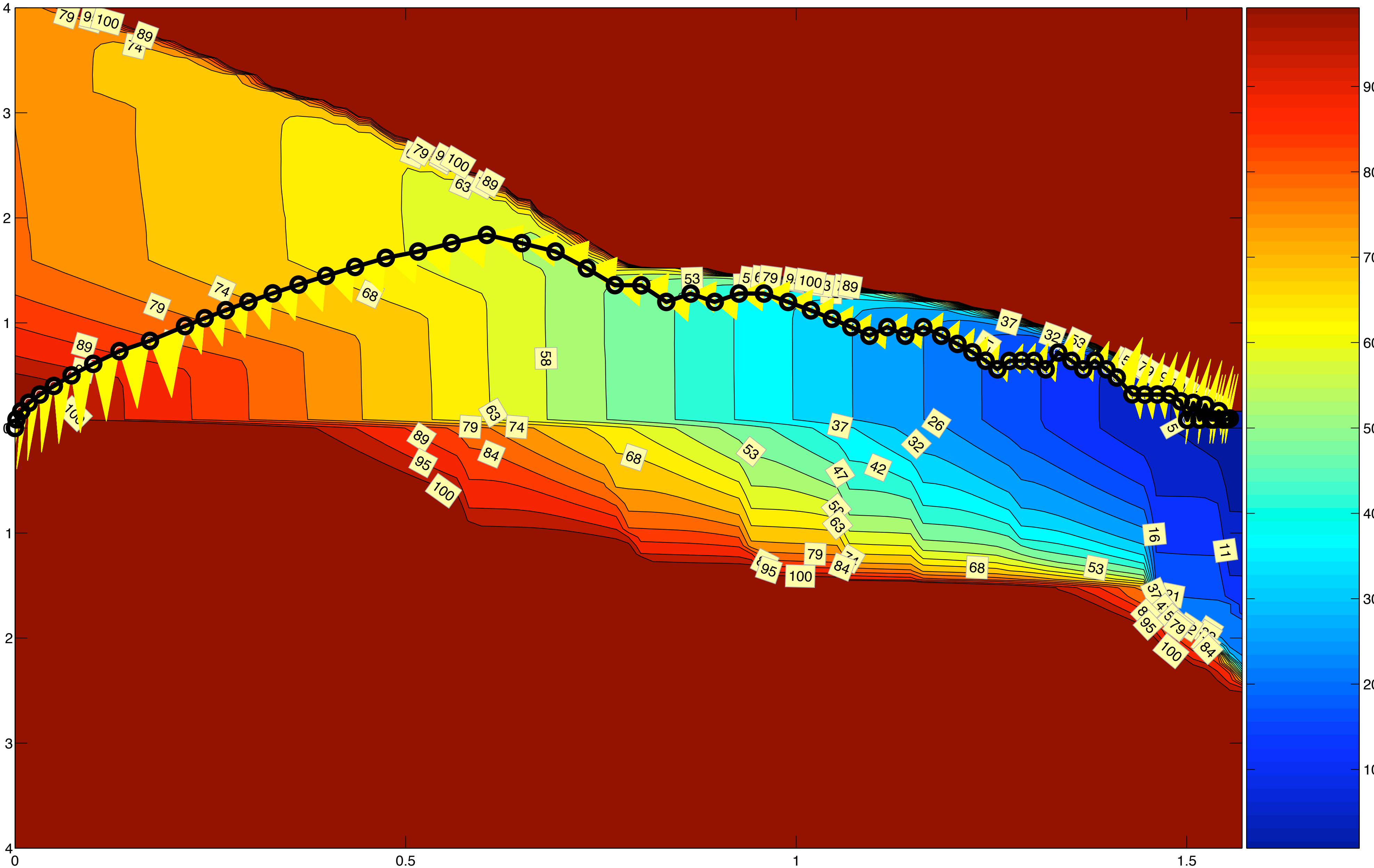
Dynamic Programming
Bellman, R.E.,
Princeton Univ. Press 1957

Dynamic programming to obtain controllers

- Discretize state and time
- Discretize controls
- Pick a cost function to optimize
- Run Backward Dynamic Programming
- Use gradient of the optimal cost function as the feedback controller



Feedback controller



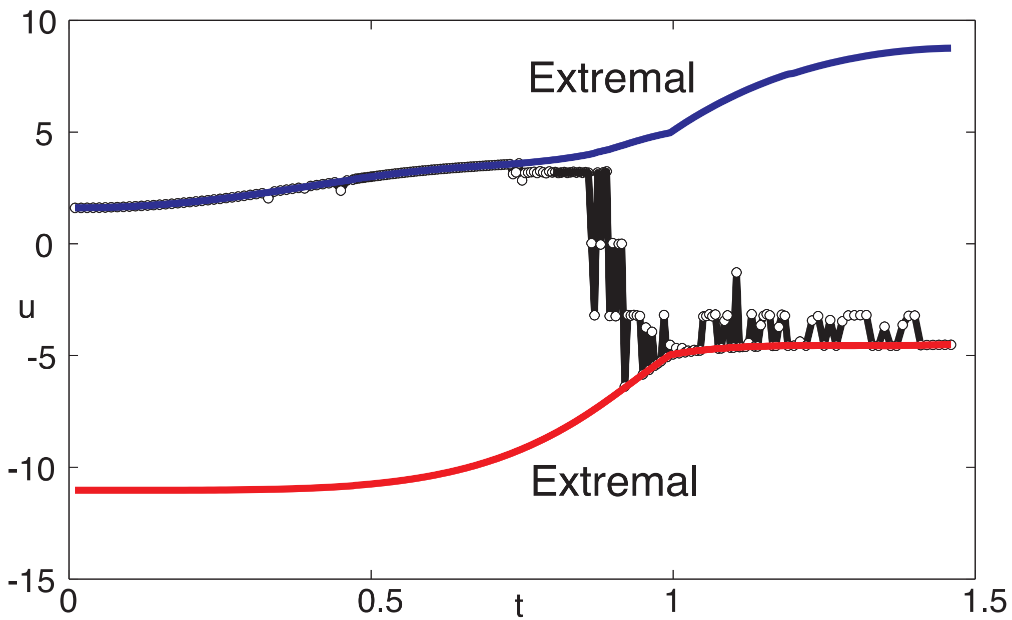
Comments

Positive

- Fast, very fast for low dimensional spaces
- Working with cost functions, easy to encode properties like
  - *Safety*
  - *Time-optimality*

Negative

- Sensitive to step length
  - *Variable resolution grids? Especially for dynamics.*
- Behavior sensitive to cost function
  - *Doing feedback on optimal paths - not a great idea?*



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Controllability

Global controllability

- The ability to reach any point in the state space from any other point in finite time
- Finding local conditions is hard

Given a manipulation system, can we find a subspace that is globally controllable?

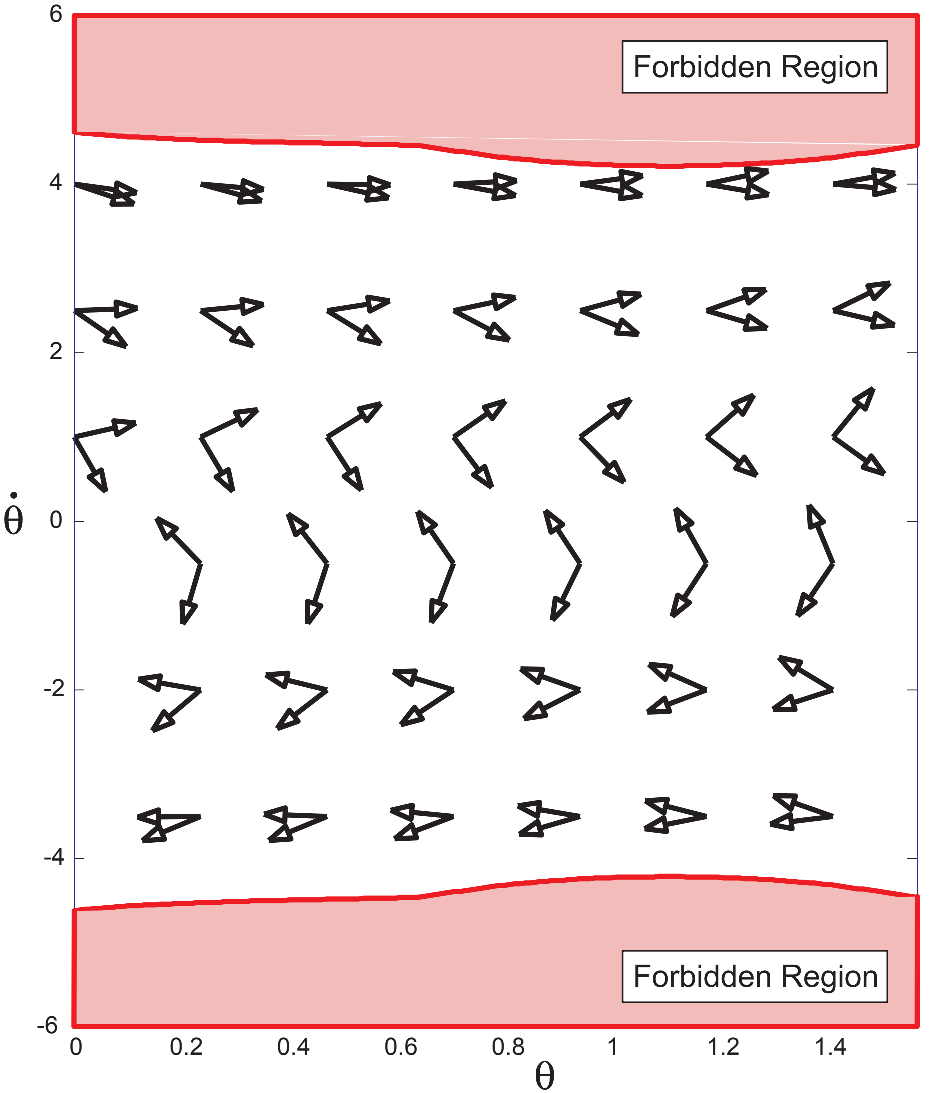
- System is nonlinear
- Unilateral constraints

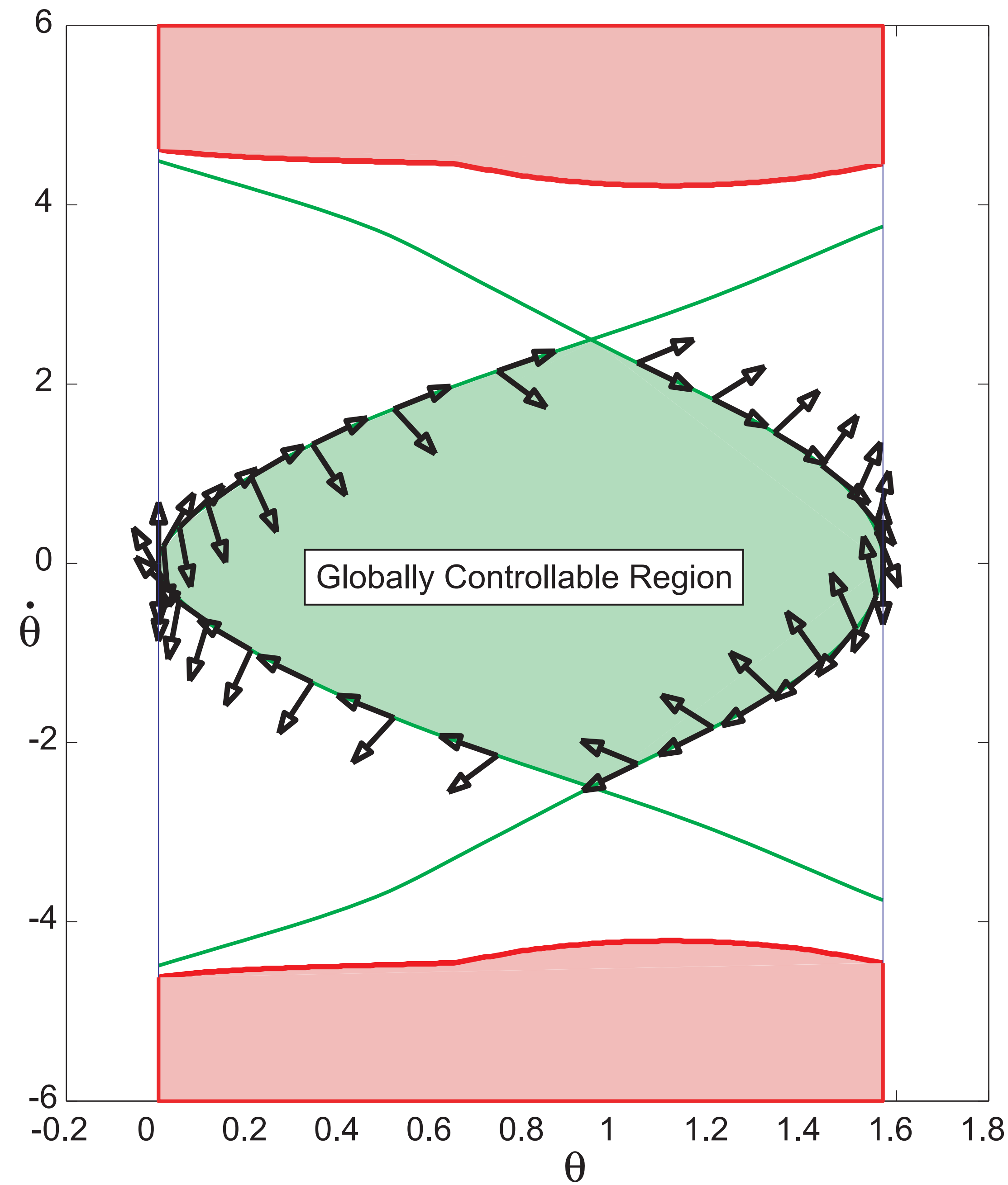
Theorem : A system is globally controllable if

- Every point satisfies the fountain condition
- There is a nontrivial orbit through every point

On the global controllability of nonlinear systems: fountains, recurrence, and applications to Hamiltonian systems
Caines, P.E. and Lemch, E.S.
SIAM J.Con 2003

Globally controllable subspace





Future work

Implementation on the Adept

- Fast but imprecise

3d manipulation

- Twirling a block about a vertex
- Higher dimensional state space
- Friction cone nonlinear

General results for global controllability and reachability

Acknowledgments

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