Fundamental Sampling Issues in Motion Planning

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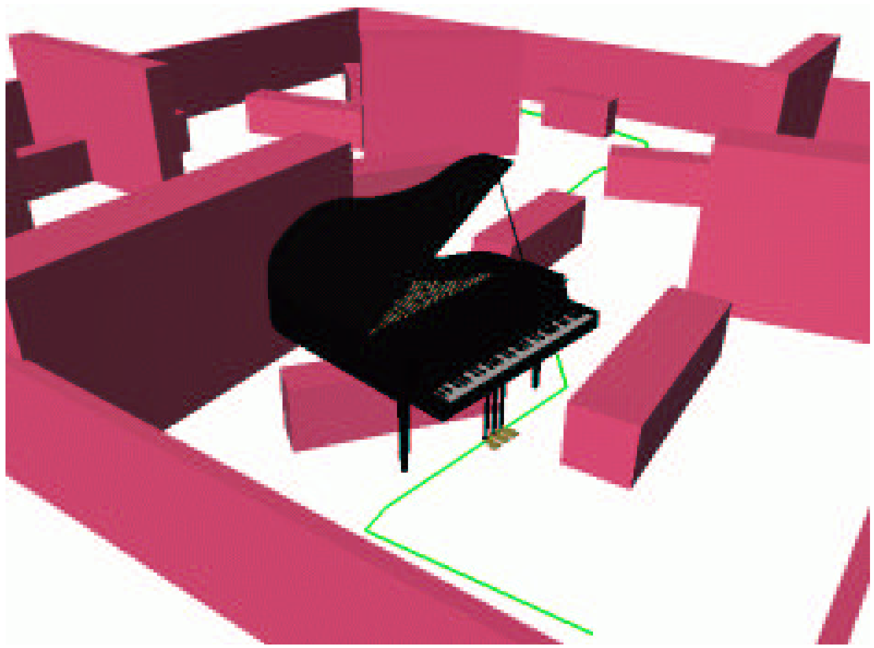
Talk Overview

1. **Motion Planning History**
2. Overview of QMC Literature
3. A Spectrum of Planners: From Grids to PRMs (w/Michael Branicky)
4. Incremental Low-Discrepancy Grids (w/Steve Lindemann)
5. Attempts to Derandomize RRTs (w/Steve Lindemann)
6. Final Thoughts

Funding: National Science Foundation (RHA & EHS)

Classical Motion Planning

"Moving Pianos"



GIVEN: $\mathcal{A}$ (geometric model of a robot)

$\mathcal{C}$ (space of configurations, q, applicable to $\mathcal{A}$)

$f_i(q) \leq 0$ (collision-free configurations)

FIND: Continuous path in $\mathcal{C}$ that avoids collision

and connects q_{init} to q_{goal}

History of Moving Pianos

Major Phases:

1. **Grid Sampling, AI Search** (beginning of time-1977)

    Experimental mobile robotics, etc.

2. **Problem Formalization** (1977-1983)

    PSPACE-hardness (Reif, 1979)

    Configuration space (Lozano-Perez, 1981)

3. **Exact Solutions** (1983-1988)

    Cylindrical algebraic decomposition (Schwartz, Sharir, 1983)

    Stratifications, roadmap (Canny, 1987)

4. **Randomized Planning** (1988-present)

    Randomized potential fields (Barraquand, Latombe, 1989)

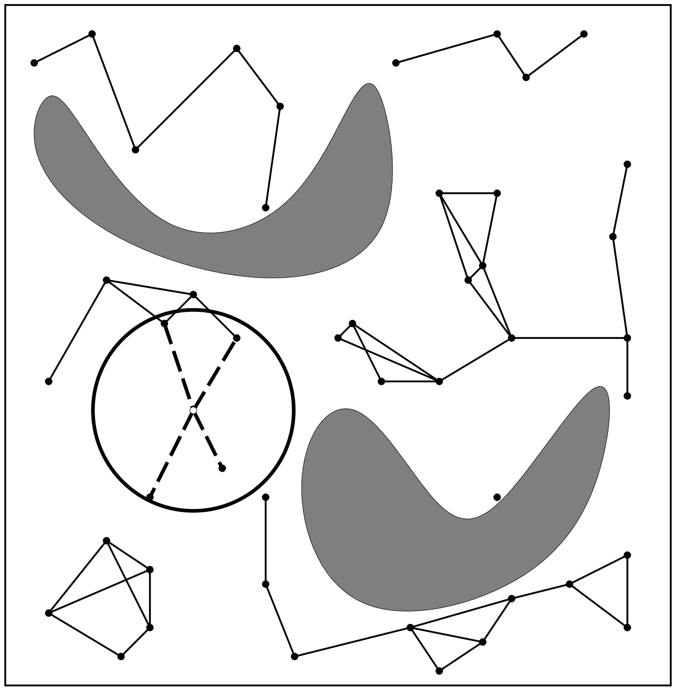
    Ariadne's clew algorithm (Ahuactzin, Mazer, 1992)

    Probabilistic Roadmaps (PRMs) (Kavraki, Svestka, Latombe, Overmars, 1994)

    Rapidly-exploring Random Trees (RRTs) (LaValle, Kuffner, 1998)

Probabilistic Roadmaps (PRMs)

Kavraki, Latombe, Overmars, Svestka, 1994



- Developed for high-dimensional spaces
- Avoid pitfalls of classical grid search
- Random sampling of $\mathcal{C}_{free}$
- Find neighbors of each sample (radius parameter)
- Local planner attempts connections
- "Probabilistic completeness" achieved

Other PRM variants: Obstacle-Based PRM (Amato, Wu, 1996); Sensor-based PRM (Yu, Gupta, 1998); Gaussian PRM (Boor, Overmars, van der Stappen, 1999); Medial axis PRMs (Wilmarth, Amato, Stiller, 1999; Pisula, Hoff, Lin, Manocha, 2000; Kavraki, Guibas, 2000); Contact space PRM (Ji, Xiao, 2000); Closed-chain PRMs (LaValle, Yakey, Kavraki, 1999; Han, Amato 2000); Lazy PRM (Bohlin, Kavraki, 2000); PRM for changing environments (Leven, Hutchinson, 2000); Visibility PRM (Simeon, Laumond, Nissoux, 2000).

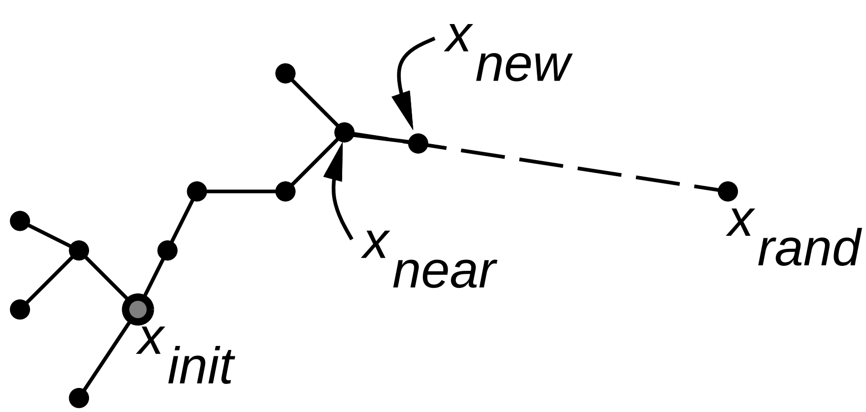
Rapidly-exploring Random Trees (RRTs)

[Movie]

BUILD_RRT(x_{init})

$\mathcal{T}$.init(x_{init});
for $k = 1$ **to** K **do**
    $x_{rand} \leftarrow$ RANDOM_STATE();
    EXTEND($\mathcal{T}, x_{rand}$);

EXTEND($\mathcal{T}, x_{rand}$)



Other RRT variants: Frazzoli, Dahleh, Feron, 2000; Toussaint, Basar, Bullo, 2000; Vallejo, Jones, Amato, 2000; Strady, Laumond, 2000; Mayeux, Simeon, 2000; Karatas, Bullo, 2001; Li, Chang, 2001; Kuffner, Nishiwaki, Kagami, Inaba, Inoue, 2000, 2001; Williams, Kim, Hofbaur, How, Kennell, Loy, Ragno, Stedl, Walcott, 2001; Carpin, Pagello, 2002; Urmson, Simmons, 2003.

Questions

- Is randomization really the reason why challenging problems have been solved?
- Is random sampling in the PRM advantageous? Even over grids?
- Which motion planning algorithms can be derandomized?

Each pseudo-random number generator has been a convenient "black box"

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5TH INTERNATIONAL CONFERENCE ON MONTE CARLO &
QUASI MONTE CARLO METHODS IN SCIENTIFIC COMPUTING
(MCQMC 2002)
25-28 NOVEMBER 2002, SINGAPORE

QMC Philosophy

- Recognize that all machine implementations produce deterministic sequences.
- View sampling as an optimization problem.
- Define criterion, and choose samples that optimize it for an intended application.

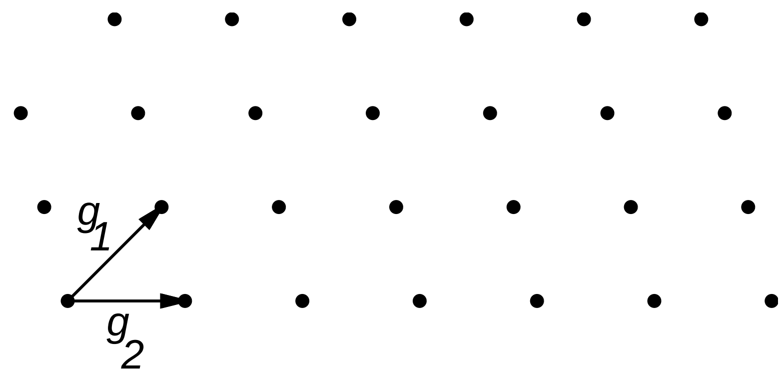
Basic Definitions

Let $X = [0,1]^d$. Let $x_i \in X$.

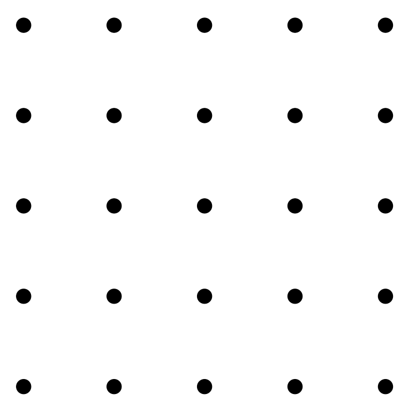
Sequence (infinite): $\langle x_1, x_2, x_3, x_4, \ldots \rangle$

Point set (finite): $\{x_1, x_2, \ldots, x_N\}$

Lattice: A point set with regular neighborhood structure.



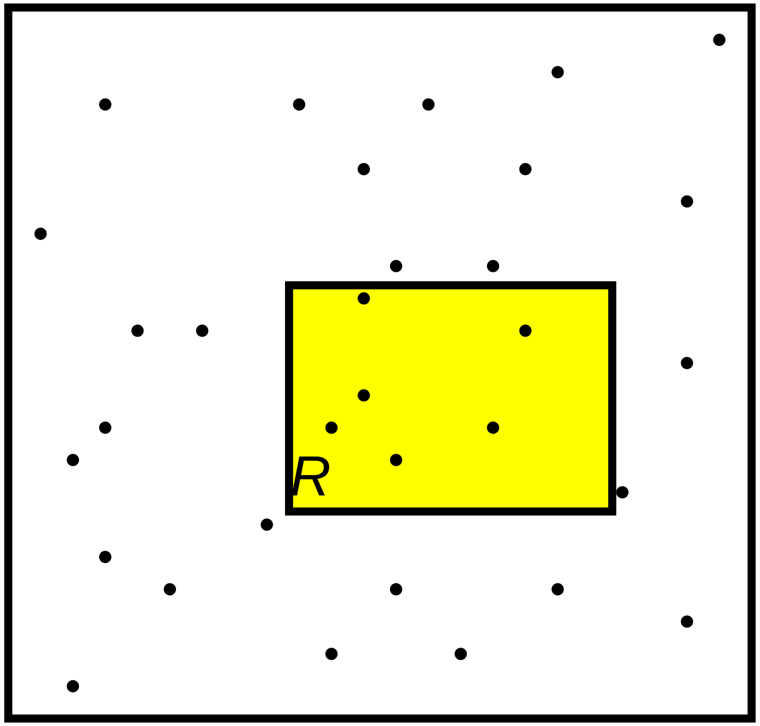
Grid: A lattice with axis-aligned generators



Measuring the (Lack of) Quality

Let $\mathcal{R}$ (range space) denote a collection of subsets of X.

Discrepancy: "maximum volume estimation error over all boxes"

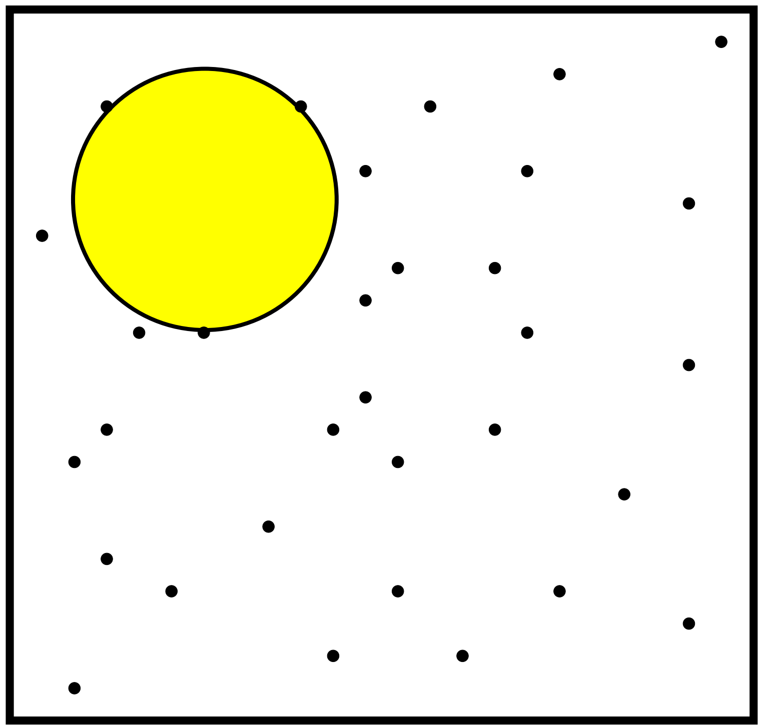


$$D(P, \mathcal{R}) = \sup_{R \in \mathcal{R}} \left| \frac{\#\text{ of samples in } R}{\text{Total }\#\text{ of samples}} - \frac{volume(R)}{volume(X)} \right|$$

Measuring the (Lack of) Quality

Let ρ denote a metric on X.

Dispersion: "radius of the largest empty ball"



$$\delta(P,\rho) = \sup_{x_1 \in X} \left[\min_{x_2 \in P} \rho(x_1, x_2) \right]$$

Optimal Quality

Low Discrepancy Sequence: $O((\log^d N)/N)$

Low Discrepancy Point Set: $O((\log^{d-1} N)/N)$

Low Dispersion: $O(1/N^{\frac{1}{d}})$

- Implied constants may depend on d.
- Low discrepancy implies good dispersion,

$$d(P, \ell^\infty) \leq D(P, \mathcal{R})^{1/d},$$

but not necessarily the best dispersion.

Literature Landmarks

1916	Weyl	Discrepancy defined
1930	van der Corput	1D low-discrepancy sequence
1951	Metropolis, Ulam	Monte Carlo integration
1959	Korobov	Low-discrepancy lattices
1960	Halton, Hammersley	ND low-discrepancy sequences, point sets
1967	Sobol'	Improved low-discrepancy sequence
1971	Sukharev	Optimal dispersion point sets
1982	Faure	Improved low-discrepancy sequence
1987	Niederreiter	(t,m,s)-nets and (t,s)-sequences
1992	Niederreiter	Seminal monograph
1998	Niederreiter, Xing	Current best low-discrepancy sequences
1998	Owen, Matousek	Randomized low-discrepancy sequences
2000	Hickernell	Extensible lattices

The van der Corput Dance

Let $X = [0, 1]$. Construct an infinite sequence in X that has better properties than a (pseudo-)random sequence.

First, try a "grid". Suppose $N = 8$.

A possible sequence: 0, 1/8, 2/8, 3/8, . . ., 7/8.

Binary decimal representation:

.000

.001

.010

.011

.100

.101

.110

.111

What if we stop at $N = 5$? How good is the coverage? What if we want more points?

The van der Corput Dance (cont'd)

The previous sequence is precisely the opposite of what we want.

.000

.001

.010

.011

.100

.101

.110

.111

The van der Corput Dance (cont'd)

Move the decimal point, and read from right to left!

000. $\leftarrow$

001.

010.

011.

100.

101.

110.

111.

This yields: 0, 1/2, 1/4, 3/4, 1/8, 5/8, 3/8, 7/8

The van der Corput Dance (cont'd)

Adding more points is easy.

0000. $\leftarrow$

0001.

0010.

0011.

0100.

0101.

0110.

0111.

1000.

1001.

1010.

This yields: 0, 1/2, 1/4, 3/4, 1/8, 5/8, 3/8, 7/8, 1/16, 9/16, 5/16, ...

[Movie]

Higher Dimensions: Halton/Hammersley

Let $v_p(i)$ denote the i^{th} point in the base p van der Corput sequence.

Halton (1960):

The i^{th} point in the *Halton sequence* is defined as

$$(v_{p_1}(i), v_{p_2}(i), \ldots, v_{p_d}(i)),$$

in which p_1, p_2, ..., p_d are relatively prime.

Hammersley (1960):

Suppose N is fixed. The *Hammersley point set*, P, is

$$\{(i/N, v_{p_1}(i), \ldots, v_{p_{d-1}}(i)) \mid i = \{0, 1, \ldots, N-1\} \}.$$

Both Halton and Hammersley achieve asymptotically optimal discrepancy and dispersion.

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Caveats

Our goal is to gain a better understanding of planning issues.

1. Importance sampling is important (Amato et al. 1998; Amato, Wu, 1996; Boor, Overmars, van der Stappen, 1999; Holleman, Kavraki, 2000; Leven, Hutchinson 2000; Pisula, Hoff, Lin, Manocha, 2000; Simeon, Laumond, Nissoux, 2000; Wilmarth, Amato, Stiller, 1999)
2. We do not recommend planners here as the fastest available.
3. Randomization is useful in many paradigms.

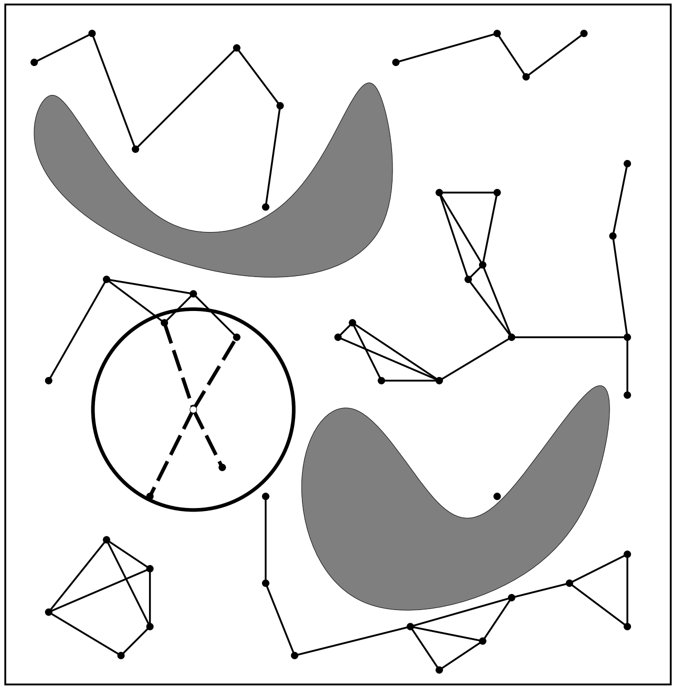
Question: Are random samples really important in the PRM framework?

Two different levels of claims:

1. The random sampling in the original PRM seems to offer no advantages over deterministic sequences, including grids.
2. There exist deterministic sequences that yield modest performance improvements.

Probabilistic Roadmaps (PRMs)

Kavraki, Latombe, Overmars, Svestka, 1994



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Lazy PRM

Bohlin and Kavraki, 2000

- Build a PRM over $\mathcal{C}$ (ignoring obstacles).
- Use A^* to search for solution from initial to goal.
- During search, collision check traversed vertices and edges. Delete any in collision.

Advantage:

Expensive collision detection is focused only on the query, as opposed to testing all edges.

Limitations:

How many vertices and edges should be generated?

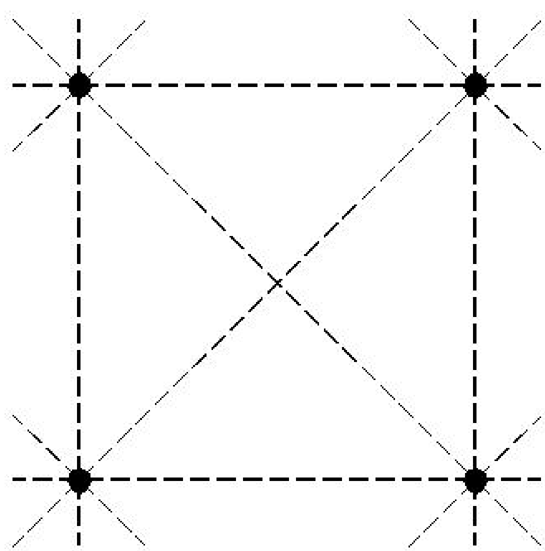
Initial PRM can be costly; yet, the graph contains no information.

A Spectrum of Planners

$$CGS - SGS - LRM - QRM - PRM$$

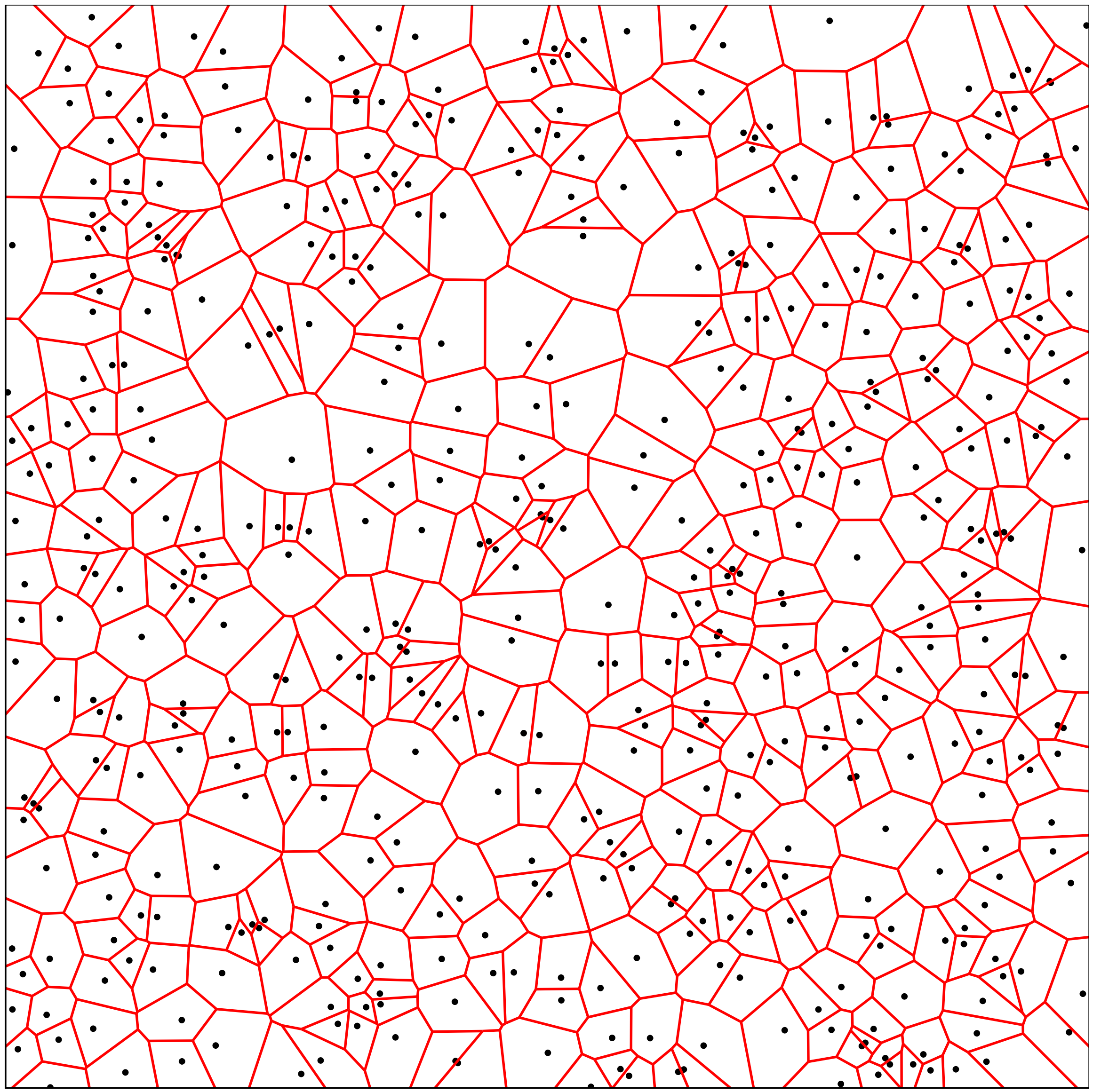
CGS	Classical Grid Search
SGS	Subsampled Grid Search
LRM	Lattice-Based Roadmap
QRM	Low-Discrepancy/Low-Dispersion (Quasi-Random) Roadmap
PRM	Probabilistic (Pseudo-Random) Roadmap

Subsampling: Collision check at finer resolution than sampling

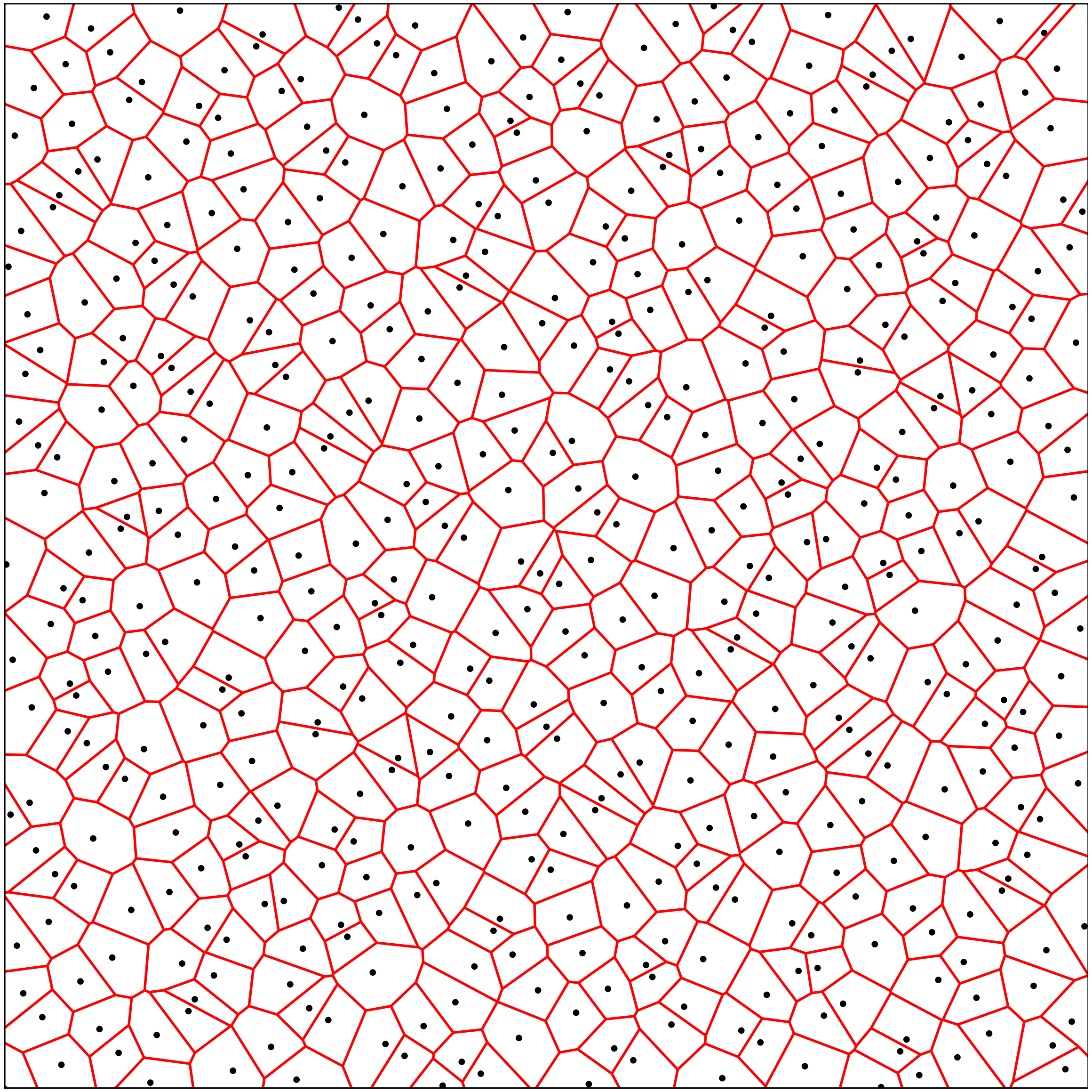


Related work: Bohlin, 2001

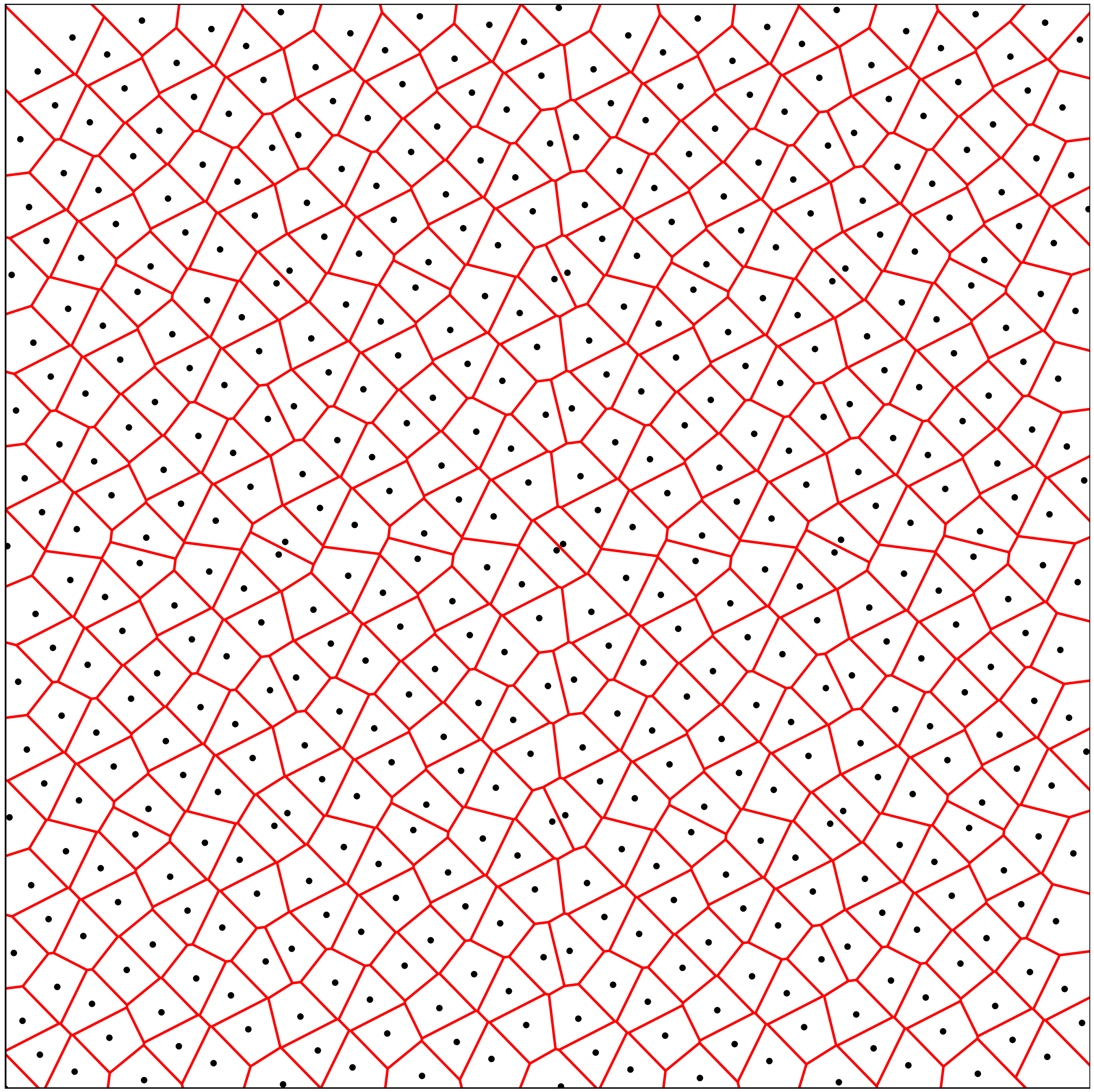
500 Pseudo-Random Samples
(nonlinear congruential)



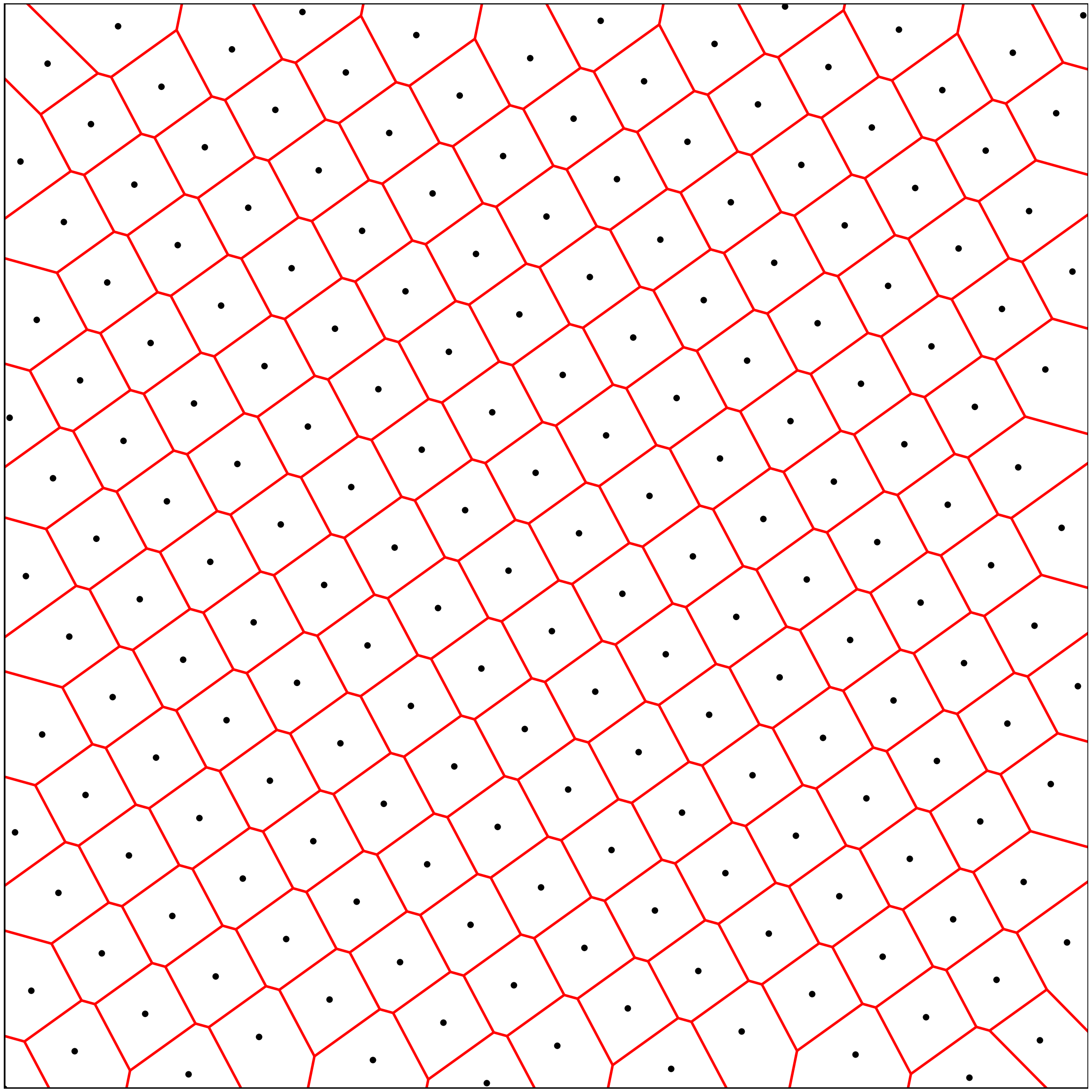
500 Halton Samples



500 Hammersley Samples



200 Lattice Samples



Sukharev Sampling Criterion

(Sukharev, 1971)

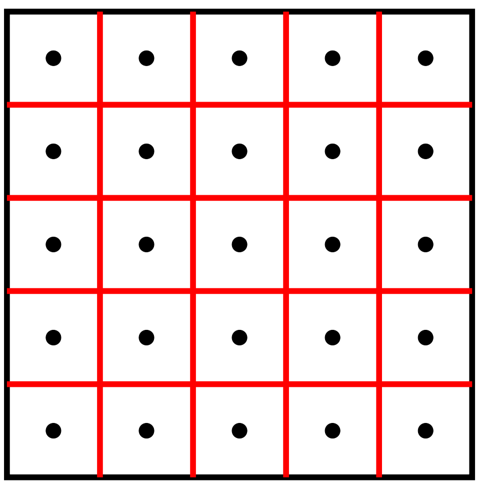
Let P be any set of N points in $[0,1]^d$.

Let $\delta(P, \ell^\infty)$ denote ℓ^∞ dispersion.

$$\delta(P, \ell^\infty) \geq \frac{1}{2\lfloor N^{\frac{1}{d}} \rfloor}$$

An exponential number of samples is needed to keep dispersion fixed, regardless of the sampling strategy.

Sukharev grid ($d = 2$, $N = 25$):



$N^{1/d}$ gives points per axis.

Lazy Planning Issues

$$CGS - SGS - LRM - QRM - PRM$$

- A lazy version can be made for every planner in the spectrum.
- Multiple-query (substantial precomputation) vs. single-query (lazy)
- Analog to multiple-query for CGS is precomputed bitmaps.

Lattices and grids are most advantageous for lazy approaches:

- The roadmap can be implicitly declared, to avoid expensive construction.

Connecting Sample Quality to Problem Difficulty

Problem	**Quality measure**	**Difficulty measure**	**Theoretical bound**
integration	discrepancy	HK bounded variation	Koksma-Hlawka Inequality
optimization	dispersion	Modulus of continuity	Niederreiter Inequality
planning	dispersion	Corridor thickness	Our analysis

- Since C-space boundaries are generally nonlinear, alignment issues handled by discrepancy seem irrelevant.
- Corridor thickness already used as a difficulty measure.
  (Barraquand, Kavraki, Latombe, Li, Motwani, Raghavan, 1996; Hsu, Kavraki, Latombe, Motwani, Sorkin, 1998)

Decidability of Configuration Spaces

Let $\Psi(x)$ be the set of $\mathcal{C}_{free}$'s, for which the width, w, of the narrowest corridor is $w \geq x$.

Theorem: *Using any low-dispersion sequence or point set, after N samples, a roadmap algorithm correctly decides all planning queries in any d-dimensional $\mathcal{C}_{free} \in \Psi(4b(d)N^{-1/d})$.*

$b(d)$ depends on the sequence or point set:

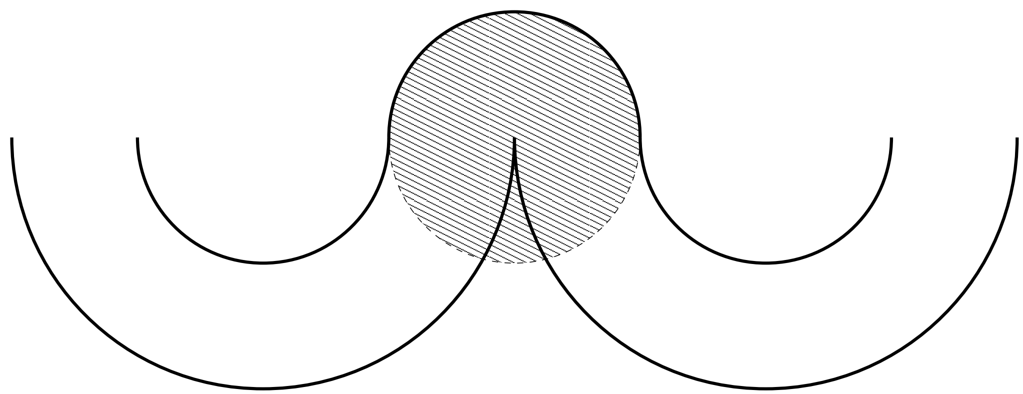
Halton	$b(d) = d$th prime
Hammersley	$b(d) = (d-1)$th prime
(t,s)-sequences	$b(d) = \text{base}^{(d+t)/d}$
(t,m,s)-nets	$b(d) = \text{base}^{(d-1+t)/d}$
Sukharev grid	$b(d) = 1/2$
Niederreiter (sequence)	$b(d) = 1/\ln 4$

The result is asymptotically optimal.

Under any sampling scheme, a roadmap requires a number of samples exponential in dimension, d, to be complete for $\Psi(\delta)$.

Comparison to Random Sampling

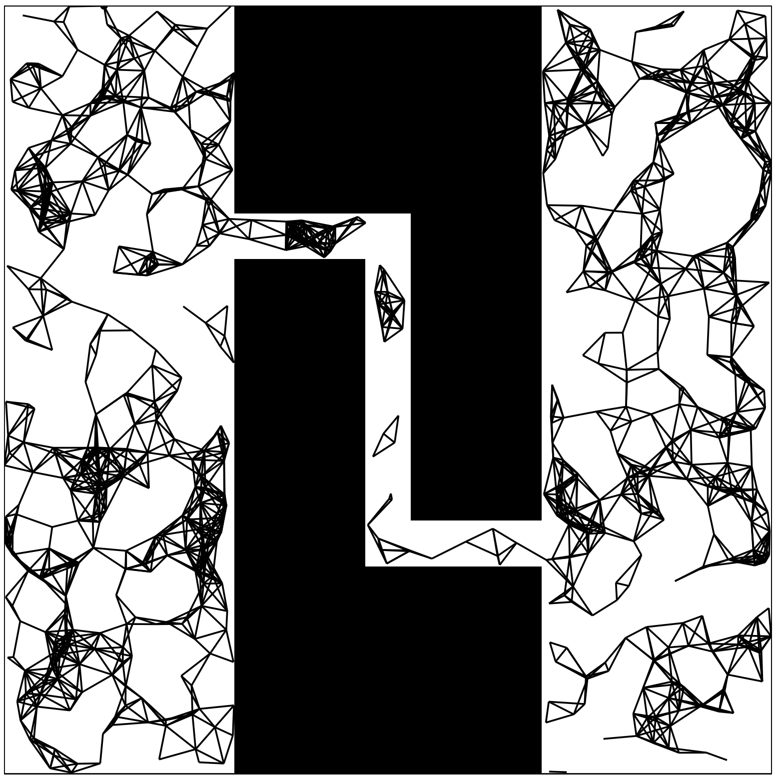
Proposition: *For any sample set, P, that has dispersion at least δ, no roadmap algorithm can be complete for $\Psi(\delta)$.*



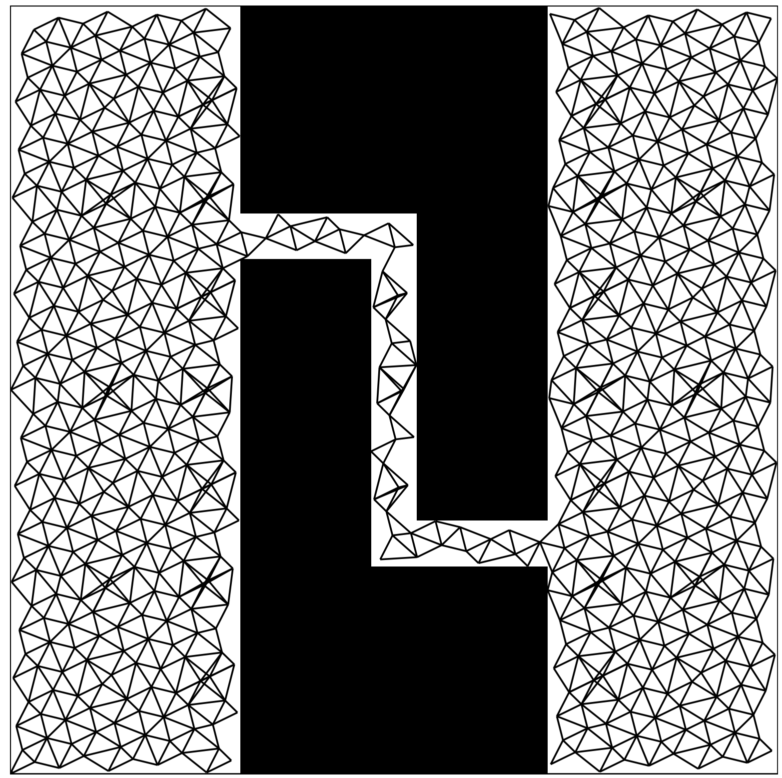
Proposition: *For a fixed d, the PRM with random sampling requires $O((\log N)^{\frac{1}{d}})$ times as many samples (with probability one) compared to low-dispersion sequences to obtain the same ℓ^∞ dispersion.*

QRM with Hammersley Sampling

Each uses 1000 samples and the same connection radius, 0.05.



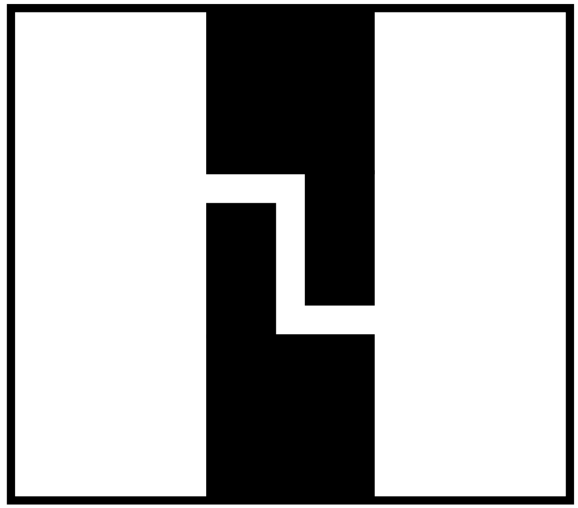
Pseudo-Random



Hammersley

Bent Corridor Experiments: Hammersley

Traversing a d-dimensional tunnel with a cubic cross section:



Dim.	Width	Rad.	Hamm.	Rand.	Factor
2	.03	.10	195	464	2.38
3	.05	.25	579	828	1.56
3	.10	.40	26	106	4.08
6	.10	.40	4052	12857	3.17
10	.25	.60	1506	1531	1.02
10	.20	.60	2101	6260	2.98

Pseudo-random results use nonlinear congruential generator, and are averaged over 100 trials.

Bent Corridor Experiments: Sukharev

- Sukharev grids were tested.
- Considered random d-dimensional rotations applied to the grid.

Dim.	Width	Rad	Min	Max	Mean	Sukh-1st	Sukh-all
2	0.02	0.5	65	652	461.97	25	441

Rotated versions: 25/324, 196/256, 49/400, 144/324, 64/289

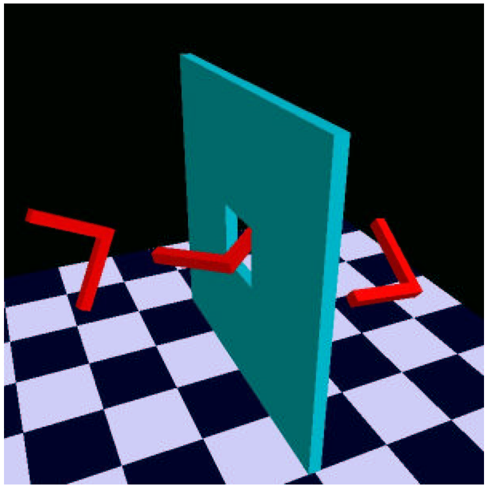
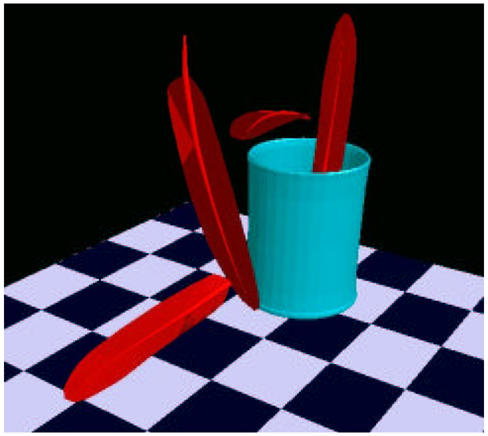
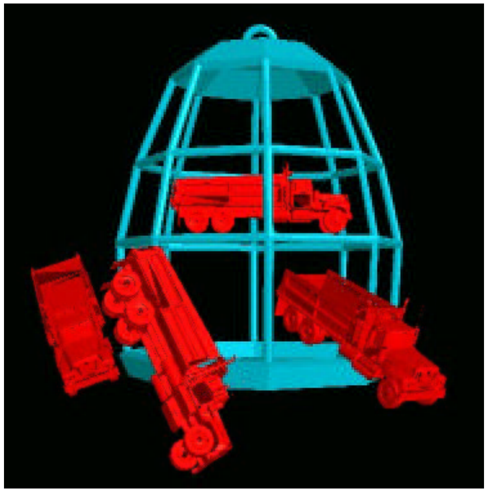
Dim.	Width	Rad	Min	Max	Mean	Sukh-1st	Sukh-all
3	0.05	0.6	131	1861	757.52	125	1331

Rotated versions: 343/729, 729/729, 216/216

Dim.	Width	Rad	Min	Max	Mean	Sukh-1st	Sukh-all
6	0.15	0.6	195	1444	1237.85	729	729

Rotated versions: 729/729, 729/729, 729/729, 729/15625, 729/729

More Performance Comparisons

Placing a feather (1184 triangles) into a cup (1632 triangles).

Getting a truck (22284 triangles) out of a cage (1032 triangles).

Prob.	Min	Max	Avg	Lattice	Sukharev
Elbow	1250	15250	4667	3963	4096 (4 ppa)
Cup	2000	12000	4800	2152	729 (3 ppa)
Truck	5000	95000	35207	5138	15625 (5 ppa)

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Important Sampling Properties

1. **Uniformity:** optimizing discrepancy or dispersion
2. **Lattice structure:** easy to find neighbors; control over vertex degree
3. **Incremental quality:** terminate at any point and maintain good uniformity

Pseudo-random: 3, but not 1 and 2 (not bad for 1, though)

Grids and lattices: 1 and 2, but not 3

Halton, Sobol', Faure, etc.: 1 and 3, but not 2

Goal: Design a sequence that achieves 1, 2 and 3.

Key Ideas

- Generalize van der Corput to d dimensions.
- Halton traded lattice structure for low discrepancy.
- Since dispersion is most important, we can use grids.
- Construct an infinite sequence that periodically looks like a full grid.

Example: Suppose $d = 2$.

After 4, 16, 64, or , 2^{2k}, points have been placed, a $2^k \times 2^k$ grid is obtained.

At all points in between, the grid is filled in incrementally, but what is the best order?

A New Generalization of van der Corput

For $d = 1$, recursive switching between intervals, $[0, 1/2)$, $[1/2, 1]$

For $d = 2$, recursive switching between quadrants

For $d = 3$, recursive switching between octants

van der Corput decides which interval based on a bit.

Our sequence decides which quadrant based on d bits.

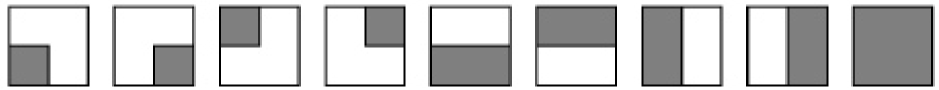
Issue: In what order should the 2^d quadrants be traversed?

All orderings yield the same dispersion.

Choose the ordering in terms of optimal incremental discrepancy reduction.

An Illustration for $d = 2$

Discrepancy optimized over *canonical rectangles*:



These are for one level of recursion.

Eventually, when there are k points per axis, canonical rectangles can have coordinates at any of k sample values.

[Simulator]

Analysis of d-Dimensional Sequence, S_d

Theorem *Take the first i elements of S_d.*

1. *This subsequence is a multiresolution grid sampling sequence of length i.*
2. *From the set of multiresolution grid sampling subsequences, it is discrepancy-optimal over canonical rectangles for the current resolution level.*

Property 1 i-th sample in S_d can be generated in $O(\log i)$ time.

Property 2 Let the number of samples taken so far be N. Then, a 1-neighbor of any of these samples can be found in $O(\log N)$ time.

Implied Constants

Optimal asymptotic dispersion, $\delta(P, \ell^\infty) = O(1/N^{\frac{1}{d}})$

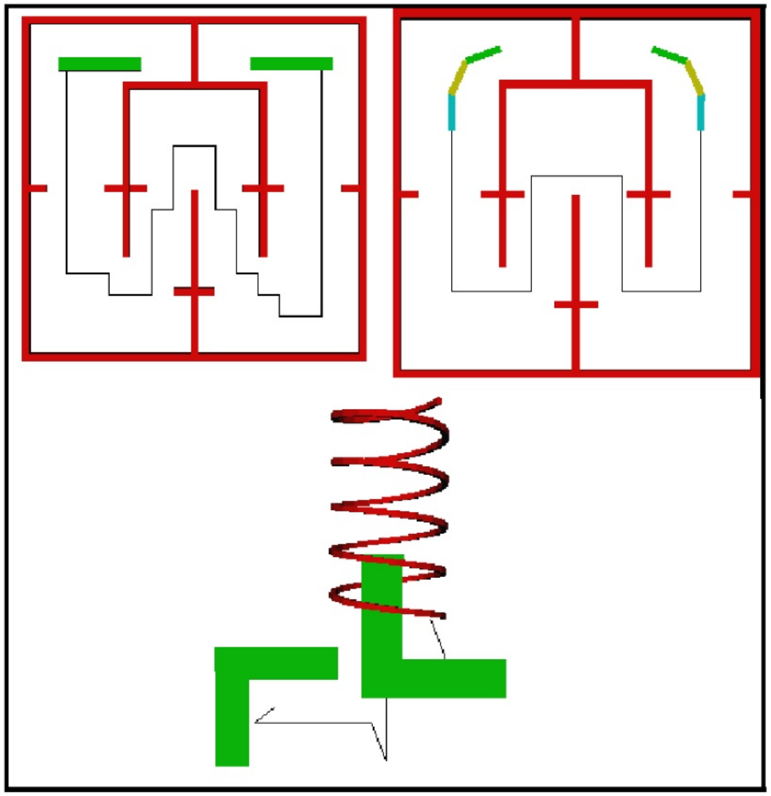
However, what is the constant?

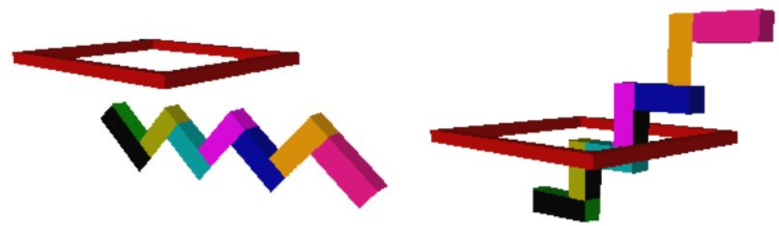
$$\limsup_{N\to\infty} N^{1/d}\delta(P, \ell^\infty)$$

- Regular grid base: 2
- Sukharev grid base: 3/2

There exist nonuniform sequences with constant $1/\ln 4$

Early Experiments





Running times (2 Ghz PC running Linux, Gnu C++)

Prob.	Dim	R_Rad	R_KNear	Scan	Opt
Bar	3	1.98	1.81	1.08	1.48
Links	5	373.26	844.58	438.00	219.10
Elbow	6	71.11	777.08	63.20	29.56
Arm	6	44.28	69.93	33.91	13.94

Talk Overview

1. Motion Planning History
2. Overview of QMC Literature
3. A Spectrum of Planners: From Grids to PRMs (w/Michael Branicky)
4. Incremental Low-Discrepancy Grids (w/Steve Lindemann)
5. **Attempts to Derandomize RRTs (w/Steve Lindemann)**
6. Final Thoughts

Derandomizing RRTs?

Harder than for PRMs because the samples do not usually appear in the graph.

Most important idea in the RRT is Voronoi bias.

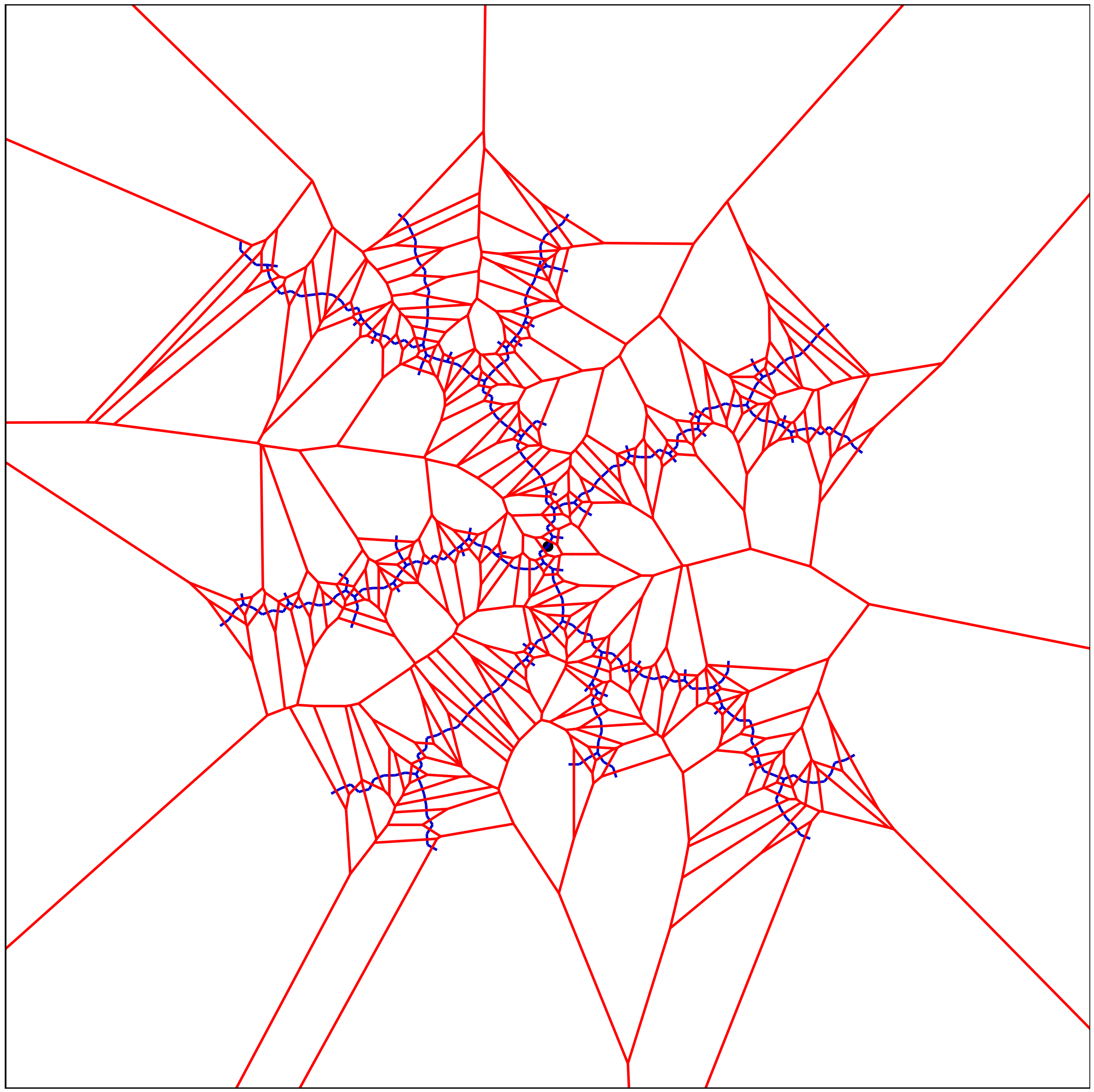
Could compute Voronoi regions to directly induce the Voronoi bias.

- Too difficult to implement!

Could replace the pseudo-random number generator with Halton sequence, etc.

- Too easy! Not much difference...

Voronoi-Biased Exploration



Multi-Sample RRTs

In each iteration, take k random samples of $\mathcal{C}$ and

1. Mark all vertices with 0.
2. For each of the k samples, increment the nearest vertex.
3. Select vertex with the most votes for extension.

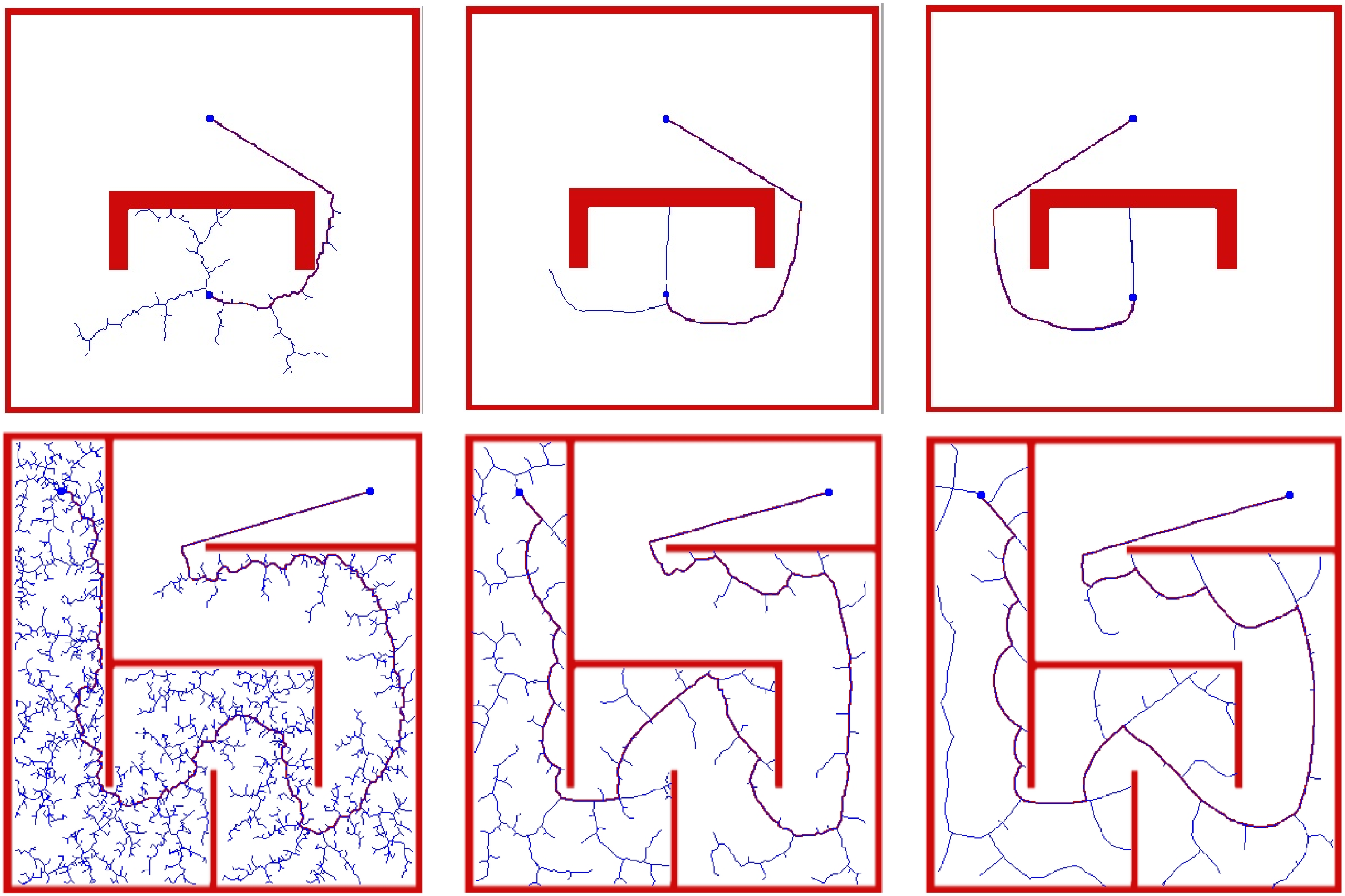
As k grows, the Voronoi volumes become approximated.

Improvement:

Instead of pseudo-random numbers, use a deterministic sequence.

- In each RRT Extend iteration, the old samples are used again.
- Each new sample has a single nearest vertex.
- Each new vertex steals votes from other samples.

Early Experiments



RRT Multi-Sample Deterministic

Progress To Date

- Appears to produce straighter paths, fewer nodes
- Appears to reduce collision checks
- Hard to avoid sticking
- Not numerically robust
- Computations are still expensive

Talk Overview

1. Overview of QMC Literature
2. A Spectrum of Planners: From Grids to PRMs
3. Theoretical Results
4. Experimental Results
5. **Final Thoughts**

Key Points

1. It is advantageous to open the sampling "black box"
2. The random sampling in the original PRM seems to offer no advantages over deterministxic sequences, including grids.
3. Deterministic sequences can offer advantages in terms of dispersion, discrepancy, and neighborhood structure.

Why do we remember grids performing so badly?

- Subsampling and Sukharev help.
- **Moore's Motion Planning Corollary:**
  Grids work well in one dimension higher every 5 years.

1970 - 1, 1975 - 2, 1995 - 6, 2005 - 8, 2050 - 17

Extensible Grids and Lattices

Grids often appear bad because N must be specified.

Consider a sequence constructed point-by-point that periodically looks like a grid.

- Embedded lattices (Sloan, Joe, 1994)
- Extensible rank-1 lattices (Hickernell, Hong, L'Écuyer, Lemieux, 2001)
- Extensible low-discrepancy lattices (Hickernell, Niederreiter, 2002)
- Extensible low-dispersion/low-discrepancy grids (Lindemann, LaValle, 2002)

Interesting connection to multiresolution planning.

What Happens in Very High Dimensions?

Assume d large (e.g., $d > 50$).

Spaces become severely undersampled.

Via Sukharev, the effective points per axis for any sequence is:

$$N^{1/d} \approx 1$$

Cannot do much better than placing one point at the center of $[0,1]^d$.

- For many problems, QMC and pseudo-random comparable.
- QMC superior for 360-dimensional financial integrals (Paskov, Traub, 1992)
- Explanation: Some problems have low "effective dimension".

Bring Back Randomization?

Desirable properties (L'Écuyer, Lemieux, 2002):

- Every element of the sequence should be uniformly, randomly distributed.
- Low-discrepancy/low-dispersion properties should be preserved under the randomization.

Can obtain BOTH probabilistic bounds and optimal deterministic bounds!

Recent developments:

- Scrambled Halton (Warnock, 1972)
- Randomized lattices (Fox, 1999)
- Randomized (t,m,s)-nets, (t,s)-sequences (Owen, 2000)
- Randomized Halton (Wang, Hicknernell, 2000)

There are sequence families over which the original sequence is worse than average.

Search Issues?

Search issues are still important.

- High-dimensional local minima is a known problem for grids.
- The problem must be the same for the PRM.

Randomized potential field over a Lazy classical grid (Barraquand, Latombe, 1989).

Could apply potential field methods, bidirectional RRTs (LaValle, Kuffner, 2000), expansive C-space planner (Hsu, Latombe, Motwani, 1997; Sanchez, Latombe, 2001), or other methods over high-resolution lazy grids.

Other Topics in Sampling-Based Motion Planning

Attempting to Unify Infinite Grid Sequences and RRTs

[Original RRT]

[A Deterministic Alternative]

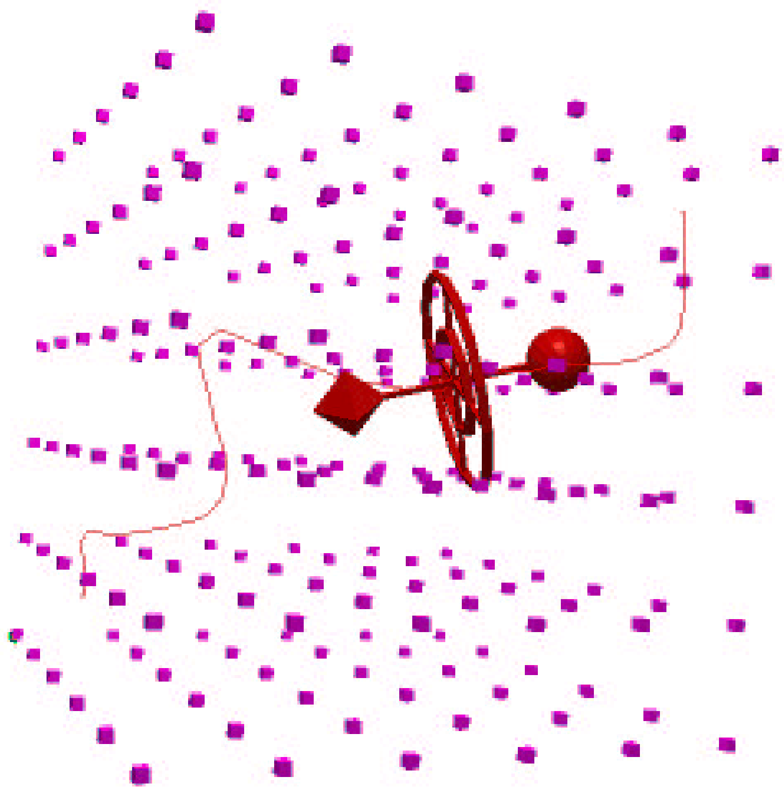
- Considering different topologies
- Relationship between sphere packing, lattices, etc.
- Differential constraints and sampling
- Sampling w.r.t. Haar measure
- Transforming sequences for importance sampling

RRTs: Trajectory Design for a Space Station

By: Peng Cheng

[Movie]

- 12D nonlinear system, including drift
- Inputs: 3 off-center bidirectional thrusters



Lattices

A family of lattices is given by (Sloan, Joe, 1994)

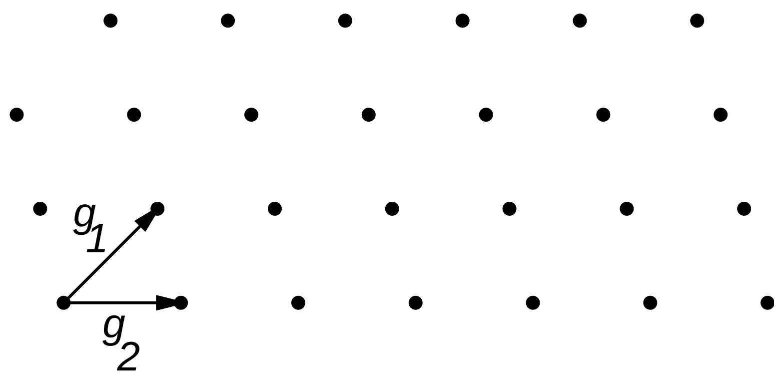
$$\frac{i\mathbf{z}}{N},$$

in which $\mathbf{z}$ is a d-dimensional vector, and all arithmetic is modulo one.

d generator vectors, g_1, …, g_d can be derived. Each lattice point can be represented as

$$x = \sum_{j=1}^{d} k_j g_j$$

in which k_j are integer coefficients.



Neighborhoods are determined automatically!