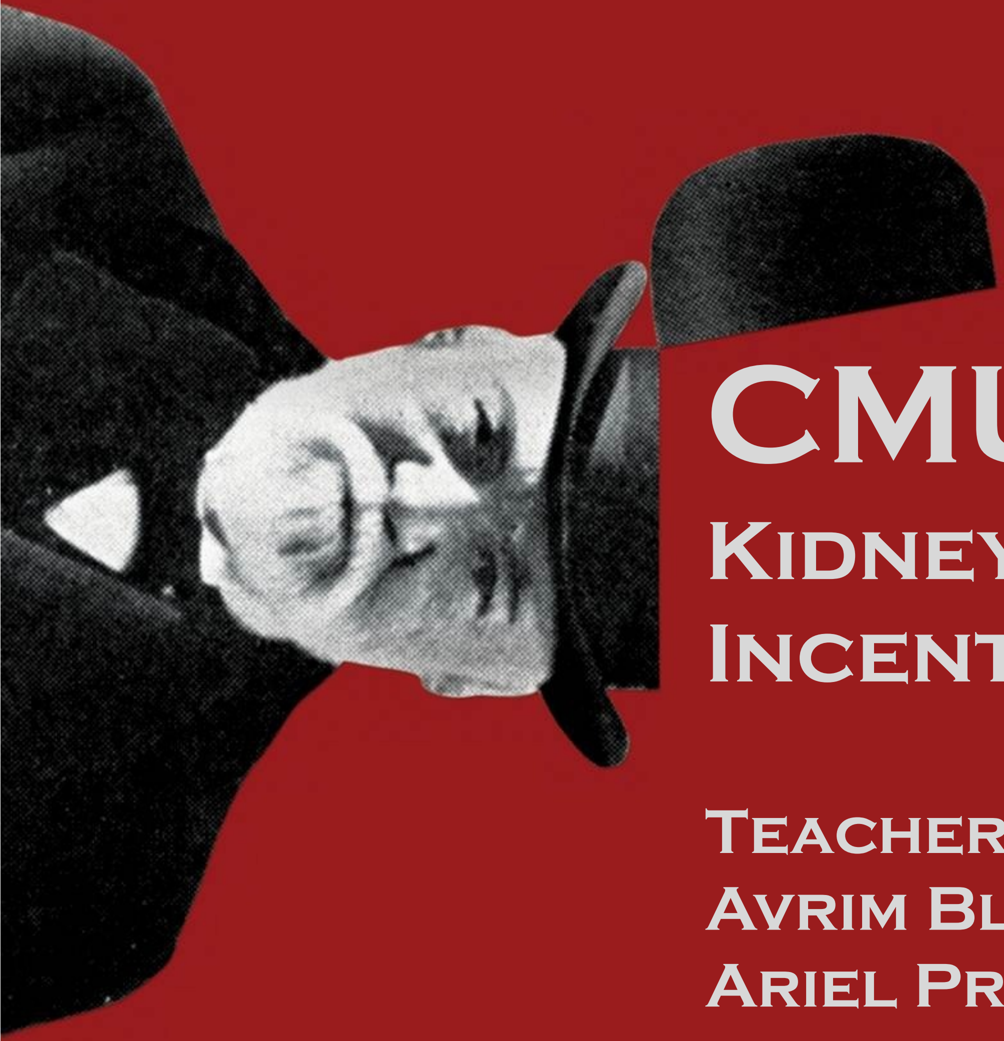

CMU 15-896

Kidney Exchange: Incentives

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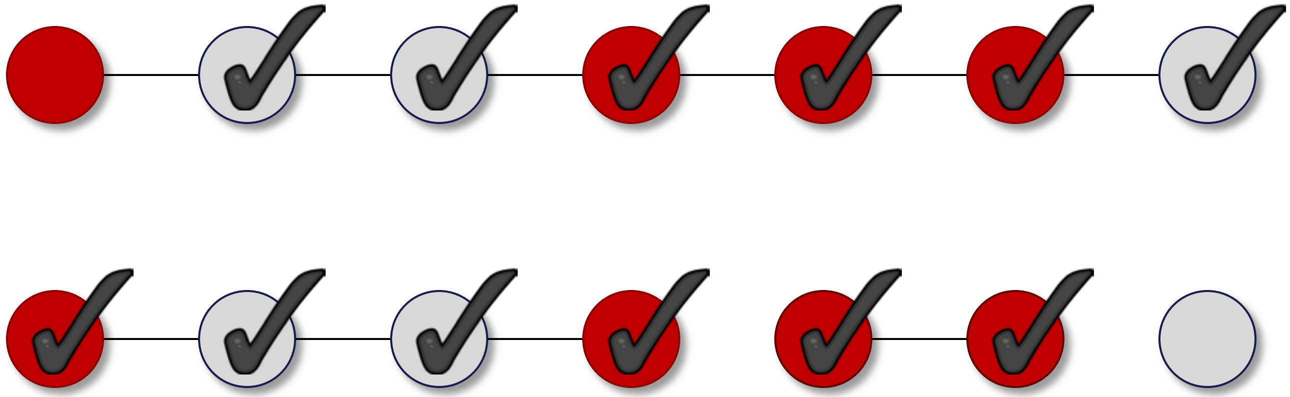
Incentives

- A few years ago kidney exchanges were carried out by individual hospitals
- Today there are nationally organized exchanges; participating hospitals have little other interaction
- It was observed that hospitals match easy-to-match pairs internally, and enroll only hard-to-match pairs into larger exchanges
- Goal: incentivize hospitals to enroll all their pairs

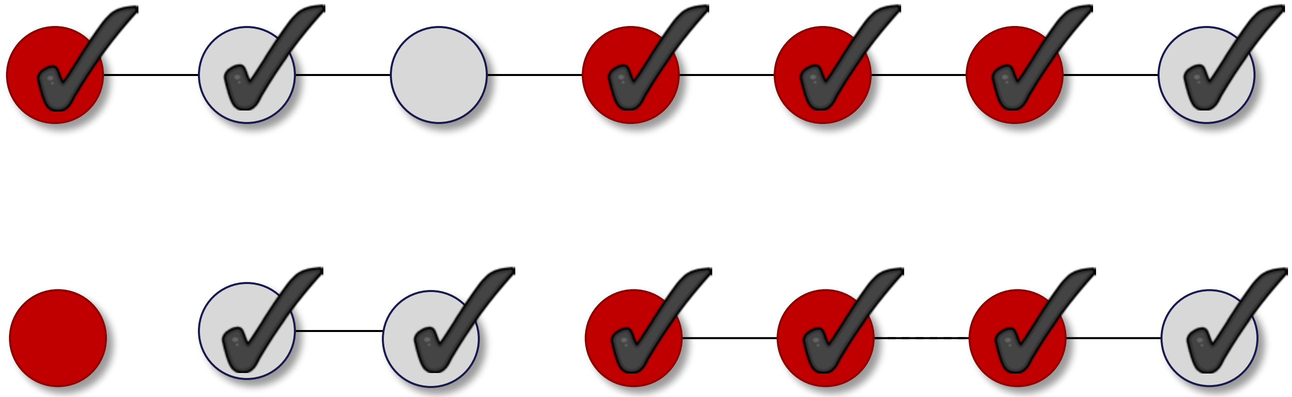
The strategic model

- Undirected graph (only pairwise matches!)
  - Vertices = donor-patient pairs
  - Edges = compatibility
  - Each player controls subset of vertices
- Mechanism receives a graph and returns a matching
- Utility of player = # its matched vertices
- Target: # matched vertices
- Strategy: subset of revealed vertices
  - But edges are public knowledge
- Mechanism is strategyproof (SP) if it is a dominant strategy to reveal all vertices

OPT is manipulable



OPT is manipulable



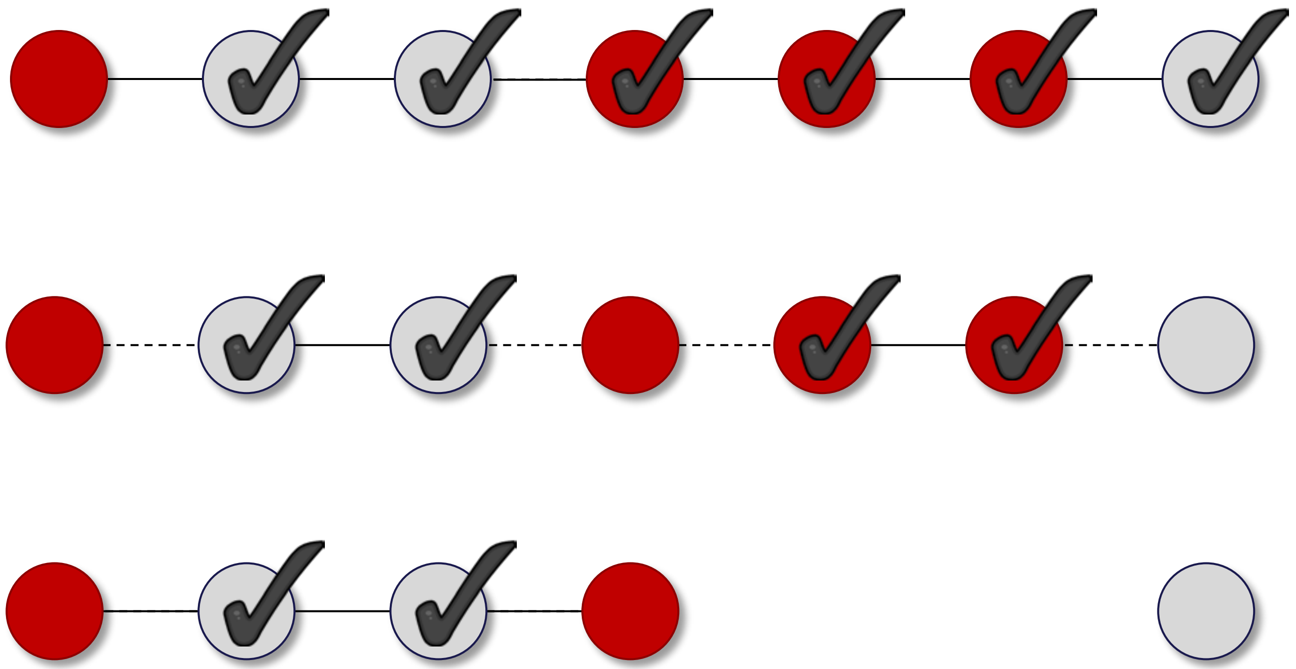
Approximating SW

- **Theorem [Ashlagi et al. 2010]:** No deterministic SP mechanism can give a $2-\epsilon$ approximation
- **Proof:** We just proved it!
- **Theorem [Ashlagi et al. 2010]:** No randomized SP mechanism can give an $8/7-\epsilon$ approximation
- **Proof:** Homework 4 q4
  - Huge bonus: improve the bound!

SP mechanism: Take 1

- Assume two players
- The $\textsc{Match}_{\{\{1\},\{2\}\}}$ mechanism:
  - Consider matchings that maximize the number of "internal edges"
  - Among these return a matching with max cardinality

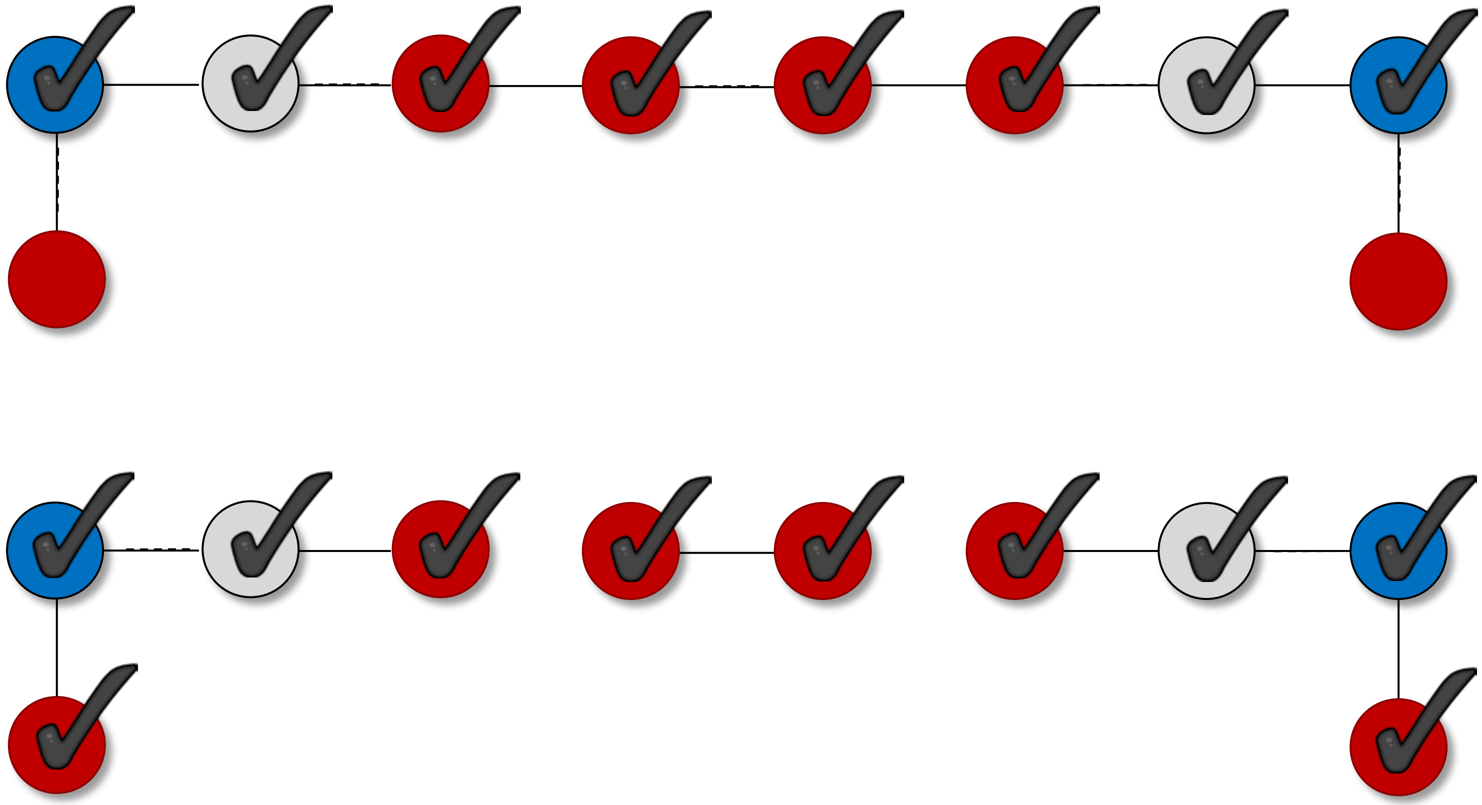
Another example



Some observations

- **Theorem (special case):** $\textsc{Match}_{\{\{1\},\{2\}\}}$ is strategyproof for two players
- We prove this on the board
- It gives a 2-approximation
  - Cannot add more edges to matching
  - For each edge in optimal matching, one of the two vertices is in mechanism's matching
- What about more than two players?

The case of 3 players



SP mechanism: Take 2

- Let $\Pi = (\Pi_1, \Pi_2)$ be a bipartition of the players
- The $\textsc{Match}_\Pi$ mechanism:
  - Consider matchings that maximize the number of "internal edges" and do not have any edges between different players on the same side of the partition
  - Among these return a matching with max cardinality (need tie breaking)

Eureka?

- **Theorem [Ashlagi et al. 2010]:** $\textsc{Match}_\Pi$ is strategyproof for any number of players and any partition Π
- For $n = 2$ $\textsc{Match}_{\{\{1\},\{2\}\}}$ guarantees a 2-approx
- Vote: approximation guarantees given by $\textsc{Match}_\Pi$ for $n = 3$ and $\Pi = \{\{1\},\{2,3\}\}$

The mechanism

- The Mix-and-Match mechanism:
  - Mix: choose a random partition Π
  - Match: Execute $\textsc{Match}_{\Pi}$
- **Theorem [Ashlagi et al. 2010]:** Mix-and-Match is strategyproof and guarantees a 2-approximation
- We prove the theorem on the board