# PAL: PRogram-aided Language Models 

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#### Abstract

Large language models (LLMs) have recently demonstrated an impressive ability to perform arithmetic and symbolic reasoning tasks when provided with a few examples at test time ("few-shot prompting"). Much of this success can be attributed to prompting methods for reasoning, such as "chain-of-thought", employ LLMs for both understanding the problem description by decomposing it into steps, as well as solving each step of the problem. While LLMs seem to be adept at this sort of step-by-step decomposition, LLMs often make logical and arithmetic mistakes in the solution part, even when the problem is correctly decomposed. In this paper, we present Program-Aided Language models (PAL): a new method that uses the LLM to read natural language problems and generate programs as the intermediate reasoning steps, but offloads the solution step to a programmatic runtime such as a Python interpreter. With PAL, decomposing the natural language problem into runnable steps remains the only learning task for the LLM, while solving is delegated to the interpreter. We demonstrate this synergy between a neural LLM and a symbolic interpreter across 12 reasoning tasks from BIG-Bench Hard and other benchmarks, including mathematical reasoning, symbolic reasoning, and algorithmic problems. In all these natural language reasoning tasks, generating code using an LLM and reasoning using a Python interpreter leads to more accurate results than much larger models, and we set new state-of-the-art results in all 12 benchmarks. For example, PaL using Codex achieves state-of-the-art few-shot accuracy on the GSM8K benchmark of math word problems when the model is allowed only a single decoding, surpassing PaLM-540B with chain-of-thought prompting by absolute $8 \%$. On GSM-HARD, a more challenging version of GSM8K that we create, PaL outperforms chain-of-thought by an absolute $40 \%{ }^{1}$


## 1 Introduction

Until as recently as two years ago, reasoning was considered to be one of the most significant challenges that large language models (LLMs) had not yet overcome (Marcus, 2018; 2020; Garcez \& Lamb, 2020). Recently, LLMs have shown impressive success on a wide range of reasoning tasks, including commonsense (Wei et al., 2021; Sanh et al., 2021; Madaan et al., 2022), mathematical (Lewkowycz et al., 2022; Wu et al., 2022), and symbolic reasoning (Yao et al., 2022; Ahn et al., 2022), using few-shot prompting (Liu et al., 2021).
This process has been accelerated by methods that require LLMs to generate their explicit reasoning steps, such as "chain-of-thought" (Wei et al., 2022), "scratchpads" (Nye et al., 2021), and "least-to-most" (Zhou et al., 2022) prompting. In particular, the widely used chain-of-thought (COT) method presents the model with

[^0]Chain-of-Thought (Wei et al., 2022)


Program-aided Reasoning (this work)
Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 tennis balls.
tennis_balls $=5$
2 cans of 3 tennis balls each is
bought_balls $=2$ * 3
tennis balls. The answer is
answer = tennis_balls + bought_balls
Q: The bakers at the Beverly Hills Bakery baked 200 loaves of bread on Monday morning. They sold 93 loaves in the morning and 39 loaves in the afternoon. A grocery store returned 6 unsold loaves. How many loaves of bread did they have left?

## Model Output

A: The bakers started with 200 loaves
loaves_baked $=200$
They sold 93 in the morning and 39 in the afternoon
loaves_sold_morning $=93$
loaves_sold_afternoon $=39$
The grocery store returned 6 loaves.
loaves_returned $=6$
The answer is
answer = loaves_baked - loaves_sold_morning

- loaves_sold_afternoon + loaves_returned
>>> print(answer)
74

Figure 1: A diagram illustrating PAL: Given a mathematical reasoning question, Chain-of-thought (left) generates intermediate reasoning steps of free-form text. In contrast, Program-aided Language models (PAL, right) generate intermediate steps and Python code. This shifts the role of running the reasoning steps from the language model to the Python interpreter. The final answer is obtained by running the generated reasoning chain. Chain-of-thought reasoning is highlighted in blue; PAL steps are highlighted in gray and pink; Python interpreter run is highlighted in black and green
the explicit intermediate steps that are required to reach the final answer. Then, the model is expected to apply a similar decomposition to the actual test example, and consecutively reach an accurate final answer (Ling et al., 2017; Amini et al., 2019). Nevertheless, while LLMs can decompose natural language problems into steps and perform simple arithmetic operations, their performance falls dramatically when dealing with complex arithmetic (Hendrycks et al., 2021; Madaan \& Yazdanbakhsh, 2022) or large numbers (Nogueira et al., 2021; Qian et al., 2022). In fact, even when fine-tuning a PaLM-based model on 164B tokens of explicit mathematical content, its two most common failures are reportedly "incorrect reasoning" and "incorrect calculation" (Lewkowycz et al., 2022).
In this paper, we propose Program-Aided Language model (PAL): a novel prompting method that uses an LLM to read natural language problems and generate programs as reasoning steps, but offloads the solution step to a Python interpreter, as illustrated in Figure 1. This offloading relies on an LLM that can decompose a natural language problem into programmatic steps, which is fortunately possible using contemporary
state-of-the-art LLMs that are pre-trained on both natural language and programming languages (Chowdhery et al., 2022; Brown et al., 2020; Chen et al., 2021a). While natural language understanding and decomposition require LLMs, solving and reasoning can be done with the external solver. This bridges an important gap in chain-of-thought-like methods, where reasoning chains can be correct but produce an incorrect answer.
We demonstrate the effectiveness of PAL across $\mathbf{1 2}$ arithmetic and symbolic reasoning tasks. In all these tasks, PaL using Codex (Chen et al., 2021a) outperforms much larger models such as PaLM-540B using chain-of-thought prompting. For example, on the popular GSM8K benchmark, PAL achieves state-of-the-art accuracy, surpassing PaLM-540B with chain-of-thought prompting by absolute $8 \%$. When we replace the concrete numbers in the GSM8K examples with larger numbers, a dataset we call GSM-HARD, PAL outperforms CoT by absolute $40 \%$. We believe that this seamless synergy between a neural LLM and a symbolic interpreter is an essential step towards general and robust AI reasoners.

## 2 Program-Aided Language Models

### 2.1 Background: Few-Shot Prompting

Few-shot prompting leverages the strength of large-language models to solve a task with a set of $k$ examples that are provided as part of the test-time input (Brown et al., 2020; Liu et al., 2021; Chowdhery et al., 2022), where $k$ is usually a number in the low single digits. These input-output examples $\left\{\left(x_{i}, y_{i}\right)\right\}_{i=1}^{k}$ are concatenated in a prompt $p \equiv\left\langle x_{1} \cdot y_{1}\right\rangle\left\|\left\langle x_{2} \cdot y_{2}\right\rangle\right\| \ldots \|\left\langle x_{k} \cdot y_{k}\right\rangle$. where "." denotes the concatenation of an input and output, and " $\|$ " indicate the concatenation of different examples. During inference, a test instance $x_{\text {test }}$ is appended to the prompt, and $p \| x_{\text {test }}$ is passed to the model which attempts to complete $p \| x_{\text {test }}$, and thereby generate an answer $y_{\text {test }}$. Note that such few-shot prompting does not change the underlying LLM.

Wei et al. (2022) additionally augment each in-context example with chain of thought (COT) intermediate steps. Specifically, each in-context example in the CoT setup is a triplet $\left\langle x_{i}, t_{i}, y_{i}\right\rangle$, where $x_{i}$ and $y_{i}$ are input-output pair as before, and $t_{i}$ is a short phrase describing the steps that are needed to arrive at the output $y_{i}$ from the input $x_{i}$. See Figure 1 for an example. With the additional "thoughts" $t_{i}$, the prompt is set to $p$ $\equiv\left\langle x_{1} \cdot t_{1} \cdot y_{1}\right\rangle\left\|\left\langle x_{2} \cdot t_{2} \cdot y_{2}\right\rangle\right\| \ldots \|\left\langle x_{k} \cdot t_{k} \cdot y_{k}\right\rangle$.
During inference, the new question $x_{t e s t}$ is appended to the prompt as before and supplied to the LLM. Crucially, the model is tasked with generating both the thought $t_{\text {test }}$ and the final answer $y_{\text {test }}$. This approach of prompting the model to first generate a reasoning process $t_{\text {test }}$ improves the accuracy of the answer $y_{\text {test }}$ across a wide range of tasks (Wang et al., 2022b; Wei et al., 2022; Zhou et al., 2022; Wang et al., 2022c).

### 2.2 Program-aided Language Models

In a program-aided language model, we propose to generate the thoughts $t$ for a given natural language problem $x$ as interleaved natural language (NL) and programming language (PL) statements. Since we delegate the solution step to the PL interpreter, we do not provide the final answers to the examples in our prompt. That is, every in-context example in PAL is a pair $\left\langle x_{i}, t_{i}\right\rangle$, where $t_{j}=\left[s_{1}, s_{2}, \ldots, s_{N}\right]$ with each $s_{i} \in \operatorname{NL} \cup \mathrm{PL}$, a sequence of tokens in either NL or PL. The complete prompt is thus $p \equiv\left\langle x_{1} \cdot t_{1}\right\rangle\left\|\left\langle x_{2} \cdot t_{2}\right\rangle\right\| \ldots \|\left\langle x_{k} \cdot t_{k}\right\rangle$.
Given a test instance $x_{\text {test }}$, we append it to the prompt, and $p \| x_{\text {test }}$ is fed to the model. We let the model generate a prediction $t_{\text {test }}$, which contains both the intermediate steps and their corresponding programmatic statements.

Example A close-up of the example from Figure 1 is shown in Figure 2. While chain-of-thought only decomposes the solution in the prompt into natural language steps such as Roger started with 5 tennis balls and 2 cans of 3 tennis balls each is, in PAL we also augment each such NL step with its programmatic statement such as tennis_balls $=5$ and bought_balls $=2 * 3$. This way, the model learns to

```
A: Roger started with }5\mathrm{ tennis balls.
tennis_balls = 5
2 cans of 3 tennis balls each is
bought_balls = 2 * 3
tennis balls. The answer is
answer = tennis_balls + bought_balls
```

Figure 2: A close-up of a single example from PAL prompt. Chain-of-thought reasoning is highlighted in blue, and PAL programmatic steps are highlighted in gray and pink
generate a program that will provide the answer for the test question as well, instead of relying on LLM to perform the calculation correctly.

We prompt the language model to generate NL intermediate steps using comment syntax (e.g. "\# . . ." in Python) such they will be ignored by the interpreter. At the end of the model's generation, we pass the generated program $t_{\text {test }}$ to its corresponding solver, we run it, and obtain the final run results $y_{\text {test }}$. In this work we use a standard Python interpreter, but this can be any solver, interpreter or a compiler.

We also ensure that variable names in the prompt meaningfully reflect their roles. For example, a variable that describes the number of apples in the basket should have a name such as num_apples_in_basket. This keeps the generated code segments semantically rich and aligned with the actual entities. In Section 5 we show that such meaningful variable names are critical. Notably, it is also possible to incrementally run the PL segments and feed the execution results back to the LLM to generate the following blocks. For simplicity, in our experiments, we limit to a single post-hoc execution.

Crafting prompts for PAL In our experiments, we leveraged the prompts of existing work whenever available, and otherwise randomly selected the same number (3-6) of examples as previous work for creating the prompt. In all cases, the authors augmented the free-form text prompts into PAL-styled prompts, leveraging programming constructs such as for loops, dictionaries, and composite functions when needed. This provides a way to efficiently and rapidly develop PAL prompts.

This work focuses on CoT-style reasoning chain, but in Appendix C we also show that PAL also improves Least-to-Most (Zhou et al., 2022) prompts, which introduce reasoning chains of that decompose a question to sub-questions.

## 3 EXPERIMENTAL SETUP

Data and in-context examples We experiment with three broad kinds of reasoning tasks from BIGBench (Srivastava et al., 2022) that require (1) symbolic reasoning (§3.1) from the BIG-Bench Hard subset (Suzgun et al., 2022); mathematical problems (§3.2) from a wide range of datasets including GSM8K (Cobbe et al., 2021), SVAMP (Patel et al., 2021), ASDIV (Miao et al., 2020), and MAWPS (KoncelKedziorski et al., 2016); and algorithmic problems (§3.3) from BIG-Bench Hard as well. Details of all datasets are shown in Appendix A. For all of the experiments, we use the same in-context examples as used by Wei et al. (2022).

Baselines We consider three prompting strategies: DIRECT prompting - the standard prompting approach using pairs of questions and immediate answers as in Brown et al. (2020); chain-of-thought (CoT) prompting (Wei et al., 2022); and PAL prompting. We performed greedy decoding from the language model using a

```
# Q: On the table, you see a bunch of objects arranged in a
    row: a purple paperclip, a pink stress ball, a brown
    keychain, a green scrunchiephone charger, a mauve fidget
    spinner, and a burgundy pen. What is the color of the
    object directly to the right of the stress ball?
# Put objects into a list to record ordering
objects = []
objects += [('paperclip', 'purple')] * 1
objects += [('stress ball', 'pink')] * 1
objects += [('keychain', 'brown')] * 1
objects += [('scrunchiephone charger', 'green')] * 1
objects += [('fidget spinner', 'mauve')] * 1
objects += [('pen', 'burgundy')] * 1
# Find the index of the stress ball
stress_ball_idx = None
for i, object in enumerate(objects):
    if object[0] == 'stress ball':
        stress_ball_idx = i
        break
# Find the directly right object
direct_right = objects[stress_ball_idx+1]
# Check the directly right object's color
direct_right_color = direct_right[1]
answer = direct_right_color
```

Figure 3: An example PaL prompt for Colored ObJECTS, which includes the intermediate chain-of-thought steps.
temperature of 0 . Unless stated otherwise, we used CODEX (code-davinci-002) as our backend LLM for both PAL, Direct and CoT, and conducted experiments using few-shot prompting. In datasets where PaLM-540B results are available from previous work, we included them and marked them as CoT ${ }_{\text {PaLm-540b }}$.

### 3.1 Symbolic Reasoning

We applied PAL to three symbolic reasoning tasks, which involve reasoning about objects and concepts, taken from BIG-Bench Hard Suzgun et al. (2022).

Colored Objects The Colored Objects task requires answering questions about colored objects on a surface. This task requires keeping track of relative position, absolute position, and the color attributes of each of the objects. Then, the model needs to answer a question regarding the provided information. Figure 3 shows an example for a question and PAL prompt.

Penguins in a Table The Penguins in a Table (Penguins) task describes a table of penguins and some additional information in natural language, and the task is to answer a question about the attributes of the penguins, for example, "how many penguins are less than 8 years old?". While both PENGUINS and Colored Object tasks require tracking objects, Penguins describes dynamics as well, since the penguins in the problem can be added or removed. Figure 10 in Appendix D. 2 shows an example for a question, a chain-of-thought prompt, and PAL prompt.

Date Understanding The Date Understanding task involves inferring dates from natural language descriptions, performing addition and subtraction of relative periods of time, and also having some global knowledge
such as "how many days are there in February", and performing the computation accordingly. Figure 17 shows example prompts.

### 3.2 Mathematical Reasoning

We also evaluate PAL on seven mathematical word problem datasets. Each question in these tasks is an algebra word problem at grade-school level. An example for a question, chain-of-thought prompt, and PAL prompt is shown in Figure 4. We found that using explicit NL intermediate steps does not further benefit these math reasoning tasks, hence we kept only the semantically meaningful variable names in the prompt.

```
# Q: Olivia has $23. She bought five bagels for $3 each. How
    much money does she have left?
money_initial = 23
bagels = 5
bagel_cost = 3
money_spent = bagels * bagel_cost
money_left = money_initial - money_spent
result = money_left
```

Figure 4: Example prompt for a mathematical task, from the GSM8k benchmark.

GSM-hard LLMs can perform simple calculations with small numbers. For example, Madaan \& Yazdanbakhsh (2022) found that $50 \%$ of the numbers that appear in the popular GSM8K dataset of math reasoning problems are integers between 0 and 8 . This raises the question of whether LLMs can generalize to larger and non-integer numbers?

We constructed a harder version of GSM8K, which we call GSM-HARD, by replacing the numbers in the questions of GSM8K with larger numbers. Specifically, one of the numbers $n$ in a question was replaced with $\hat{n}$ drawn uniformly from $\hat{n} \sim \mathcal{U}\left(1,10^{7}\right)$. Further, for $80 \%$ of the converted numbers $\hat{n}$, we converted $\hat{n}$ into a decimal number by adding a random fractional part, setting $\hat{n} \leftarrow \hat{n}+\mathcal{N}(0,1)$. More details regarding this dataset creation are provided in A. 1

### 3.3 AlGORITHMIC TASKS

```
# Q: I have a chair, two potatoes, a cauliflower, a lettuce
    head, two tables, a cabbage, two onions, and three fridges.
    How many vegetables do I have?
# note: I'm not counting the chair, tables, or fridges
vegetables_to_count = {
    'potato': 2,
    'cauliflower': 1,
    'lettuce head': 1,
    'cabbage': 1,
    'onion': 2
}
result = sum(vegetables_to_count.values())
```

Figure 5: Object Counting task: prompts used by CoT (top) and PaL (bottom).

Finally, we compare PAL and CoT on algorithmic reasoning. We experiment with two algorithmic tasks: Object Counting and Repeat Copy.

Object Counting involves answering questions about the number of objects belonging to a certain type. For example, as shown in Figure 5: "I have a chair, two potatoes, a cauliflower, a lettuce head, two tables, ... How many vegetables do I have?" ).

REPEAT COPY requires generating a sequence of words, according to instructions. For example, as shown in Appendix D.6: "Repeat the word duck four times, but halfway through also say quack").

## 4 RESULTS

|  | Colored Objects | Penguins in a Table | Date Understanding |
| :--- | :---: | :---: | :---: |
| DIRECT | 75.7 | 71.1 | 49.9 |
| COT | 86.3 | 79.2 | 64.8 |
| PAL | $\mathbf{9 5 . 1}$ | $\mathbf{9 3 . 3}$ | $\mathbf{7 6 . 2}$ |

Table 1: Solve rate on 3 reasoning datasets, Colored Objects, Penguins and Date. In all datasets, PaL achieves a much higher accuracy than chain-of-thought.

Symbolic Reasoning Results for symbolic reasoning tasks are shown in Table 1. In Colored Objects, PAL improves over the strong CoT by $8.8 \%$, and by $19.4 \%$ over the standard direct prompting. In PENGUINS, PAL provides a gain of absolute $14.1 \%$ over CoT. In DATE, PAL further provides $11.4 \%$ gain over the strong CoT.

Math Results Table 2 shows the following results: across all tasks, PaL using Codex outperforms both CoT +Codex and CoT +PaLM-540B, setting a new few-shot state-of-the-art when allowed only a single decoding across all datasets. ${ }^{2}$

Interestingly, CoT also benefits from using Codex over PaLM-540B in some of the datasets such as ASDIV, but performs worse than PaLM-540B in others such as SVAMP. Yet, using PAL further improves the solve rate on all datasets.

On the GSM-HARD dataset (Table 2), the accuracy of CoT drops dramatically from $65.6 \%$ to $20.1 \%$ (a relative drop of almost $70 \%$ ), while the accuracy of PAL remains stable at $61.5 \%$, dropping by only $14.3 \%$. This shows how PAL provides not only better results on the standard benchmarks, but also better robustness to larger and non-integer number. In fact, since PAL offloads the computation to the Python interpreter, any complex computation can be performed accurately given the correctly generated program.

Are these failures primarily due to failure of LLM to do arithmetic, or do large numbers in the question affect the output generated by the language models? To investigate, we evaluate the outputs generated by CoT for the two versions of the same question (with and without large numbers). We find that in 16 out of 25 cases we analyze, CoT generates nearly identical thought, indicating that the primary failure mode is the inability to perform arithmetic. Sample outputs are shared in Table 7.

Algorithmic Tasks Table 3 shows that PaL is close to solving Object Counting, reaching $96.7 \%$ and improving over CoT by absolute $23.7 \%$. Similarly, PAL vastly outperform CoT by absolute $21.8 \%$ on REpEAT COPY.

[^1]|  | GSM8K | GSM-HARD | SVAMP | ASDIV | SINGLEEQ | SINGLEOP | ADDSUB | MULTIARITH |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| DIRECT | 19.7 | 5.0 | 69.9 | 74.0 | 86.8 | 93.1 | 90.9 | 44.0 |
| COT $_{\text {COT }}^{\text {PaLM-540B }}$ | 65.6 | 56.9 | 20.1 | 74.8 | 76.9 | 89.1 | 91.9 | 86.0 |
| PAL | $\mathbf{7 2 . 0}$ | $\mathbf{6 1 . 7}$ | 79.0 | 73.9 | 92.3 | 94.1 | 91.9 | 95.9 |

Table 2: Problem solve rate (\%) on mathematical reasoning datasets. The highest number on each task is in bold. The results for DIRECT and PaLM-540B are from Wei et al. (2022), and we reproduced the results on CoT. Since PaLM-540B is not publicly available, we only include reported results.

|  | Object Counting | Repeat Copy |
| :--- | ---: | ---: |
| Direct | 37.6 | 81.3 |
| CoT | 73.0 | 68.8 |
| PaL | $\mathbf{9 6 . 7}$ | $\mathbf{9 0 . 6}$ |

Table 3: The performance of PaL vs. CoT on the algorithmic Object Counting and Repeat Copy tasks.

Surprisingly, Direct prompting performs better than CoT on Repeat Copy. Still, CoT improves over Direct by $9.3 \%$ in Repeat Copy and by $59.1 \%$ in Object Counting.

## 5 ANALYSIS



Figure 6: Left to right: Colored Objects, Date, Penguins
Figure 7: Ablation study of PAL prompt formats. We consider the original PaL prompt, it with natural language comments removed ( $\mathrm{PAL}_{- \text {comment }}$ ), and further variable names replaced with random character (PAL $\left.{ }_{-c o m m e n t}^{-v a r}\right)$. As a reference, we also show the CoT performance (blue).

In all our experiments, we always tried to use meaningful variable names, to ease the model's grounding of the variables to the entities they represent. For example, to calculate "How many apples does John have left?", we used a temporary variable john_apples_left. For the Python interpreter, however, variable names are meaningless, and the computation would result in the same answer even if the variable was named, for example, $x$. In this section, we analyze whether variable names are useful, and whether explicit intermediate steps in NL are helpful, given that the code has meaning variable names.

To measure the importance of meaningful variable names, we conducted the following ablation study. On three datasets, Colored Objects, Date, and Penguins, we considered two prompts variants:

1. $\mathrm{PAL}_{-\mathrm{comment}}$ - the PAL prompt without intermediate NL comments.
2. PAL ${ }_{-c o m m e n t}^{-v a r}$ - the PAL prompt without intermediate NL comments and with variable names substituted with random characters.

The results are shown in Figure 7. In Colored Objected and Date, removing intermediate NL comments but keeping meaningful variable names ( $\mathrm{PAL}_{- \text {comment }}$ ) - slightly reduces the results compared to the full PAL prompt, but it still achieves higher accuracy than the baselines CoT. Removing variable names as well (PAL-comment) further decreases accuracy, and performs worse than COT. Since variable names have an important part in code quality (Gellenbeck \& Cook, 1991; Takang et al., 1996), it is only expected that meaningful variable names will ease reasoning for Codex, which was trained on mostly meaningful names, as was also found by Madaan et al. (2022).
Surprisingly, in PENGUINS, both ablations ( $\mathrm{PAL}_{- \text {comment }}$ and $\mathrm{PAL}_{-\mathrm{comment}}^{-\mathrm{var}}$ ) result in only a minor decrease in accuracy, and both ablations result in much higher accuracy than CoT. We believe that the reason is that in Penguins there is usually less ambiguity regarding the mapping between entities in the NL text and the generated code. There is usually only a single variable that represents the table of penguins, an "addition" or "removal" of a penguin that is applied to the same table, and a final query applied to the same table. That is, there is more logic, fewer local variables, and thus PAL works well even without meaningful variable names. An example from Penguins is shown in Figure 10.

Ablation experiments on GSM8K show a similar trend, and we include those results in Appendix G.

## 6 Related Work

Prompting Techniques With the advent of LLMs, few-shot prompting (Liu et al., 2021) has been a popular approach to tackle tasks ranging from text-generation (Gehrmann et al., 2021; Reif et al., 2021; Wei et al., 2021; Sanh et al., 2021) to code (Chen et al., 2021b) and dialog generation (Thoppilan et al., 2022). Techniques like chain-of-thought prompting (COT) have further boosted the performance of such models, attracting several proposed improvements. Notable examples include least-to-most prompting (Zhou et al., 2022) and self-ask (Press et al., 2022), which decomposes a complicated question into a set of sub-questions. The sub-questions are then evaluated to obtain the final answer. As discussed in the paper, these works are in the space of reasoning with natural language, focusing on formulating better solutions to reasoning problems. Nevertheless, they all still suffer from the failure modes in LLMs dealing with arithmetic and procedural tasks. Another line of work attempts to improve on chain-of-thought prompting by generating multiple reasoning chains (Li et al., 2022; Wang et al., 2022c) and 0-shot chain-of-thought prompting Kojima et al. (2022). PaL is complementary to them, but as our experiments show, it can outperform such methods on a wide range of tasks with just a top-1 sample. Yao et al. (2022) propose to enhance the execution of situated agents through natural language reasoning, PAL seeks to improve natural language reasoning using structured code.

Post-hoc processing Equipping neural models with specialized modules for post hoc processing is a promising direction for improving reasoning capabilities. For example, (Cobbe et al., 2021) employ a calculator for performing arithmetic operations, and (Demeter \& Downey, 2020) adds specialized modules for generating cities and dates. Unlike these works, our method produces a complete executable code in a formal programming language (Python), obviating the need to employ ad-hoc fixes.

Models of code for structured generation With increasing abilities of code-LLMs, there is a growing interest in applying them to structured generation tasks. Such methods typically re-formulate their desired
outputs to programs, and leverage a code-generation model to solve the task. Recent examples include script graph, explanation graph, entity state tracking generation (Madaan et al., 2022), and event argument extraction (Wang et al., 2022a). Unlike these works, the generated program in this work is not an end goal. Instead, a program is a formalism to drive better intermediate reasoning. Our work shares the broad idea of extracting structure from free-form text, and thus is similar in spirit to semantic parsing.

Semantic parsing Our work share certain similarities with the long-established semantic parsing task (Zelle \& Mooney, 1996; Zettlemoyer \& Collins, 2009; Liang, 2016, inter alia). Semantic parsing focuses on mapping and grounding the concepts in natural language to the primitives in the target representation (e.g., lambda calculus, dataflow) and to the entries in the target environment (e.g., a column name in a database), Our work shares the similar spirit of mapping and grounding, but focuses on abstract and complex natural language reasoning. We ground reasoning concepts to variables and manipulate them through any operations defined by a programming language. We believe the discoveries in this work, such as the importance of meaningful text, could also inspire works on semantic parsing with code-LLMs (Shin et al., 2021; Shin \& Van Durme, 2021; Cheng et al., 2022).
Andor et al. (2019) propose to generate simple arithmetic operations for reading comprehension tasks. For example, instead of directly generating the difference between two numbers $n_{0}$ and $n_{1}$, they propose to generate an expression like diff $n_{0}-n_{1}$, which is fed to a calculator for evaluation. Gupta et al. (2019) design neural modules such as count to deal with arithmetic operations. In contrast, PaL leverages state-of-the-art LLMs on code to produce a complete step-by-step solution as a Python program, generating expressions for not only arithmetic operations but also leveraging features like loop and conditional evaluation.

## 7 Conclusion

In this work, we introduce PAL, a new method for natural language reasoning, using programs as intermediate reasoning steps. Differently from existing LM-based reasoning approaches, the main idea is to offload solving and calculating to an external Python interpreter, instead of using the LLM for both understanding the problem and solving. This results in a final answer that is guaranteed to be accurate, given the correctly predicted programmatic steps. We demonstrate this seamless synergy between NL, PL and execution through runtime across 12 tasks from BIG-Bench Hard and other benchmarks. In all these benchmarks, PAL outperforms the popular "chain-of-thought" method that uses huge LLMs such as PaLM-540B, and sets new state-of-the-art accuracy on 11 of them. We believe that these results unlock exciting directions for future neuro-symbolic AI reasoners. To this end, we make all our code and prompts available at http://reasonwithpal.com/.

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## Part I

## Appendix

## Table of Contents

A Datasets ..... 18
A. 1 Creating GSM-HARD ..... 18
B GSm-hard Analysis ..... 20
C Generalization of PAL to Least-to-Most Prompting ..... 20
D Prompts ..... 22
D. 1 Reasoning about Colored Objects ..... 22
D. 2 Penguins in a Table ..... 23
D. 3 Date Understanding ..... 24
D. 4 Math ..... 25
D. 5 Object Counting ..... 27
D. 6 Repeat Copy ..... 28
E Penguins in a Table ..... 29
F Date Understanding ..... 29
G Ablation Study for GSM8K ..... 31

## A Datasets

In the following tables (Table 4, Table 5, Table 6), we presents statistics and examples for the datasets we considered.

| Dataset | N | Example |
| :---: | :---: | :---: |
| Reasoning about Colored Objects | 2000 | On the table, you see a bunch of objects arranged in a row: a purple paperclip, a pink stress ball, a brown keychain, a green scrunchiephone charger, a mauve fidget spinner, and a burgundy pen. What is the color of the object directly to the right of the stress ball? |
| Penguins in a Table | 149 | Here is a table where the first line is a header and each subsequent line is a penguin: name, age, height (cm), weight (kg) Louis, 7, 50, 11 Bernard, 5, 80, 13 Vincent, 9, 60, 11 Gwen, 8, 70, 15 For example: the age of Louis is 7, the weight of Gwen is 15 kg , the height of Bernard is 80 cm . We now add a penguin to the table: James, 12, 90, 12 How many penguins are less than 8 years old? |
| Date Understanding | 369 | 2015 is coming in 36 hours. What is the date one week from today in MM/DD/YYYY? |

Table 4: Reasoning datasets about everyday objects and concepts.

| Dataset | N | Example |
| :--- | :--- | :--- |
| Object Counting | 1000 | I have a chair, two potatoes, a cauliflower, a lettuce head, two tables, a <br> cabbage, two onions, and three fridges. How many vegetables do I have? <br> Repeat the word duck four times, but halfway through also say quack. |

Table 5: Reasoning datasets about algorithmic problems.

## A. 1 CREATING GSM-HARD

While replacing numbers in the question is easy using pattern matching, a more challenging aspect is recalculating the correct answer. GSM8K evaluation set contains 1319 samples, which is prohibitively expensive to perform manual re-calculation. Instead, we leverage PAL to assist obtaining the correct answers. For $71 \%$ of the examples where PAL is correct on GSM8K, we utilize the generated program and replace the initial value with the larger values. For example, if we create a harder version of the problem in Figure 4 by replacing $\$ 23$ dollars with $\$ 15687$ dollars, we correspondingly replace money_initial=23 to money_initial=15678. Running the program could automatically produce the correct answer of the harder question. Notably, this annotation process assumes that a program that produces a correct answer to a GSM8K question indicates the correctness of the program itself. While this is not guaranteed due to possible spurious correlations, we manually checked 25 programs and found all of them are correct. For the incorrect $29 \%$ of the cases, we run PAL again and perform nucleus sampling (Holtzman et al., 2019) with temperature 0.7 , and repeat the above process if any correct solution is found. Finally, the authors manually annotate the correct answer for 50 remaining cases that PAL was not able to solve after 100 iterations.

| Dataset | N | Example |
| :--- | ---: | :--- |
| GSM8K (Cobbe et al., 2021) | 1319 | Olivia has \$23. She bought five bagels for \$3 each. How <br> much money does she have left? |
| SVAMP (Patel et al., 2021) | 1000 | Each pack of dvds costs 76 dollars. If there is a discount <br> of 25 dollars on each pack. How much do you have to pay <br> to buy each pack? |
| ASDIV (Miao et al., 2020) | 2096 | Ellen has six more balls than Marin. Marin has nine balls. <br> How many balls does Ellen have? |
| SINGLEOP (Koncel-Kedziorski et al., 2016) | 562 | If there are 7 bottle caps in a box and Linda puts 7 more <br> bottle caps inside, how many bottle caps are in the box? |
| SINGLEEQ (Koncel-Kedziorski et al., 2016) | 508 | Benny bought a soft drink for 2 dollars and 5 candy bars. <br> He spent a total of 27 dollars. How much did each candy <br> bar cost? |
| ADDSUB (Koncel-Kedziorski et al., 2016) | 395There were 6 roses in the vase. Mary cut some roses from <br> her flower garden. There are now 16 roses in the vase. |  |
| MULTIARITH (Koncel-Kedziorski et al., 2016) | 600 | How many roses did she cut? <br> The school cafeteria ordered 42 red apples and 7 green <br> apples for students lunches. But, if only 9 students wanted <br> fruit, how many extra did the cafeteria end up with? |

Table 6: Examples from math reasoning datasets used in this work. Table from Wei et al. (2022). Datasets vary in number of words per question, with 47 words per question in GSM8K, 21 words in SINGLEOP, and $\sim 30$ words in the other datasets.

## B GSM-HARD ANALYSIS

Table 7 shows thoughts generated with CoT on GSM8K and GSM-HARD. A manual analysis reveals that a majority of the generated thoughts ( $16 / 25$ ) were identical for GSM8K and GSM-HARD, indicating that larger numbers primarily diminish performance due to failure of LLM to do arithmetic..

## C Generalization of PAL to Least-to-Most Prompting

```
Q: Four years ago, Kody was only half as old as Mohamed. If Mohamed is
\hookrightarrowurrently twice 30 years old, how old is Kody?
A: To answer the question "How old is Kody?", we need to know: "How old is
\hookrightarrow Mohamed?", "How old was Mohamed four years ago?", "How old was Kody
four years ago?".
```

(a) Least-to-Most Math Reducing Prompt

```
Four years ago, Kody was only half as old as Mohamed. If Mohamed is
currently twice 30 years old, how old is Kody?
Q: How old was Mohamed four years ago?
A: We were told that Mohamed is currently twice 30 years old, so he is
currently 30 * 2 = 60 years old. That means that four years ago he must
\hookrightarrow have been 60-4 = 56 years old. The answer is 56.
Q: How old is Kody?
A: Four years ago, Kody was half as old as Mohamed, so Kody must have been
\hookrightarrow6 / 2 = 28 years old then. Since Kody was 28 years old four years ago,
s she must now be 28 + 4 = 32 years old. The answer is 32.
```

(b) Least-to-Most Math Solving Prompt

```
# Four years ago, Kody was only half as old as Mohamed. If Mohamed is
    currently twice 30 years old, how old is Kody?
# How old was Mohamed four years ago?
mohamed_age_current = 30*2
mohamed_age_4_years_ago = mohamed_age_current - 4
# Final Question: How old is Kody?
kody_age_4_years_ago = mohamed_age_4_years_ago / 2
kody_age_current = kody_age_4_years_ago + 4
answer = kody_age_current
```

(c) PaL Math Solving Prompt

Figure 8: Prompts for Math data sets.

Previous experiments focus on the CoT technique. In this section, we also examine if PAL generalizes to other prompt types. We consider a strong alternative prompting strategy LEAST-TO-MOST Zhou et al. (2022). LEAST-TO-MOST solves problems in two stages, problem reducing and problem solving. Problem reducing stage turns the problem to sub-problems and the solving stage solves them sequentially. It keeps two prompts, each for an individual stage. To patch Least-TO-Most prompts with PaL, we adopt a simple and straightforward approach: we note that problem reduction require logically thinking in NL while solving

| Question |
| :--- |
| 90 single use contacts come |
| in 1 box and will last Pete 45 |
| days. Each box is $\$ 100.00$ |
| and currently $10 \%$ off. If he |
| buys 2 boxes of contact, how |
| much will each pair of con- |
| tacts cost? |

A Reddit group has 1000 members. If each member posts an average of 3 posts per day, what's the total number of posts that the group will have in March?

A bag has a 5\% discount. If it is marked $\$ 140$, how much will you pay after the discount?

A basket of green food costs $\$ 25$ and a basket of red food costs $\$ 18$. If you buy 3 baskets of green food and red food, how much will you have to pay in total if you get $\$ 2$ off for each basket of red food?

A bumper car rink has 12 red cars. They have 2 fewer green cars than they have red cars. They have 3 times the number of blue cars as they have green cars. The rink also has yellow cars. If the rink has 75 cars in total how many yellow cars do they have?


The discount is $5 \%$. So you will pay $95 \%$ of the price. $95 \%$ of 140 is 133 . The answer is 133 .
3 baskets of green food will cost $3 * 25$ $=75$ dollars. 3 baskets of red food will cost $3 * 18=54$ dollars. But you get \$2 off for each basket of red food. So you get $3 * 2=6$ dollars off. So you pay $75+54$ $6=123$ dollars. The answer is 123 .
There are 12 red cars. There are 2 fewer green cars than red cars. So there are 12-2 = 10 green cars. There are 3 times as many blue cars as green cars. So there are $3 * 10=30$ blue cars. There are $12+10+30=52$ cars that are not yellow. So there are ${ }^{175}$ $-52=23$ yellow cars. The answer is 23 .

| GSMHard-Cot | Identical? |
| :--- | :--- |
| 90 single use contacts come | Yes | in 1 box. So 1 box will last Pete 45 days. Each box is $\$ 5451440$ and currently $10 \%$ off. So each box is $\$ 5451440$ * $0.9=\$ 4906296$. If he buys 2 boxes of contact, he will have $2 * 90=180$ contacts. So each pair of contacts will cost $\$ 4906296$ / $180=\$ 2725849$. The answer is $\$ 2725849$

There are 1000 mem- Yes bers. Each member posts 913437.3469 posts per day. So the total number of posts is 1000 * $913437.3469=$ 913437346.9. The answer is 913437346.9

The discount is $5 \%$. So you will pay $95 \%$ of the price. $95 \%$ of $\$ 3921772.0722$ is $\$ 3730654.0682$. The answer is $\$ 3730654.0682$
A basket of green food costs 9412763 dollars. A basket of red food costs 18 dollars. If you buy 3 baskets of green food and red food, you will have to pay 9412763 * $3+$ $18 * 3=28238280$ dollars. If you get 2 dollars off for each basket of red food, you will have to pay 28238280-18* $2=28238244$ dollars. The answer is 28238244
The rink has 9318738 red cars. They have 2 fewer green cars than red cars. So they have $9318738-2=$ 9318736 green cars. They have 3 times the number of blue cars as green cars. So they have $3 * 9318736=$ 27956208 blue cars. The total number of cars is 75 . So the number of yellow cars is 75-9318738-9318736 $-27956208=-55,828,829$. The answer is $-55,828,829$

Yes

No
Yes

Table 7: Analysis of thoughts generated by CoDEX. Manual analysis of 25 randomly generated thoughts reveals that 16 out of 25 thoughts were identical, whereas others were close paraphrases.

| Dataset (500 examples) | LEAST-TO-Most | LEAST-TO-MOST + PAL |
| :--- | :---: | :---: |
| GSM8K | 67.2 | $\mathbf{7 2 . 8}$ |
| SVAMP | 75.2 | $\mathbf{7 8 . 2}$ |

Table 8: Results on GSM8K and SVAMP with Least-TO-MOST and LEAST-TO-MOST with PaL solving prompt.
requires the precision that PL offers. We therefore keep the original reducing prompts while only turn solution segments in the solving scripts in PL. We show example reducing prompt, original solving prompt and PAL solving prompt in Figure 8. Note that one unique property of PaL solving is able to naturally use previous questions' answers as the symbol values are shared. In comparison, the original solving script needs to explicitly re-cite answers from previous answers.
For our analysis, we consider the Math data sets GSM8K and SVAMP as Zhou et al. (2022) found Least-toMost helps solving complex math problems. We patch the GSM8K prompt from the Zhou et al. (2022) into PAL. Note that the other tasks in Zhou et al. (2022) like "concatenating last letters" from several words require simple routines and are trivially solvable by PAL. We experiments with subsets of 500 examples and record results in Table 8. Here we see PaL can take advantage of the problem decomposition offered by the LEAST-TO-MOST reducing and further leverage the arithmetic capability in the Python runtime to achieve additional performance gains.

## D PROMPTS

We show here example PAL prompts we used for each data set. We show one example for each of the few-shot prompts. The full prompt can be found in our released code.

## D. 1 Reasoning about Colored Objects

```
# Q: On the table, you see a bunch of objects arranged in a row: a purple
    paperclip, a pink stress ball, a brown keychain, a green scrunchiephone
    charger, a mauve fidget spinner, and a burgundy pen. What is the color
    of the object directly to the right of the stress ball?
# Put objects into a list to record ordering
objects = []
objects += [('paperclip', 'purple')] * 1
objects += [('stress ball', 'pink')] * 1
objects += [('keychain', 'brown')] * 1
objects += [('scrunchiephone charger', 'green')] * 1
objects += [('fidget spinner', 'mauve')] * 1
objects += [('pen', 'burgundy')] * 1
# Find the index of the stress ball
stress_ball_idx = None
for i, object in enumerate(objects):
    if object[0] == 'stress ball':
        stress_ball_idx = i
        break
# Find the directly right object
direct_right = objects[stress_ball_idx+1]
# Check the directly right object's color
direct_right_color = direct_right[1]
answer = direct_right_color
```


## D. 2 Penguins in a Table

```
"""Q: Here is a table where the first line is a header and each subsequent
    line is a penguin: name, age, height (cm), weight (kg) Louis, 7, 50,
    1 1 ~ B e r n a r d , ~ 5 , ~ 8 0 , ~ 1 3 ~ V i n c e n t , ~ 9 , ~ 6 0 , ~ 1 1 ~ G w e n , ~ 8 , ~ 7 0 , ~ 1 5 ~ F o r ~ e x a m p l e : ~
    the age of Louis is 7, the weight of Gwen is 15 kg, the height of
    Bernard is 80 cm. We now add a penguin to the table: James, 12, 90, 12
How many penguins are less than }8\mathrm{ years old?
" ""
# Put the penguins into a list.
penguins = []
penguins.append(('Louis', 7, 50, 11))
penguins.append(('Bernard', 5, 80, 13))
penguins.append(('Vincent', 9, 60, 11))
penguins.append(('Gwen', 8, 70, 15))
# Add penguin James.
penguins.append(('James', 12, 90, 12))
# Find penguins under }8\mathrm{ years old.
penguins_under_8_years_old = [penguin for penguin in penguins if penguin[1]
    < 8]
# Count number of perguins under 8.
num_penguin_under_8 = len(penguins_under_8_years_old)
answer = num_penguin_under_8
```

Figure 10
D. 3 DATE UndERSTANDING

```
# Q: 2015 is coming in 36 hours. What is the date one week from today in
    MM/DD/YYYY?
# If 2015 is coming in 36 hours, then today is 36 hours before.
today = datetime(2015, 1, 1) - relativedelta(hours=36)
# One week from today,
one_week_from_today = today + relativedelta(weeks=1)
# The answer formatted with %m/%d/%Y is
one_week_from_today.strftime('%m/%d/%Y')
```


## D. 4 MATH

```
#Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does
    she have left?
money_initial = 23
bagels = 5
bagel_cost = 3
money_spent = bagels * bagel_cost
money_left = money_initial - money_spent
print(money_left)
#Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On
    wednesday, he lost 2 more. How many golf balls did he have at the end of
    wednesday?
golf_balls_initial = 58
golf_balls_lost_tuesday = 23
golf_balls_lost_wednesday = 2
golf_balls_left = golf_balls_initial - golf_balls_lost_tuesday -
    golf_balls_lost_wednesday
print(golf_balls_left)
#Q: There were nine computers in the server room. Five more computers were
    installed each day, from monday to thursday. How many computers are now in
    the server room?
computers_initial = 9
computers_per_day = 5
num_days = 4 # 4 days between monday and thursday
computers_added = computers_per_day * num_days
computers_total = computers_initial + computers_added
print(computers_total)
#Q: If there are 3 cars in the parking lot and 2 more cars arrive, how many
    cars are in the parking lot?
cars_initial = 3
cars_arrived = 2
total_cars = cars_initial + cars_arrived
print(total_cars)
#Q: Leah had 32 chocolates and her sister had 42. If they ate 35, how many
    pieces do they have left in total?
leah_chocolates = 32
sister_chocolates = 42
total_chocolates = leah_chocolates + sister_chocolates
chocolates_eaten = 35
chocolates_left = total_chocolates - chocolates_eaten
print(chocolates_left)
```

Figure 12: Prompt used for mathematical reasoning (1/2)

```
#Q: Jason had 20 lollipops. He gave Denny some lollipops. Now Jason has 12
    lollipops. How many lollipops did Jason give to Denny?
jason_lollipops_initial = 20
jason_lollipops_after = 12
denny_lollipops = jason_lollipops_initial - jason_lollipops_after
print(denny_lollipops)
#Q: There are 15 trees in the grove. Grove workers will plant trees in the
    grove today. After they are done, there will be 21 trees. How many trees
    did the grove workers plant today?
trees_initial = 15
trees_after = 21
trees_added = trees_after - trees_initial
print(trees_added)
#Q: Shawn has five toys. For Christmas, he got two toys each from his mom and
    dad. How many toys does he have now?
toys_initial = 5
mom_toys = 2
dad_toys = 2
total_received = mom_toys + dad_toys
total_toys = toys_initial + total_received
print(total_toys)
```

Figure 13: Prompt used for mathematical reasoning (2/2)

## D. 5 Object Counting

```
# Q: I have a chair, two potatoes, a cauliflower, a lettuce head, two
    tables, a cabbage, two onions, and three fridges. How many vegetables
    do I have?
# note: I'm not counting the chair, tables, or fridges
vegetables_to_count = {
    'potato': 2,
    'cauliflower': 1,
    'lettuce head': 1,
    'cabbage': 1,
    'onion': 2
}
print(sum(vegetables_to_count.values()))
# Q: I have a drum, a flute, a clarinet, a violin, four accordions, a
    piano, a trombone, and a trumpet. How many musical instruments do I
    have?
musical_instruments_to_count = {
    'drum': 1,
    'flute': 1,
    'clarinet': 1,
    'violin': 1,
    'accordion': 4,
    'piano': 1,
    'trombone': 1,
    'trumpet': 1
}
print(sum(musical_instruments_to_count.values()))
# Q: I have a chair, two ovens, and three tables. How many objects do I
    have?
objects_to_count = {
    'chair': 1,
    'oven': 2,
    'table': 3
}
print(sum(objects_to_count.values()))
```

Figure 14: Prompt used for Оbject Counting.

## D. 6 Repeat Copy

```
# Q: Repeat the word duck four times, but halfway through also say quack
result = []
for i in range(1, 5):
    result.append("duck")
    if i == 2:
            result.append("quack")
print(" ".join(result))
# Q: Print boolean eleven times, but after the 3rd and 8th also say correct
result = []
for i in range(1, 12):
    result.append("boolean")
    if i == 3 or i == 8:
            result.append("correct")
print(" ".join(result))
# Q: say java twice and data once, and then repeat all of this three times.
result = []
tmp = ["java", "java", "data"]
for i in range(3):
    result.extend(tmp)
print(" ".join(result))
# Q: ask a group of insects in what family? four times. after the fourth
        time say The happy family
result = []
tmp = []
for i in range(1, 5):
    tmp.append("a group of insects in what family?")
tmp.append("The happy family")
result.extend(tmp)
print(" ".join(result))
```

Figure 15: Prompt used for Repeat Copy.
By manually examining the the model outputs, we observe that PAL often performs better on questions that involve counting objects that satisfy one or several conditions. For example, CoT fails in the following example: "On the desk, you see a bunch of items arranged in a row: a gold textbook, a purple puzzle, a teal necklace, and a silver pencil. How many non-gold items do you see to the right of the pencil?". With pure NL reasoning, a LLM can easily lose track of the objects and output the wrong answer of "The number of non-gold items to the right of the pencil is two. So the answer is two."
In comparison, PAL is able to accurately construct the object lists with correct order and attributes. Further, it can precisely leverage the simple yet complete PL syntax: it composes routines with functional operators from elementary builtin operation/operators. Figure 3 lists the last a few lines of the solution to the question described above generated by PAL, which perform filtering over objects. PAL can further compose such

```
# Find non-gold items to the right of the pencil
non_gold = [object for object in objects[i+1:] if object[1] != 'gold']
```

(a) Snippet of PAL doing a filter operation.

```
# Remove all pink objects
non_pink = [object for object in objects if object[1] != 'pink']
# Count number of crayons
crayons = [object for object in non_pink if object[0] == 'crayon']
answer = len(crayons)
```

(b) Snippet of PAL composing multiple operations.

Figure 16: Example model generation snippets on Reasoning about Color Object.
operations across multiple reasoning steps as shown in Figure 16b, where it first filters out all pink objects and then scans the remaining to count crayons. Other reasoning such as "how many objects are left after filtering" can be easily tackled precisely with an runtime.

## E Penguins in A TABLE

In this task, we found a typical failure mode of the language is its inability to stably tracks states in its past generation. ?? lists the generations of COT and PAL to the question that contains the removal of a penguin ("...We then delete the penguin named Bernard from the table. How many penguins are less than 8 years old?"). Although CoT picks up the critical information that "penguins that are less then 8 years old are Louis and Bernard", and that "Bernard is deleted". It still fails to aggregate the information properly and infer that there is one penguin less then 8 left in the end. In comparison, PaL expresses this dynamic through manipulating a penguins list by filtering out the penguin whose name is "Bernard", and maintaining a penguins_under_8_years_old list. It offloads tracking exact the values (in this case, the length of the list) to the deterministic Python runtime.

## F Date Understanding

We found this especially common when the time deltas are across month boundary. We show an example in Figure 17. Here with CoT prompting, the LLM expresses the knowledge of the 28-day-long February yet it still outputs $02 / 29 / 2017$ as final answer. With PAL, the actual calendar is accurate as the operation is handled by a program.

```
Q: Jane was born on the last day of Feburary in 2001. Today is her
@ 16-year-old birthday. What is the date 24 hours later in MM/DD/YYYY?
A: The last day of February is the 28th, so Jane was born on 02/28/2001.
Today is her 16-year old birthday, so today is 02/28/2017. So 24 hours
\hookrightarrow later is 02/29/2017. So the answer is 02/29/2017.
```

```
# Q: Jane was born on the last day of Feburary in 2001. Today is her
```


# Q: Jane was born on the last day of Feburary in 2001. Today is her

    16-year-old birthday. What is the date 24 hours later in MM/DD/YYYY?
    16-year-old birthday. What is the date 24 hours later in MM/DD/YYYY?
    
# If Jane was born on the last day of Feburary in 2001 and today is her

# If Jane was born on the last day of Feburary in 2001 and today is her

    16-year-old birthday, then today is }16\mathrm{ years later.
    16-year-old birthday, then today is }16\mathrm{ years later.
    today = datetime(2001, 2, 28) + relativedelta(years=16)
today = datetime(2001, 2, 28) + relativedelta(years=16)

# 24 hours later,

# 24 hours later,

later = today + relativedelta(hours=24)
later = today + relativedelta(hours=24)

# The answer formatted with %m/%d/%Y is

# The answer formatted with %m/%d/%Y is

later.strftime('%m/%d/%Y')

```
later.strftime('%m/%d/%Y')
```

Figure 17: Example model generation on Date Understanding.

| Setting | CoT | PAL-var | PAL-var + comms | PAL |
| :--- | :--- | :--- | :--- | :--- |
| Solve Rate | 63.1 | 59.0 | 69.0 | 71.8 |

Table 9: Role of text: including text either as informative variable names (PAL) or comments is important (PAL - var + comms). Uninformative variable names PAL - var cause a drastic drop in performance, indicating that just structure is not sufficient. The corresponding prompts are shown in Figure 18.

## G Ablation Study for GSm8k

```
a=23
b}=
c}=
d = b * c
e =a - d
print(e)
```

(a) Structured explanation with uninformative variable names (PAL - var)

```
# Olivia has $23
a=23
# number of bagels bought
b}=
# price of each bagel
c = 3
# total price of bagels
d = b * c
# money left
e = a - d
print(e)
```

(b) Structured explanation with uninformative variable names, but useful comments (PAL-var + comms)

```
money_initial = 23
bagels = 5
bagel_cost = 3
money_spent = bagels * bagel_cost
money_left = money_initial - money_spent
result = money_left
print(result)
```

(c) PaL prompts

Figure 18: Role of text in PAL: three different reasoning steps for the question Olivia has $\$ 23$. She bought five bagels for $\$ 3$ each. How much money does she have left? Uninformative variable names (left), Uninformative variable names with useful comments (left), and PAL. Including text description

For mathematical problems, since our standard prompts do not use much comment, we start by creating alternative prompts where the informative variable names are replaced with single-letters (Figure 18). The results in Table 9 shows a considerable performance drop: from an average of $71.8 \%$ to $59 \%$. Note that the ablation where structured outputs are completely removed in favor of purely text explanations is precisely the CoT setting, which achieves a solve rate of $63 \%$. These results underscore the importance of text but
more importantly show that combining both text and procedural statements leads to higher performance gains-either is sub-optimal.


[^0]:    *The first three authors contributed equally.
    ${ }^{1}$ Our code and data are publicly available at http://reasonwithpal.com/.

[^1]:    ${ }^{2}$ Lewkowycz et al. (2022) and Wang et al. (2022c) showed that an even higher accuracy can be achieved by generating up to 256 solutions, and selecting the answer according to majority voting.

