Decimal Point: Designing and Developing an Educational Game to Teach Decimals to Middle School Students

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Abstract

Research shows that U.S. students are falling behind in mathematics. Concurrently, computer and video games transfix students. Unfortunately, little academic content is typically presented in such games. Researchers have seized the opportunity to develop *educational games*, a promising new area of research exploring learning and motivational benefits. We are designing, developing, and evaluating an educational game that explores how erroneous examples can help middle school students learn decimals. Erroneous examples are an instructional technique for learning mathematics that show step-by-step examples of how to solve problems in which one or more of the steps are incorrect. These types of examples have recently been investigated within educational technology as a means for instructing students on mathematics, with promising learning results. The puzzle-like challenge of erroneous examples – finding and correcting errors – meshes nicely with the concept of game playing. Our game, “Decimal Point, the Fantastically Fabulous World of Fractional Fun,” is based on an amusement park metaphor. Players travel to different areas in the park and play a variety of short web-based computer games to learn about decimals. Players watch different fantasy characters play the games and make common errors in erroneous example mode. The students are prompted to find the errors, and also to self explain and fix them. In direct play mode, a student plays the game him or herself. Prompted self-explanation questions support the student’s conceptual understanding of decimals. This paper presents a detailed view of our game design process, which follows from the nascent literature on developing educational games. Our goal is to present a consolidated view of our process so that we – and others – can leverage and build on it in the future.

Keywords: game design, educational game, erroneous examples, decimals

1 Introduction

U.S. students are falling behind their peers around the world in mathematics, as shown on a variety of international studies (e.g., Mullis, Martin, & Foy, 2008; Fleischman, Hopstock, Pelczar, & Shelley, 2010). At the same time, kids are clearly transfixed by computer and video games. Unfortunately, little academic content is typically presented in such games; most games are created without a deep knowledge of the Learning Sciences and educational objectives. Educational technologists have seized the opportunity to develop educational games, a promising new area of research but one that, so far, lacks much scientific evidence that kids learn academic content while playing, despite clear evidence of motivational benefits.

We are designing, developing, iteratively testing, and evaluating an educational game, based on an amusement park metaphor, that explores how erroneous examples can help middle school students learn decimals. Students play “mini-games” within different theme areas of the amusement park that target common decimal misconceptions, using erroneous examples as a mechanism to engage the learner. Erroneous examples are a novel instructional technique for learning mathematics that show step-by-step examples of how to solve problems in which one or more of the steps are incorrect. Such examples are seldom used in classrooms, and have only recently been empirically investigated within educational technology as a means for instructing students on mathematics, with promising learning results. The puzzle-like challenge of erroneous examples – finding and correcting errors – is very game-like and meshes nicely with game playing. Diagnosing and fixing problems in fantasy worlds is not uncommon in games; in fact, this type of challenge is precisely one of the characteristics that entices kids to games.

With this work, we explore the hypothesis that an educational game can motivate students and help them learn more if it is designed with a stakeholder-centered, iterative interaction design process. We build on the literature in game and educational game design (Gee, 2003; Aleven et al, 2010; Lomas et al, 2013); frameworks for the educational design community (Aleven et al, 2010; Echeverria et al, 2011); and work that explains the game design process in more detail (Schell, 2008). Our process, which combines user-centered design methods for gathering data from stakeholders (e.g., students, teachers), visual and interaction design methods, and play-testing evaluation methods, is detailed below.

2 Erroneous Examples and Decimals

Theoreticians in mathematics education have proposed that studying errors can help students learn (cf. Borasi, 1996). Borasi argues that a student's understanding of mathematics could be improved by promoting critical thinking about errors and motivating reflection and inquiry. According to Borasi, directly confronting students with errors and encouraging them to reflect can lead to the eradication of the errors, similar to what has been shown in the learning research on misconceptions (Bransford, Brown, Cocking, 1999). On the other hand, other theory (and some empirical work) supports the notion that allowing errors to occur, or exposing learners to errors, hurts learning, presumably because errors would then be more likely to occur during later answer retrieval or problem solving (Skinner, 1938).

Thus, the question of if and how erroneous examples are beneficial to learning is still very much open and controversial. Despite this, there have been relatively few controlled, empirical studies that have studied the effects of erroneous examples on learning. We are aware of only a few studies that have investigated the benefits or detriments of learning with erroneous examples (Grosse & Renkl, 2007; Durkin & Rittle-Johnson, 2012; McLaren et al, 2012; Siegler, 2002; Tsovaltzi et al, 2010). The results have been mixed, with some studies indicating that only students with higher prior knowledge benefit from erroneous examples (Gross & Renkl, 2007; Tsovaltzi, Melis, & McLaren, 2012) while other studies have shown only a delayed learning benefit to erroneous examples (McLaren et al, 2012).

Decimals are a surprisingly difficult topic for students to master (Glasgow et al, 2000; Rittle-Johnson, Siegler, & Alibali, 2001), and, in fact, student difficulties with decimals often persist into adulthood (Stacey et al., 2001). One reason for these difficulties is that students have common and persistent misconceptions (e.g., Glasgow et al., 2000; Irwin, 2001). For instance, students often treat decimals as if they are whole numbers (e.g. they think 0.25 is greater than 0.8, since 25 is greater than 8). These persistent decimal misconceptions must be overcome so that students can master more advanced mathematics.

From a learning science perspective, our project will address the following key research question: Can erroneous examples increase learning of decimals and add motivation to an educational game?

3 Overview of the Game

Our game, "Decimal Point, the Fantastically Fabulous World of Fractional Fun," is based on an amusement park metaphor. Players travel to different theme areas in the park (e.g., Old-time Amusement Park; Haunted House; Wild West; Space Adventure) and play a variety of mini-games targeted at learning decimals and relevant to that area's theme. Players see a map of the amusement park (Figure 1). In our initial version and studies with the game, players will be led through all theme areas in sequence and will be prompted to play all of the mini-games. Ultimately, our goal is to provide more user control, allowing students to choose a path through the game and the specific mini-games they will play. The amusement park map also allows the student (and teacher) to monitor progress throughout the game.

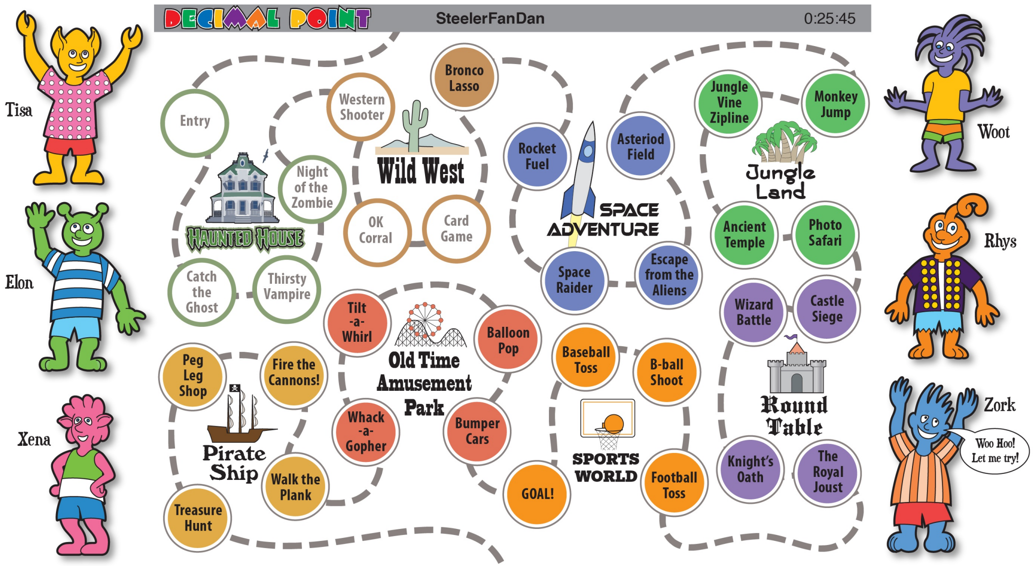

Figure 1. Overview of the Decimal Point Amusement Park game.

Players engage with mini-games focused on conceptual understanding (specific decimal misconceptions that include prompted self-explanation). In erroneous examples play mode (see Figure 2), players watch a fantasy character (all of the game's fantasy characters are shown on the left and right side of Figure 1) play the game and make common errors. Students then find and fix the errors and respond to prompted self-explanation questions about their answers. In direct play mode players solve decimal problems by playing the game themselves and respond to prompted self-explanation questions about their answers.

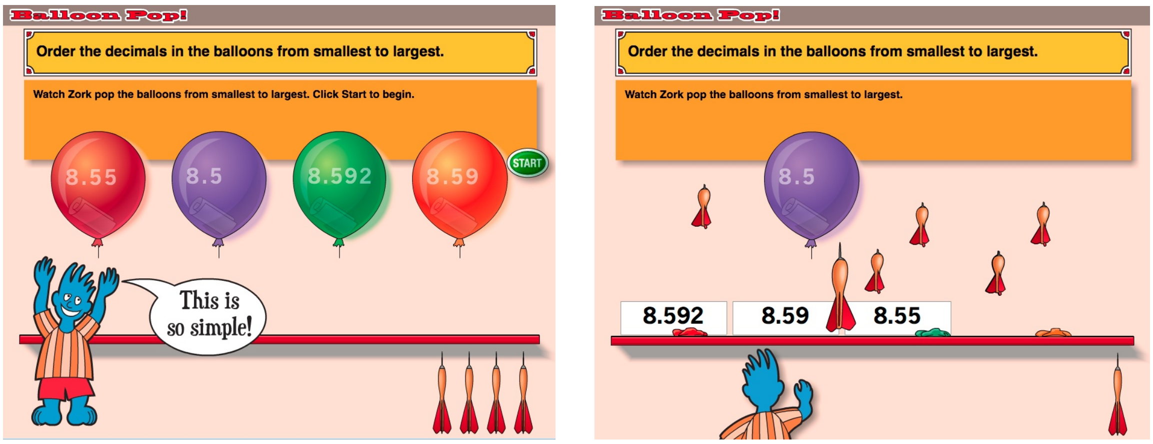

Figure 2. Balloon Pop, erroneous example play mode with fantasy character Zork playing

Each mini-game is built as a self-contained intelligent tutor within the Cognitive Tutor Authoring Tools (CTAT) (Aleven et al, 2009). The user interface of each mini-game is implemented in Adobe Flash and CTAT's Flash widgets. CTAT provides a set of widgets for constructing intelligent tutors, along with back-end services to, for instance, deploy tutors on the web and log student interactions. So-called "behavior graphs" model how a student might respond to steps of the decimal problems so that CTAT can track a student's progress and provide appropriate feedback: correct/incorrect, move on to the next problem step, etc. Behavior graphs also contain the content of the decimal problems. Different behavior graphs model different decimal problems and different misconceptions. For the purpose of this project, we have developed a common set of CTAT components – like self-explanation questions – that are shared across all of the mini-games. Other common components for the mini-games include a component for feedback messages and for the appearance and behavior of the fantasy characters that appear frequently in the mini-games (see Figure 2). This approach supports common stylistic elements across all of the mini-games.

4 Development Team
Our team – the authors of this paper – consists of one learning scientist, one interaction designer, one programmer, one illustrator, and three undergraduate student programmers from computer and learning science who work part time to help develop the game. We have placed a great deal of emphasis on documentation of process, code, playtests and interim prototypes, so that new members of the project team can easily come up to speed.

5 Early Design and Concept Development
To begin our game design process, we conducted a competitive analysis of games for education and fun designed for children aged 11-13 (6^{th} through 8^{th} grade). We gathered examples from over 100 educational and entertainment games. We analyzed the games to reveal the following design patterns. First, adaptivity is a common theme in games. Players are typically offered different game experiences, especially different levels of difficulty, to optimize challenge for everyone, and/or to adjust educational problems to a player's level of skill or knowledge. For example, in the Donkey Kong game different levels, with increasing difficulty, include jumping over obstacles; destroying objects with a hammer; and collecting items such as hats, parasols, and purses. Second, games often offer unsolicited, but optional, help. For instance, the game Egg Baby offers players help, but players can turn off notifications if they desire. Third, games often offer on-demand help when a player asks for it. For example, the game Zelda provides extensive help resources that players can search within or outside of the context of the game. Fourth, games provide tutorials in the form of worked examples or other forms of instruction about what to do in the game. Fifth, feedback in many forms is quite common. For example, feedback might immediately reveal whether an action was successful or not, as in the Pocket Law Firm game, and this feedback can be folded into the structure of the game or given in a more didactic fashion, for example, as green (for correct) or red (for incorrect) text on the screen. These game patterns served as guidelines in our design and conception of the Decimal Point game.

From (Schell, 2008), we knew that our game needed to have a goal, and had to provide challenge while, at the same time, delivering educational content. Based on ideas about entertainment and education goals, motivation, and overall game experience, we designed three initial game concepts to vet with students and teachers:

- The first was an "Obstacle Course" game, where players try to complete an obstacle course in the shortest time possible. They cross rivers, jump ledges, and squeeze through tiny spaces.
- The second was a "Plan a Party" game, where players are running out of time in preparation for a friend's birthday party. Once they have accomplished all of the tasks, they win the game.
- The third was an "Explore a City" game, where players move from place to place in a city and explore, investigate, and do urban things such as baking a cake, fixing a tire, and creating a painting. After finishing all the events, they have deep knowledge of the city and win the game.

6 Playtesting Co-Design Session
In early phases of user-centered design, it is often helpful to involve stakeholders directly in the ideation and design process. This co-design process has been adopted for educational and purely fun games, following from Gee's suggestion that students can be producers as well as consumers of games (Gee, 2003).

We conducted a co-design session of our initial concepts with thirty-two 7^{th} grade children in two Pittsburgh area schools. Students spent a total of 42 minutes, in two 21-minute sessions, providing input to our game designs. We conducted a warm-up activity, which consisted of freeform discussions about what games students actually play (see Table 1). We next talked about what makes a game fun, prompting the discussion with two examples, Angry Birds (a game for pure fun) and Sushi Monster (a game that teaches basic math). Finally, students provided feedback on the three game concepts listed above (see Table 2), closing with open discussion about what they believe makes an ideal game.

Mine craft	**Angry Birds**	**Temple Run**	**Bubble Trouble**	**Candy Crush**	**Web kinz**	**Happy Wheels**	**Truck Loader**	**Cube field**	**Black Ops**
12	7	7	6	3	3	3	2	2	2

Table 1: Games mentioned (and number of times each) in answer to "What are your top three favorite or online games?"

	All Students (N=32)		
Game	**I like the objective of this game**	**I would like to Play**	**Others would like to Play**
Obstacle Course	2.3 (N=31)	2.0 (N=31)	2.3 (N=32)
Plan a Party	3.7 (N=32)	3.7 (N=31)	4.1 (N=32)
Explore a City	2.7 (N=31)	2.6 (N=31)	2.9 (N=32)

Table 2: Average student responses to three questions about the three game concepts. Ratings run from 1 (very much) to 7 (not at all), i.e., lower scores indicate more positive responses. (Note: In some cases, students skipped questions, hence N=31 for some responses)

From these sessions, the key information we learned was:

- Students mentioned a total of 54 different games as being one of their "top three favorite online games," with the most common favorites being Mine Craft (12), Angry Birds (7), and Temple Run (7) (See Table 1).
- Students liked the "Obstacle Course" game best, but clearly preferred "Explore a City" over the "Plan a Party" game (See Table 2).
- The students particularly enjoyed action, colors, and graphics, particularly to keep games interesting. This was revealed both in the favorite games cited by the students and in the open discussion at the end.
- Finally, students seemed to enjoy games that rely on familiar metaphors, places, and events. This was primarily revealed in the open discussion session.

7 Deciding on the Game Concept and the Iterative Game Design and Development Process

Based on the findings of the playtesting co-design session, we settled on an amusement park game for a variety of reasons. First, since students seemed drawn to common metaphors and places, the idea of an amusement park game was natural. Clearly, virtually all students of middle school age are familiar with amusement parks and the rides, attractions, and arcade games that are typical of such an experience. Second, the amusement park idea seemed to have parallels to the two favorite game concepts of the students, "Obstacle Course" and "Explore a City", in that the discrete and different mini-games of an amusement park parallel, in concept, the discrete and different stations of the two favored games (e.g., obstacles in an obstacle course are like the rides and attractions in an amusement park). Third, there are many gender-neutral activities in an amusement park, so our game would have the potential to be equally interesting to girls and boys. Finally, the amusement park metaphor also addresses student interest in action (e.g., popping balloons, shooting at western-themed objects, whacking gophers) and color (all of the amusement park mini-games are very colorful, see Figure 2 for instance).

Once this choice was made, our iterative game design and development process unfolded over about a year's time, involving the following steps. First, we created a grid of theme areas and mini-games that fit the amusement park metaphor and that could support the learning of decimals and presentation of erroneous examples. Next, we brainstormed the specific mini-game scenarios to support these problems and discussed the interaction for each mini-game. Led by our illustrator, we collectively brainstormed, designed, and sketched the interaction for and visualization of each mini-game. As mini-games were built in Flash and CTAT, we reviewed the mini-games during weekly meetings, discussing whether each mini-game seemed fun and engaging, that interaction worked as designed, that learning of the decimal content seemed feasible, that games were consistent in behavior, and that no bugs existed.

7.1 Mapping of Erroneous Example (EE) Types to Game Problems

The erroneous examples used in our mini-games are modeled from existing erroneous example problem types used in the classroom (McLaren et al., 2012). These problems focus on demonstrating errors around four of the most common misconceptions prior research has shown students have in decimal learning (Isotani et al. 2011):

- Megz (“longer decimals are larger”, e.g. 0.23 > 0.7),
- Segz (“shorter decimals are larger”, e.g. 0.3 > 0.57),
- Negz (“decimals less than 1.0 are less than zero”, e.g. 0.5 < 0),
- Pegz (“the numbers on either side of a decimal are separate and independent numbers”, e.g. 11.9 + 2.3 = 13.12).

These misconceptions lend themselves toward certain types of math activity to demonstrate the misconception (see Table 3).

An erroneous example would then visually display one of the fantasy characters exhibiting one of the misconceptions by making an error in playing that game. Erroneous examples are carefully worded to tell the student that he or she should watch the character and then identify and correct the error. Upon seeing the error, the student is first asked to explain it. Then the student is asked to correct the error, interactively within the visual of the demonstrated error. After correcting the error, the student is asked a second question about what advice he or she would give a person making the error to avoid the error in the future. It is this general game flow that we incorporated into all of our mini-games.

Game & Problem Type	**Misconceptions Covered**	**Decimal Math Activity**	**Example Game**
Addition – Adding two decimals together	Pegz	Solve a decimal addition problem.	Thirsty Vampire (Haunted House) – determine if vampire can drink enough blood to survive by adding quantities from his victims.
Bucket – Putting values in less than and greater than “buckets”	Megz, Segz, Negz	Group set of decimal numbers into two “buckets” based on whether each number is greater or less than a given number.	Basketball Shoot (Sports World) – shoot basketballs with decimal numbers into one of two baskets, based on whether they are greater or less than a given number.
Number Line – Placing a number correctly on a number line	Megz, Negz, Pegz, Segz	Place a point on a number line for a given decimal number.	Lasso the Bronco (Wild West) – select the point at which to lasso the bucking bronco.
Ordering Values – Taking decimal numbers and putting them in order	Megz, Negz, Pegz, Segz	Sort a set of decimal numbers in increasing or decreasing order.	Balloon Pop (Old Time Amusement Park) – pop the balloons in increasing order of the numbers on the balloons.
Sequence – Adding decimal numbers to a starting sequence of three numbers	Pegz	Complete a sequence of decimal numbers that is created via addition.	Alien Escape (Space Adventure) – complete the sequence of code numbers that open the sliding doors to get to the escape shuttle.

Table 3: Mapping of common decimal misconceptions to math activities that demonstrate the misconceptions, along with game examples that incorporate the activity.

7.2 Create Scenarios and Write Content For Each Problem

As part of planning and designing Decimal Point, we synthesized lists of theme areas and rides/attractions/games at many popular real-life amusement parks and used these to envision scenarios of gameplay around the decimal math activities. Scenarios and content were brainstormed iteratively with all members of the design and development team, as described above.

7.3 Sketch, Create Visual and Interaction Design

We then created visual and interaction design for both regular and erroneous example versions of each game. We worked to ensure several things: first, that problems were the same level of complexity, with roughly the same number of steps and density of instructions; that games were consistent in the use of EE characters and feedback, and that games did not give solutions away by demonstrating the value or size of numbers visually. For example, the Enter if You Dare game features a haunted house that a player can enter if he or she places a given decimal value onto a number line shown in the game (e.g. “You must be this tall to ride”, with a vertical number line shown). The number is displayed on a broken-down fence in front of the haunted house near the entry gate. In this case, we were careful not to make any visual correlation between the location of the sign with the number on the fence (indicating a spatial-decimal relationship) and the vertical number line displayed beside the gate.

The general flow of the EE games has been designed as follows:

- A student is prompted to click a “Start” button to begin the game. A focused interaction by the student to initiate the game was needed because the next step is that the error is demonstrated to the student.
- A mini-game specific fantasy character appears who demonstrates the error by playing part of the game.
- Two other game characters appear expressing dismay that the error occurred.
- The student is prompted with a multiple-choice question (MCQ) to explain why the character made the error. The question is designed to have the student recognize the essence of the misconception displayed. The questions are designed such that the student gets feedback as to whether or not their answer is correct or incorrect and must continue to answer after and incorrect response until they correctly answer the question before continuing.
- The student is prompted to correct the error by completing the decimal game element of the mini-game (for example: getting the decimal numbers in order). They get feedback on each attempt to complete the game, and cannot progress until they correctly solve the decimal game.
- An animated ensemble of game characters appears and cheers for the student’s success in solving the decimal game.
- The student is prompted with a MCQ to explain to the original character how to avoid the error in the future. As with the previous MCQ, it is designed to get at the essence of the misconception, the student gets correct/incorrect feedback and continues to answer until a correct answer is entered.

8 Prototype Play Testing

We are currently preparing to playtest a subset of our developed games with middle school students. We created our playtest by extending techniques discussed in (Eladhari, 2010 and Schell, 2008). Our goals are 1) to assess whether the game creates the experience that our design team intended; 2) to assess whether the characters and game narrative are fun for students, and will engage them over time; and 3) to ensure that there are no significant usability problems with the visual and interaction design of the games.

To conduct the playtest, we will have the students begin by recalling the last time they played a game and what they liked and disliked about it. This will be followed by 15 minutes of gameplay, accompanied by thinking aloud about what they are doing. The playtest will conclude with a free-form discussion of impressions of the game.

9 Guidelines for the Design of an Educational Game

As in the many domains of design, designers first strive to create general design guidelines for a product or service they are designing, then begin to add detail in the form of design recommendations as more cases within the domain are generated (Forlizzi et al., 2009). Here, we present what we found to be the three most critical design guidelines for the design of an educational game.

The first guideline is to *regularly and rigorously share decisions, goals, and values in the game design process* among members of the game design team. Our weekly meetings, scaffolded by the emerging shared view of the game narratives and goals, the game characters, and the amusement park metaphor, ensured that we could develop new aspects of the game that all team members could critically discuss and come to agreement about.

The second guideline is to *balance educational content and game mechanics* (after Linehan et al, 2011). Although our design team knew that it would be easy to simply “gamify” the problems that already existed from prior work, we strove to create new game narratives that posed similar decimal problems while taking into account the needs and desires of a player and his or her experience, as uncovered during our initial playtesting co-design session.

The third guideline is to *provide goals at different scale to motivate gameplay* (after Swartout and Van Lent, 2003). Currently, The Decimal Point game features goals for each mini-game and goals for the overarching metaphor of the game. Future work will explore how other goals, such as scoring and leaderboards, affect gameplay and, subsequently, learning.

10 Conclusions and Future Work

This paper has presented a case study of the ongoing design and development of Decimal Point, an educational game that teaches middle school students decimals. As we continue to further flesh out and test our educational game, we will refine our design guidelines for educational games in general, and design recommendations for decimal games in particular. We will also investigate the potential for applying our game concept to other areas of mathematics besides decimals. The notion of learning from erroneous examples has been applied to various mathematical topics – e.g., statistics (Grosse & Renkl, 2007) and fractions (Tsovaltzi, Melis, & McLaren, 2012) – so it is not hard to imagine that the game approach outlined here could also be applied to other areas of mathematics where misconceptions are common and recurring.

11 Acknowledgements

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