Intrinsic Tactile Sensing For Mobile Manipulation II

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Foundations Seminar Talk

July 30 2003

Outline

Recap of Matt's talk

- Intrinsic Tactile Sensing
- System Inversion

Differential Algebraic Equations

Solution using Orthogonal Basis Functions

Noise Rejection

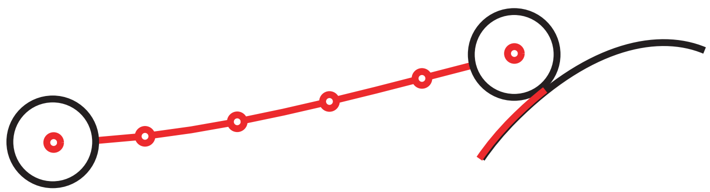
Summary

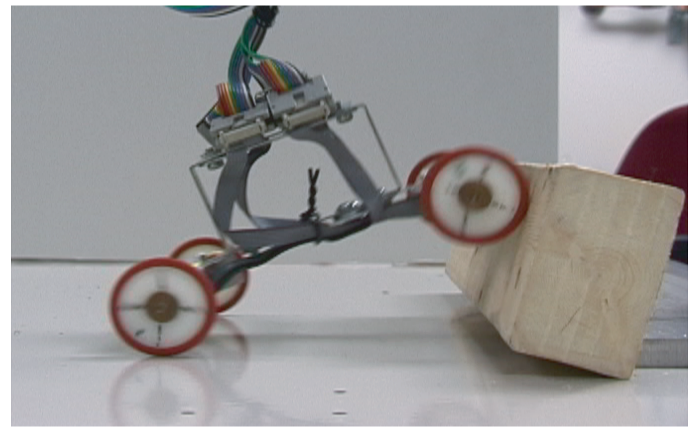
Intrinsic Tactile Sensing

Sensing internal forces and inferring externally applied forces

Force-torque sensing wrists for robot arms

Mobile Manipulation





Goals

- Dynamic out-of-plane manipulation
- Terrain estimation

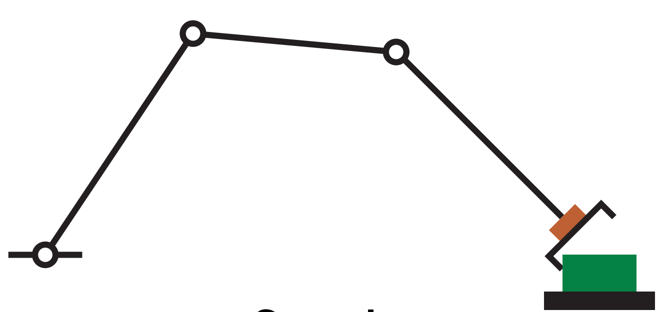
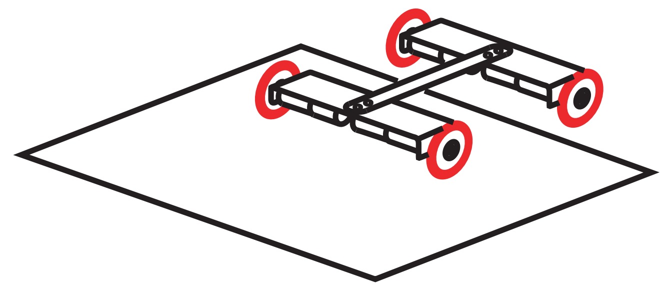
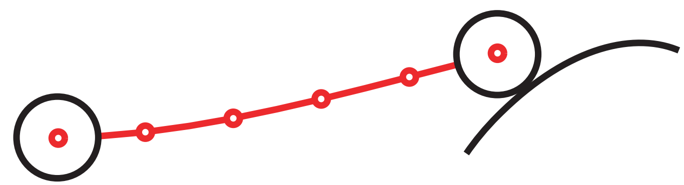
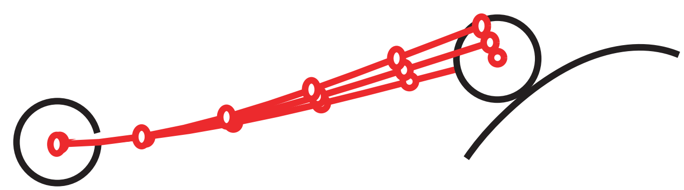
Need

- Location of wheel contact
- Force exerted at the contact

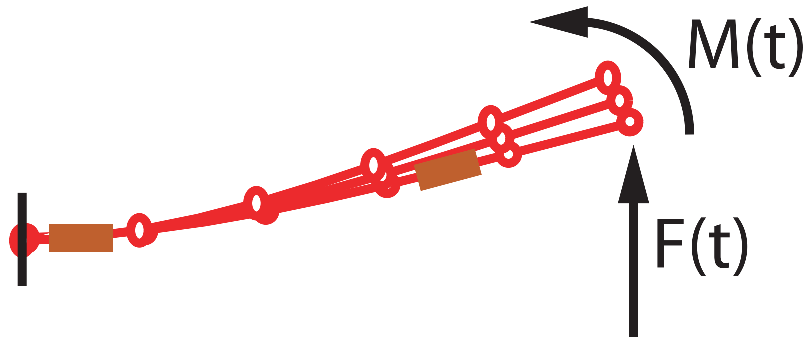
The Answer

- Mount a force-torque sensor on the robot chassis and infer external forces

Challenges

Robot Arms	Mobile Manipulators
 Serial	 Parallel
Rigid	 Flexible
Static	 Dynamic

Framework



The Problem

- Let *q* be the mechanical configuration, i.e. shape, of the deformable structure;
- Let $x = (q, \dot{q})$ be the mechanical state of the structure;
- Let *u* = *(F(t),M(t))* be the unknown input;
- Let *y* be the measured output from the sensors;

Solution

- Model beam dynamics as a linear time-invariant (LTI) system :

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- Use system inversion to reconstruct input

Optimal design of dynamic multi-axis force/torque sensor
Bicchi, Caiti and Prattichizzo
Proc. CDC 1999

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System Inversion

Let S be a system with state $x(t)$, input $u(t)$ and output $y(t)$ defined over some t in $[0,T]$. Assume that $x(0)$ is known.

S is invertible if given $y(t)$ and $x(0)$ we can uniquely compute $u(t)$.

L-integral inverse - $u(t)$ can be composed from $y(t)$ and its first L derivatives.

For LTI systems :

- System must have at least as many outputs as inputs
- System and its inverse must be stable
- Existence - rank condition on a big matrix M composed of A,B,C and D
  - *Brockett and Mesarovic (1965)*
  - *Sain and Massey (1966)*
- Hairy algorithm to compute L-integral inverse
  - *Silverman (1969)*

Outline

Recap of Matt's talk

Differential Algebraic Equations

- Equivalence to system inversion
- Kronecker Weierstrass transformation
- Fast subsystems

Solution using Orthogonal Basis Functions

Noise Rejection

Summary

Numerical Inversion

Original System :

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Rewrite as a Differential Algebraic Equation (DAE):

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{u} \end{pmatrix} = \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} y$$

$$E\dot{z} = Fz + Gy$$

Proposition : Inverting the LTI system is equivalent to solving an initial value problem (IVP) for the DAE

Equivalence

To prove equivalence, we will show that the transfer functions are inverses of each other.

For the original system :

$$\frac{Y}{U} = G(s) = C[sI - A]^{-1}B$$

For the DAE :

$$\frac{Z}{Y} = \hat{G}(s) = \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\frac{U}{Y} = \frac{\begin{bmatrix} 0 & I \end{bmatrix} Z}{Y} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} sI - A & -B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$= \left[C[sI - A]^{-1}B\right] = G^{-1}(s)$$

Numerical solutions of DAEs extremely well researched

Differential Algebraic Equations

What's with the *algebraic*?

$$E\dot{z} = Fz + Gy$$

If *E* were full-rank, we would have an ODE :

$$\dot{z} = E^{-1}Fz + E^{-1}Gy$$

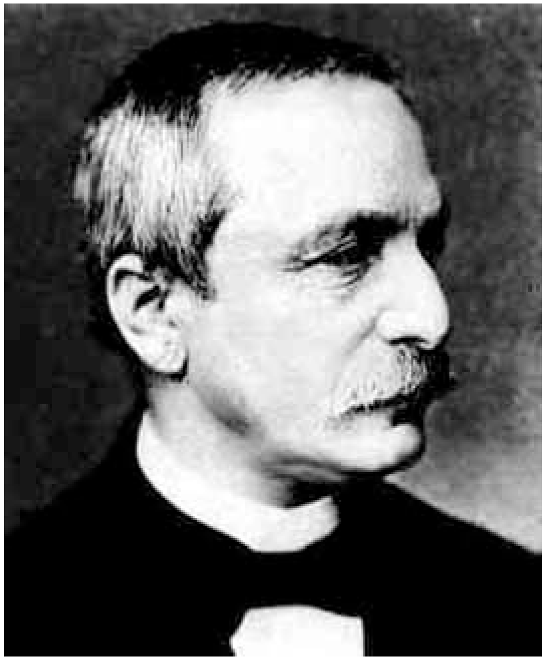
If E were rank-deficient, some of the elements of *z* must *also* satisfy an algebraic relation

The decomposition of the DAE into an ODE and an algebraic equation is called the Kronecker-Weierstrass transformation.

But, before all that, ... here are some pictures of Kronecker and Weierstrass

DAEs are not ODEs
Petzold L. R.
SIAM J. Sci. and Statist. Comp., 1982

K and W visit Pittsburgh



Kronecker

Weierstrass ...
happy, I think

After a Primanti Bros.
sandwich

Slow and Fast Subsystems

Let *k* be the nullity of *E*.

The KW transformation decomposes the DAE into

Slow subsystem : $\dot{z}_1 = F_1 z_1 + G_1 y$

Fast subsystem : $N\dot{z}_2 = z_2 + G_2 y$

where *N* is of the form :

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

m=(k+1)

$N^m=0$

m is called the global index of the DAE

Computation of Kronecker's canonical form of a singular pencil
P. Van Dooren.
Lin. Alg. Appl.,1979

ODE methods for the solution of DAEs
Gear C.W. and Petzold L. R.
SIAM J. Numerical Anal.,1984

Example

$$\begin{aligned} \dot{x} &= x + u \\ y &= x \end{aligned} \qquad\qquad x, u, y \in \mathbf{R}$$

The input *u* and the state *x* are given by :

$$\begin{aligned} u &= \dot{y} - y \\ x &= y \end{aligned}$$

The system has a 1-Integral inverse.

Example

The DAE can be written as :

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{u} \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y$$

$$E\dot{z} = Fz + Gy$$

Nullity(E) = 1. Global index m = 1+1 = 2.

The KW transform of the DAE looks like :

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} u \\ x \end{pmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} y$$

$$N\dot{z}_2 = z_2 + G_2 y$$

where N^2=0.

Nullity(E) = (m-1) = L

No slow subsystem

What makes the fast subsystem fast?

If we differentiate the fast subsystem, we get :

$$N{z_2}^{(2)} = z_2^{(1)} + G_2 y^{(1)}$$

$$N^2{z_2}^{(2)} = N z_2^{(1)} + N G_2 y^{(1)}$$

$$= z_2 + G_2 y + N G_2 y^{(1)}$$

If we differentiate *m* times :

$$N^m G_2 z_2^{(m)} = z_2 + G_2 y + \cdots + N^{m-1} G_2 y^{(m-1)} = 0$$

`

$$z_2 = -\sum_{i=0}^{m-1} N^i G_2 y^{(i)}$$

The fast subsystem depends only on *y* and its derivatives

It does not depend on the initial value $z_2(0)$

It is extremely sensitive to changes in *y*

Example II

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{z}_{21} \\ \dot{z}_{22} \\ \dot{z}_{23} \end{pmatrix} = \begin{pmatrix} z_{21} \\ z_{22} \\ z_{23} \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} y$$

The solution :

$$\begin{aligned} z_{23} &= y \\ z_{22} &= \dot{z}_{23} = \dot{y} \\ z_{21} &= \dot{z}_{22} = \ddot{y} \end{aligned}$$

Fast subsystem is merely a cascade of differentiators.

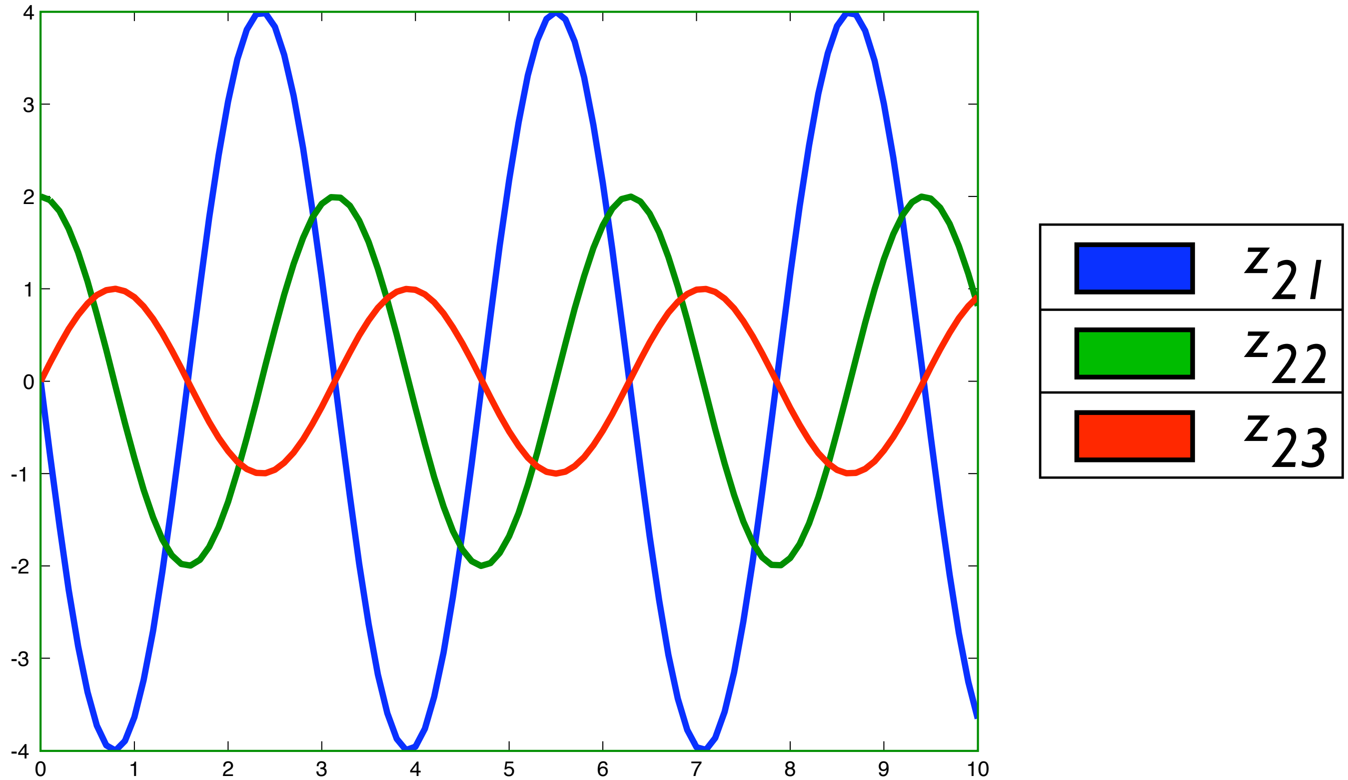
Need to do (m-1) differentiations.

Example II

Let's solve for z_2 for y = $sin(2t)$.

Let's do it numerically using Forward Euler.

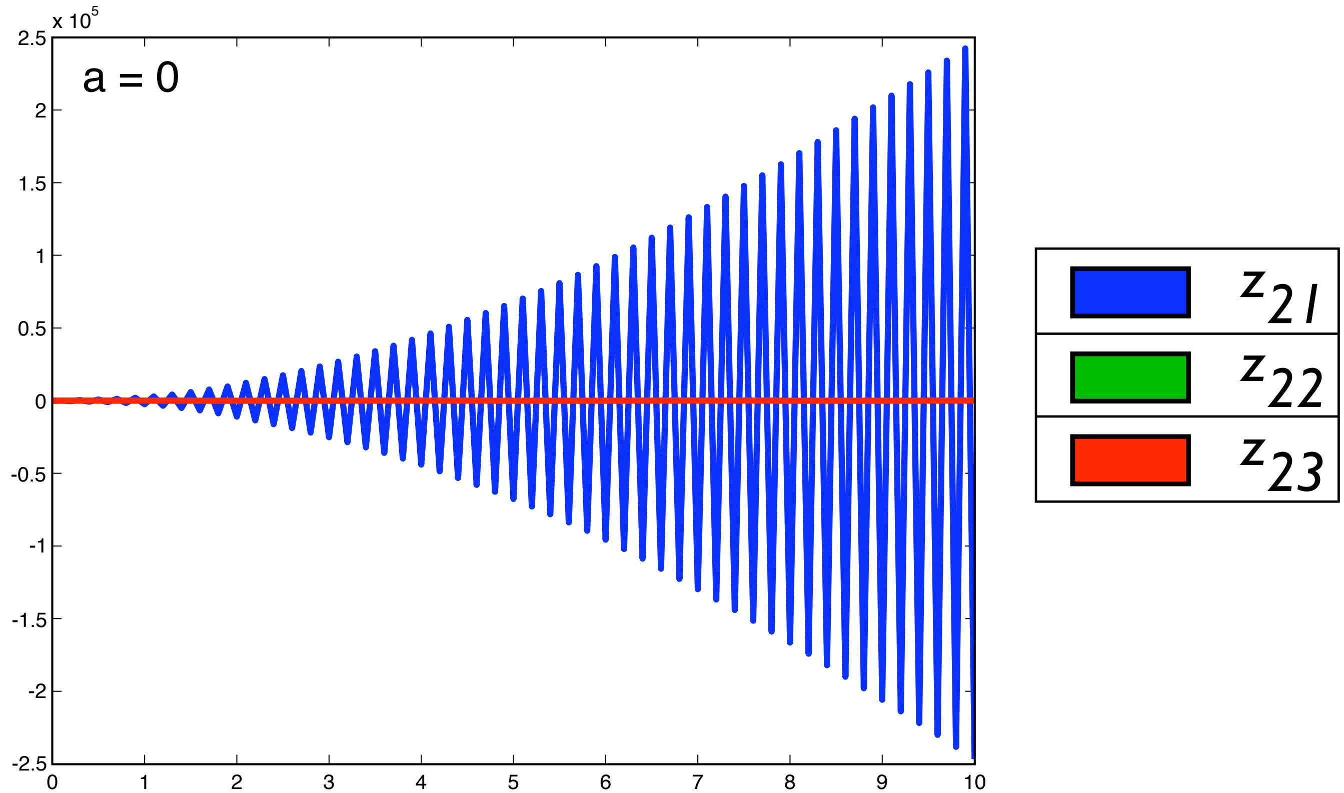
$$\dot{y}_n = \frac{y_{n+1} - y_n}{h}$$



Example II

Let's solve for z_2 for a noisy $y = sin(2t) + 0.05\ n(t)$.

Forward Euler is unstable with noise.



Summary of DAEs

System inversion is equivalent to IVP for DAEs.

DAEs are composed of fast and slow subsystems.

The slow subsystem is an ODE.

Fast subsystem

- A cascade of differentiators.
- Does not depend on its initial value. Luenberger called it a pure predictor.
- Extremely sensitive to noise and changes in y.

Need a principled method of solving DAEs with guaranteed noise rejection.

Orthogonal Basis Functions

Aim - Find the solution of the IVP :

$$E\dot{z}(t) = Fz(t) + Gy(t) \quad t \in [0, T]$$
$$z(0) = z_0$$

Method - Convert the DAE into an algebraic equation.

Define a set of orthogonal basis functions :

$$\Phi_N(t) = [\phi_1(t), \phi_2(t), \cdots \phi_N(t)]$$

The basis functions are chosen such that :

$$\int_0^t \Phi_N(\tau)\mathrm{d}\tau = P_N \Phi_N(t)$$

P_N - Operational Matrix of Integration

Functions which satisfy closure property - Fourier, Hermite, Chebyshev, Block-Pulse, ...

Analysis of Singular Systems using Orthogonal Functions
Lewis F.L. and Mertzois B.G.
IEEE Trans. Auto. Control, 1987

Projection and Solution

Project *z(t)*, *y(t)* and *z(0)* onto the basis functions :

$$z(t) \approx Z_N \Phi_N(t)$$
$$y(t) \approx Y_N \Phi_N(t)$$
$$z(0) \approx Z_{0N} \Phi_N(t)$$

Integrate the DAE :

$$\int_0^t E\dot{z}(\tau)\mathrm{d}(\tau) = \int_0^t Fz(\tau)\mathrm{d}(\tau) + \int_0^t Gy(\tau)\mathrm{d}(\tau)$$

$$E(z(t) - z(0)) \approx F \int_0^t Z_N \Phi_N(\tau)\mathrm{d}(\tau) + G \int_0^t Y_N \Phi_N(\tau)\mathrm{d}(\tau)$$

$$E(Z_N - Z_{0N}) = F Z_N P_N + G Y_N P_N$$

We end up with an algebraic equation for Z_N

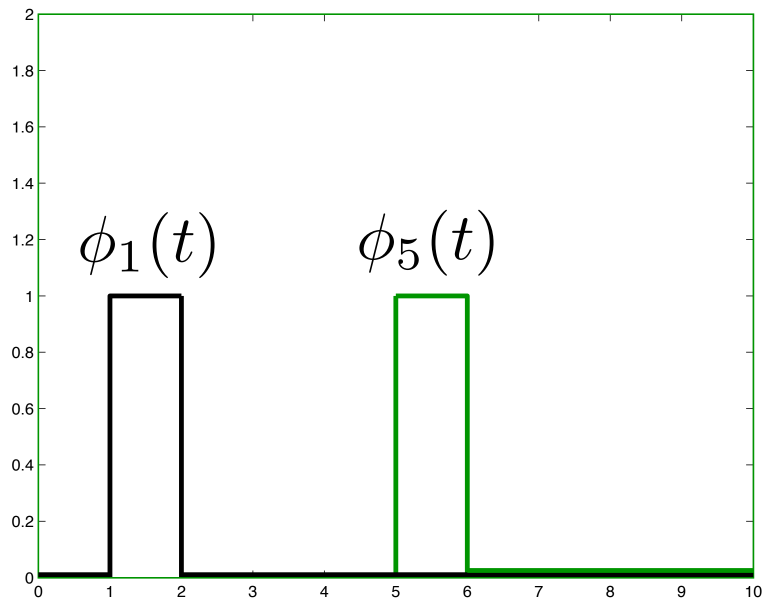
Block-Pulse Functions

We pick Block-Pulse Functions (BPF) as our basis.

Why?

- Less online computation.
- Noise rejection.

$$\phi_i(t) = \begin{cases} 1 & i\frac{T}{N} \leq t < (i+1)\frac{T}{N} \\ 0 & otherwise \end{cases}$$



Stabilization of Spectral Methods for the Analysis of Singular Systems Using Piecewise Constant Basis Functions
Caiti A. and Cannata G.
Cir. Systems Sig. Proc., 1995

Noise Rejection

Applying the BPFs to the algebraic equation :

$$E(Z_N - Z_{0N}) = FZ_N P_N + GY_N P_N$$

The solution is of the form :

$$P_N Z_i = (Y_i + known) = V_i \qquad i = 1 \cdots N$$

$$Z_i = P_N^{-1}(V_i)$$

Let V_{i0} be the noise-free input and Z_{i0} the solution :

$$Z_{i0} = P_N^{-1}(V_{i0})$$

Now consider the case :

$$V_i = V_{i0} + \epsilon$$

How does the method react to the added noise?

Noise Rejection

$$Z_i = P_N^{-1}(V_i + \epsilon)$$

Define the error *e* as :

$$e = \| Z_i - Z_{i0} \|$$

$$e = \| P_N^{-1}(V_i + \epsilon) - P_N^{-1}(V_i) \| \leq \| P_N^{-1} \| \| \epsilon \|$$

Define the matrix norm as :

$$\| A \| = \max_j \sum_i | a_{ij} |$$

$$e \leq \frac{(4N-2)}{h} \| \epsilon \| \qquad h = \frac{T}{N}$$

Error increases as N increases.

This is because we are differentiating noise.

Regularization

Define a regularized operator as :

$$(P_N + aI)Z_{ia} = V_i + \epsilon$$

a - regularization parameter

Equivalent to a low-pass filter applied to the solution.

$$e = \| Z_{ia} - Z_{i0} \| = \| (P + aI)^{-1} \| (a \| Z_{i0} \| + \| \epsilon \|)$$

Regularization adds to the error.

"After an easy, though boring, computation ... "

$$e \leq \begin{cases} \| Z_{i0} \| + \frac{1}{a} \| \epsilon \| & 0 < a < \frac{h}{2} \\ \frac{4}{h+2a}(a \| Z_{i0} \| + \| \epsilon \|) & \frac{h}{2} \leq a \end{cases}$$

The error is independent of N.

Regularization Techniques for the Analysis of Singular Dynamic Systems
Caiti A. and Cannata G.
IEEE CDC, 1990

Numerical Solution

Substituting the regularized operator back into the algebraic equation, we get :

$$\left[E - h\left(\frac{1}{2} + a\right)F\right]Z_{i+1} = \left[E + h\left(\frac{1}{2} - a\right)F\right]Z_i + hG\left[\left(\frac{1}{2} + a\right)(Y_{i+1} - Y_i) + Y_i\right]$$

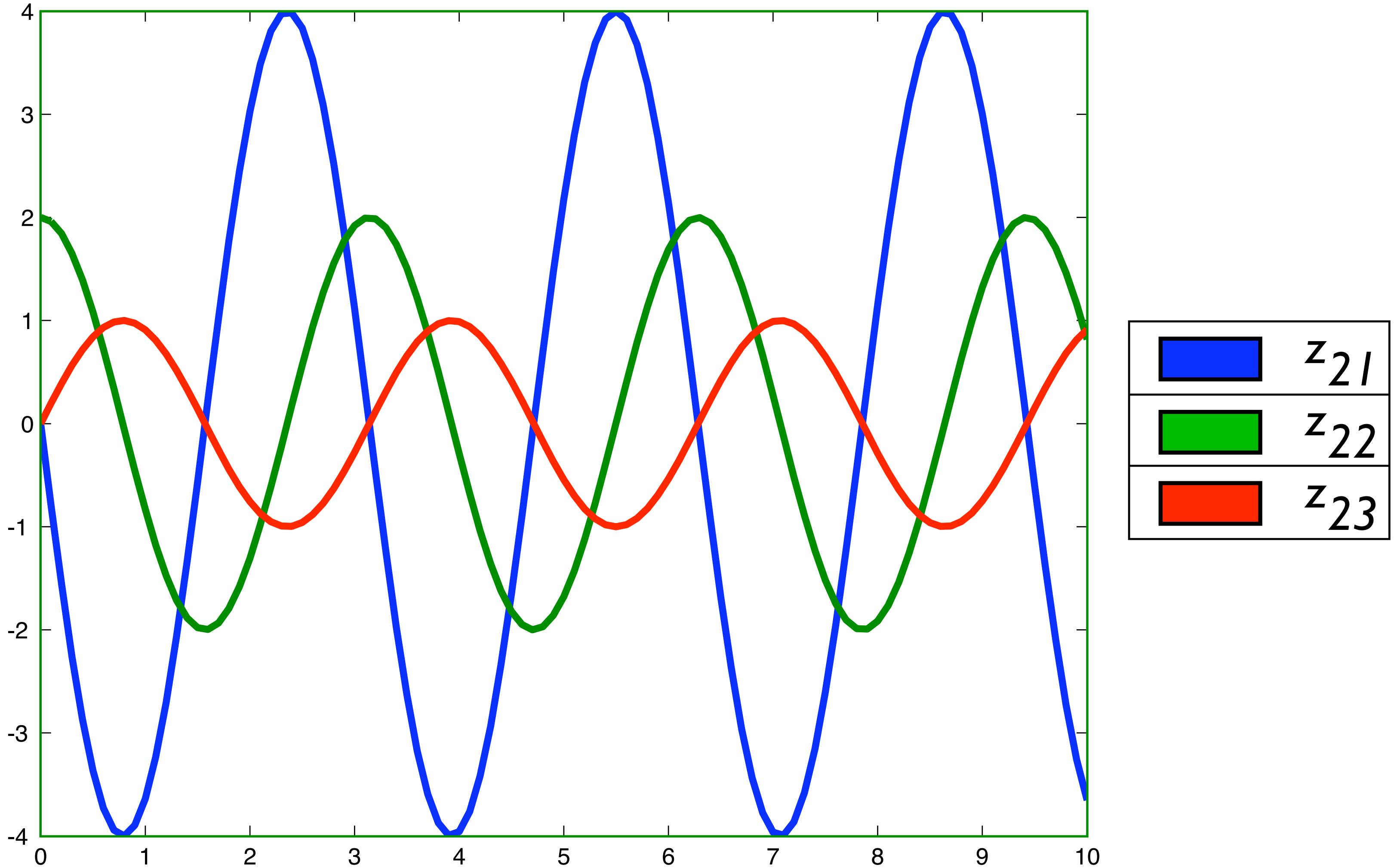
Solve for Z_{i+1}

a = 0 - Forward Euler

a = 0.5 - Backward Euler

Increasing *a* - smoother solution, but, less accurate.

Results



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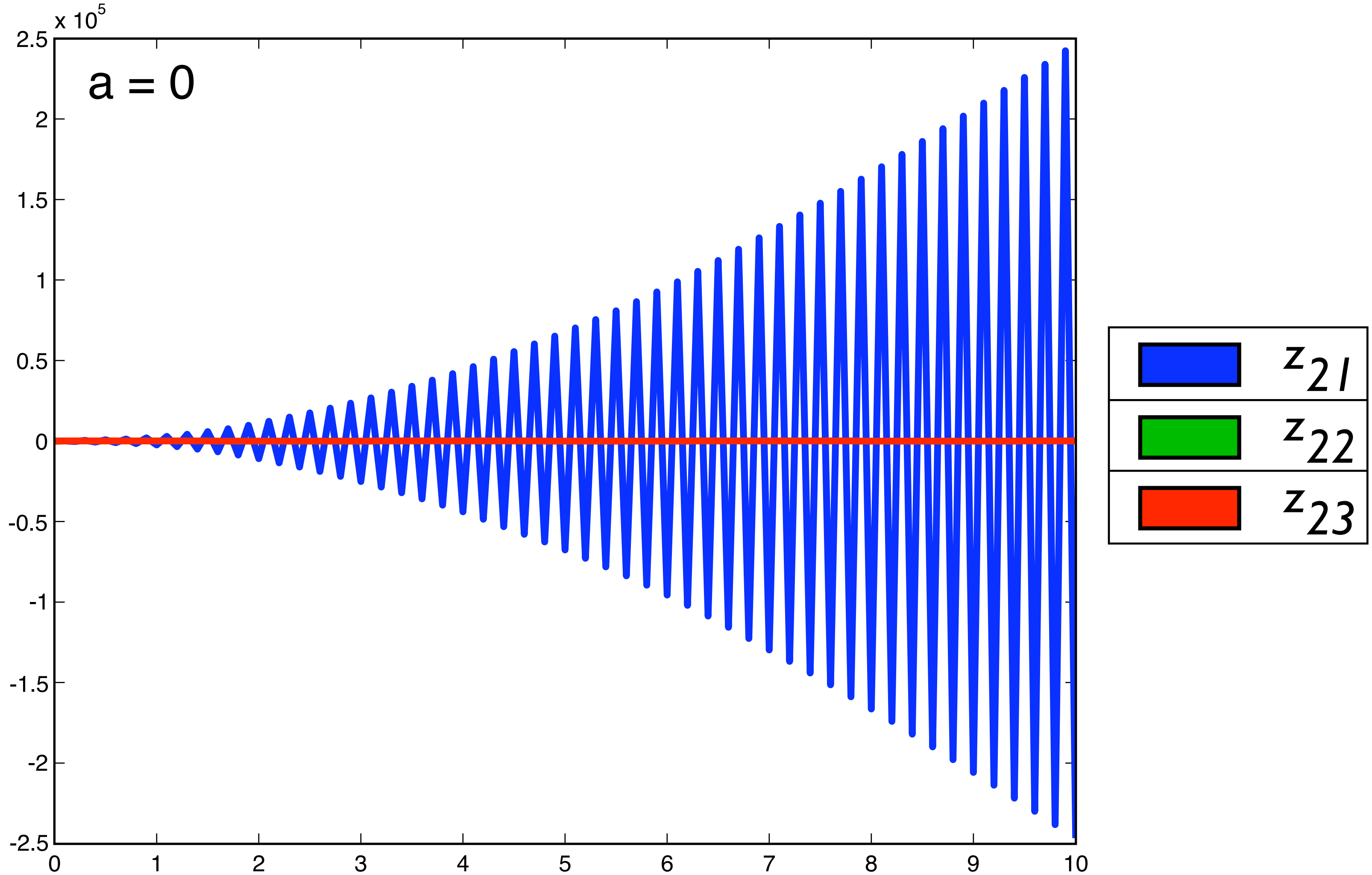

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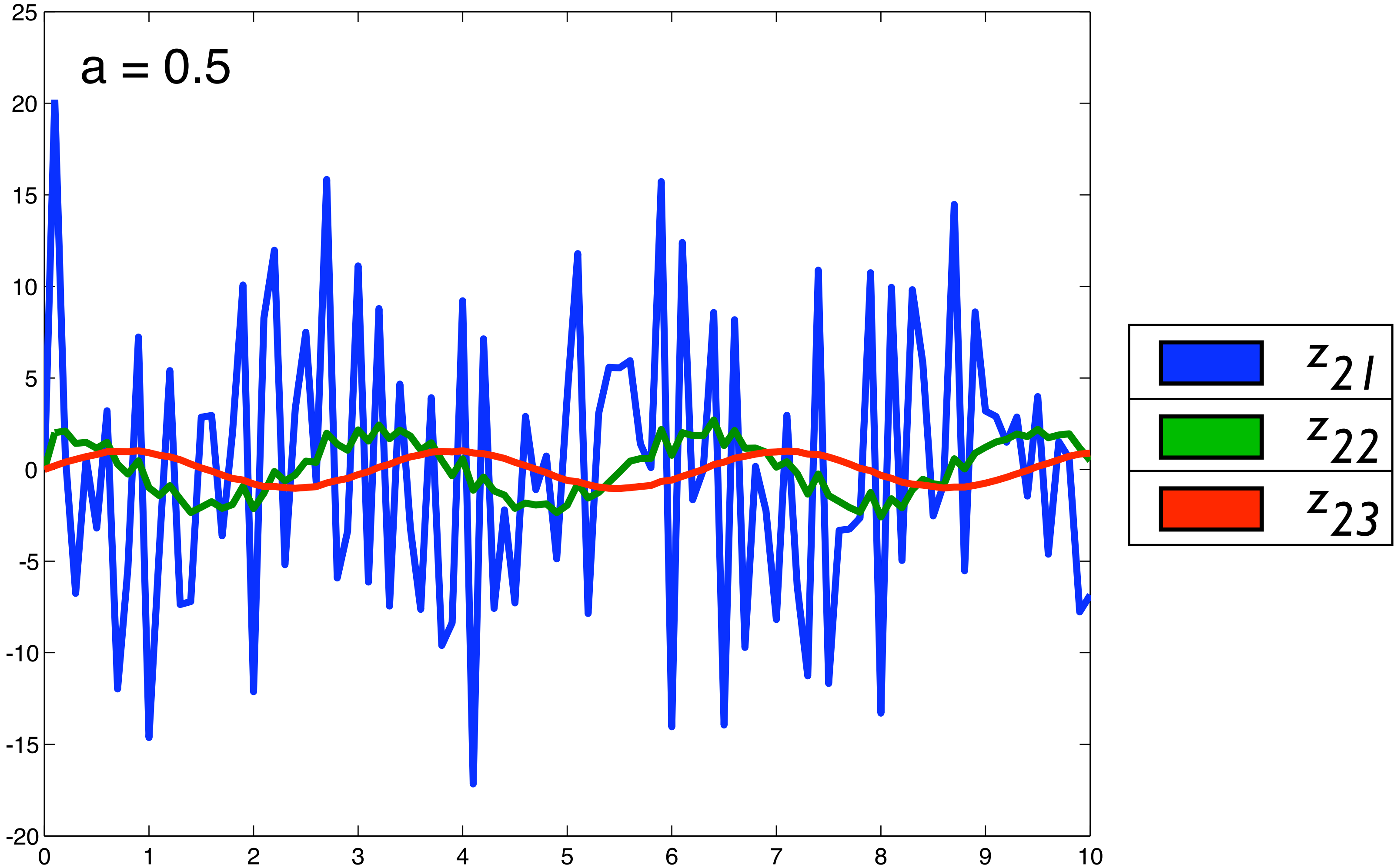

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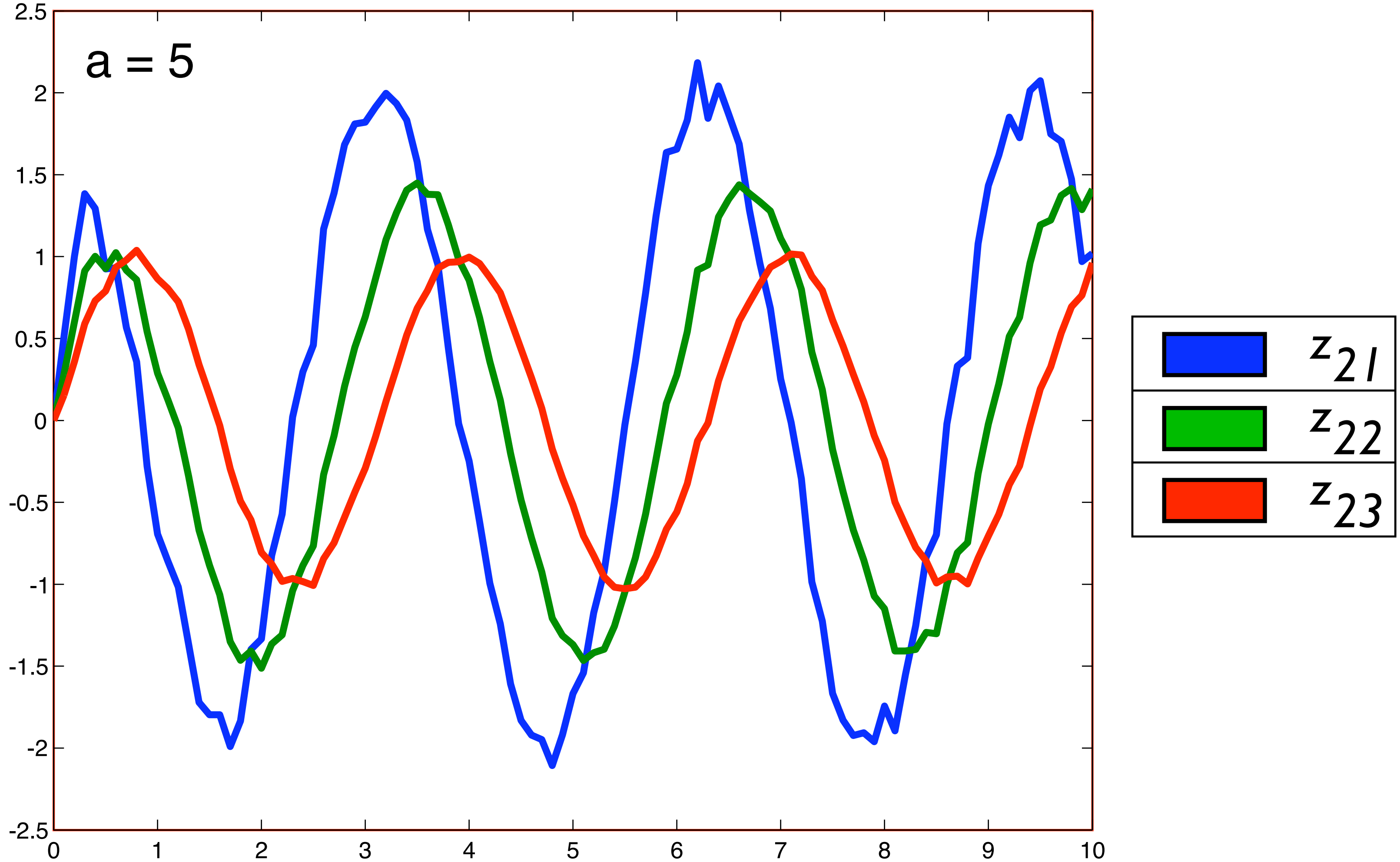

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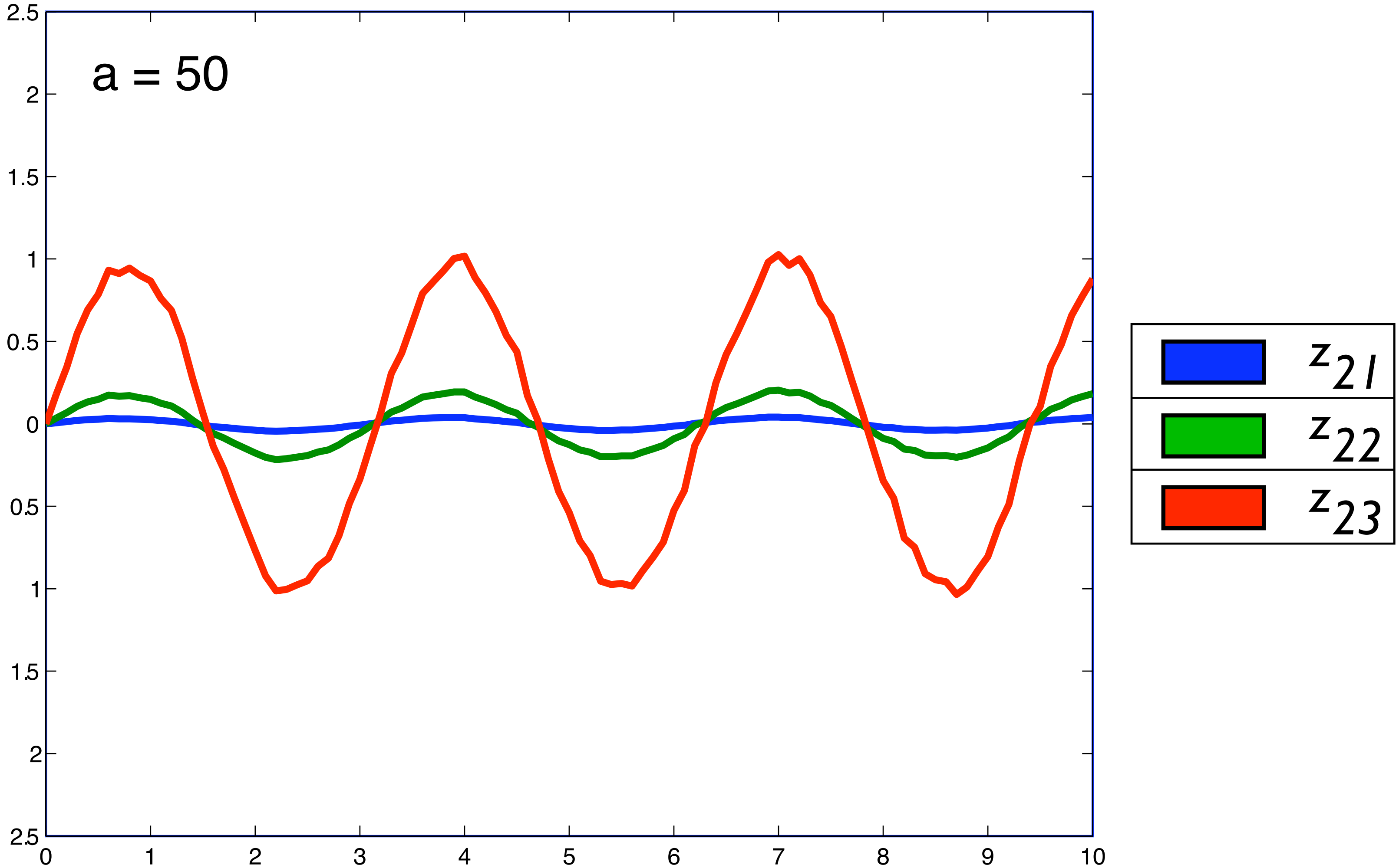

Conclusion

System inversion can be framed as an IVP for a DAE.

The fast subsystem is extremely sensitive to noise.

Numerical solutions obtained using Block-Pulse functions.

Noise rejection can be controlled by regularization.

- Advantage - Smoother solution
- Disadvantage - Deviation from real solution

Pick a judiciously.

The End