

# Predicate-based Test Coverage and Generation

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Analysis of Software Artifacts

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These slides prepared by Thomas Ball, with additional material from M. Young, A. Memon and MSR's FSE group. Used by permission.

# MSIL Unit Test Tool

—————→ *a hybrid helper*

- Goal
  - capture developer knowledge **ASAP**
  - via a **strong** set of unit tests
  - to form a **specification** of the code's behavior
- How
  - generate tests based on analysis of MSIL
  - symbolic execution + constraint satisfaction
  - runtime analysis to check complicated invariants
- Facets
  - complements specification-based test generation
  - positive feedback cycle with programmer

What criteria should  
guide unit test generation?

# Predicate-complete Testing

- Predicates
  - relational expression such as  $(x < 0)$
  - the expression  $(x < 0) \parallel (y > 0)$  has two predicates
  - predicates come from program and safe runtime semantics
- Consider a program with  $m$  statements and  $n$  predicates
  - predicates partition input domain
  - $m \times 2^n$  possible *observable* states  $S$
- Goal of Predicate-complete Testing:
  - cover all *reachable* observable states  $R \subseteq S$

# PCT Coverage

L2: if (A || B) S else T

L3: if (C || D) U else V

- PCT requires covering all logical combinations over {A,B,C,D} at
  - L2 and L3
  - S, T, U and V
- Some combinations may not be reachable

# PCT Coverage Subsumes Statement, Edge Coverage

- Statement coverage
  - Must cover all statements with all predicates
- Edge coverage
  - Must cover if statements with all predicates
  - Therefore touch both branches
- Path coverage
  - Must cover every path through program
  - Infinite number of them!

# PCT Coverage does not imply Path Coverage

```
L1:  if (x<0)      (L1, x<0)  (L1, !(x<0))
L2:    skip;      (L2, x<0)
      else
L3:    x = -2;    (L3, !(x<0))
L4:    x = x + 1; (L4, x<0)
L5:    if (x<0)  (L5, x<0)  (L5, !(x<0))
L6:      A;      (L6, x<0)
```

# PCT Coverage does not imply Path Coverage

```
L1:  if (x<0)
L2:    skip;
     else
L3:    x = -2;
L4:    x = x + 1;
L5:    if (x<0)
L6:      A;
```

Handwritten annotations in parentheses:

- $(L1, x < 0)$  (blue)
- $(L1, \neg(x < 0))$  (red)
- $(L2, x < 0)$  (blue)
- $(L3, \neg(x < 0))$  (red)
- $(L4, x < 0)$  (purple)
- $(L5, x < 0)$  (red)
- $(L6, x < 0)$  (red)
- $(L5, \neg(x < 0))$  (blue)

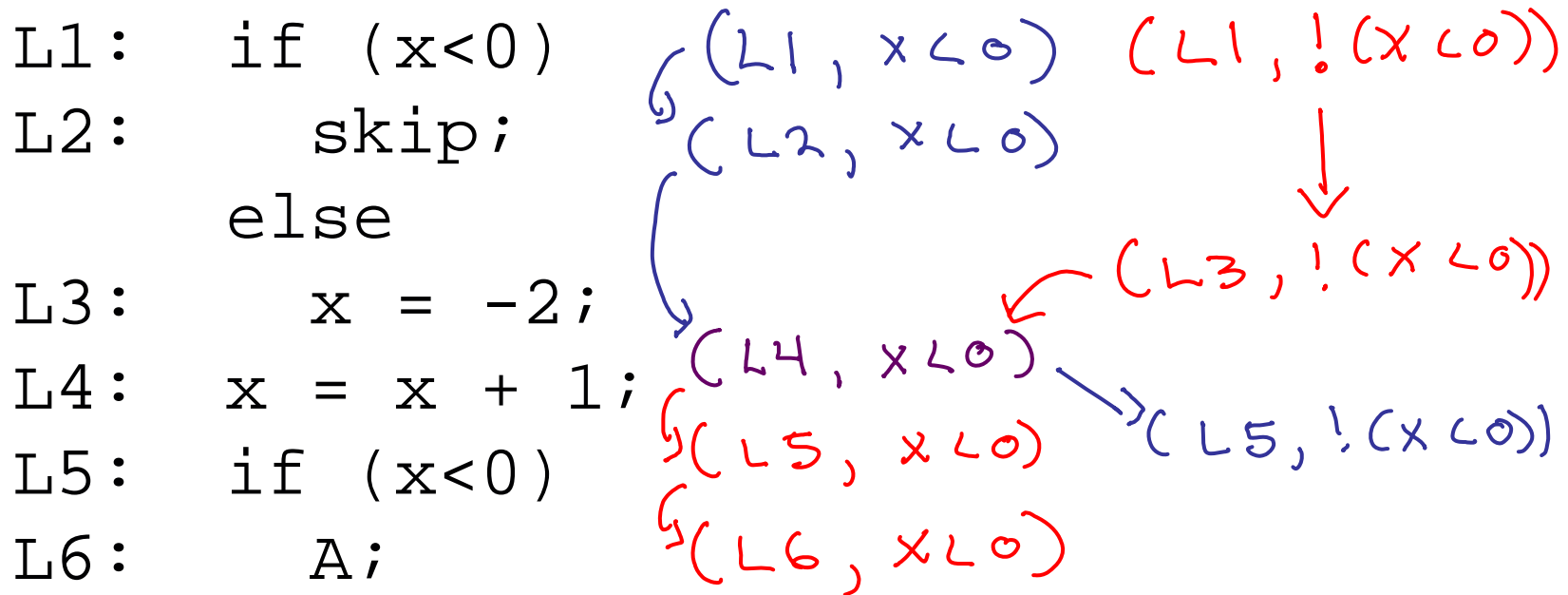
Arrows indicate flow from L1 to L2 and L3, and from L4 to L5.

Three tests

$x \rightarrow -1$



# PCT Coverage does not imply Path Coverage



Three tests

x → -1

x → 2

# PCT Coverage does not imply Path Coverage

L1: if (x<0)

L2: skip;

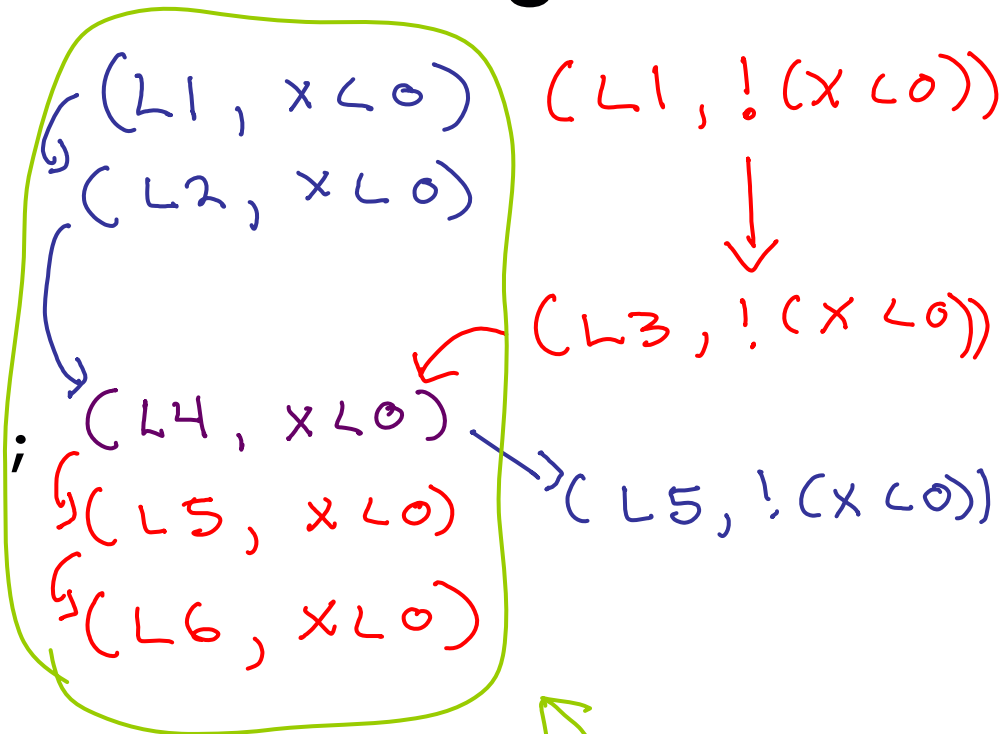
else

L3: x = -2;

L4: x = x + 1;

L5: if (x<0)

L6: A;



Three tests

x → -1

x → 2

x → -3

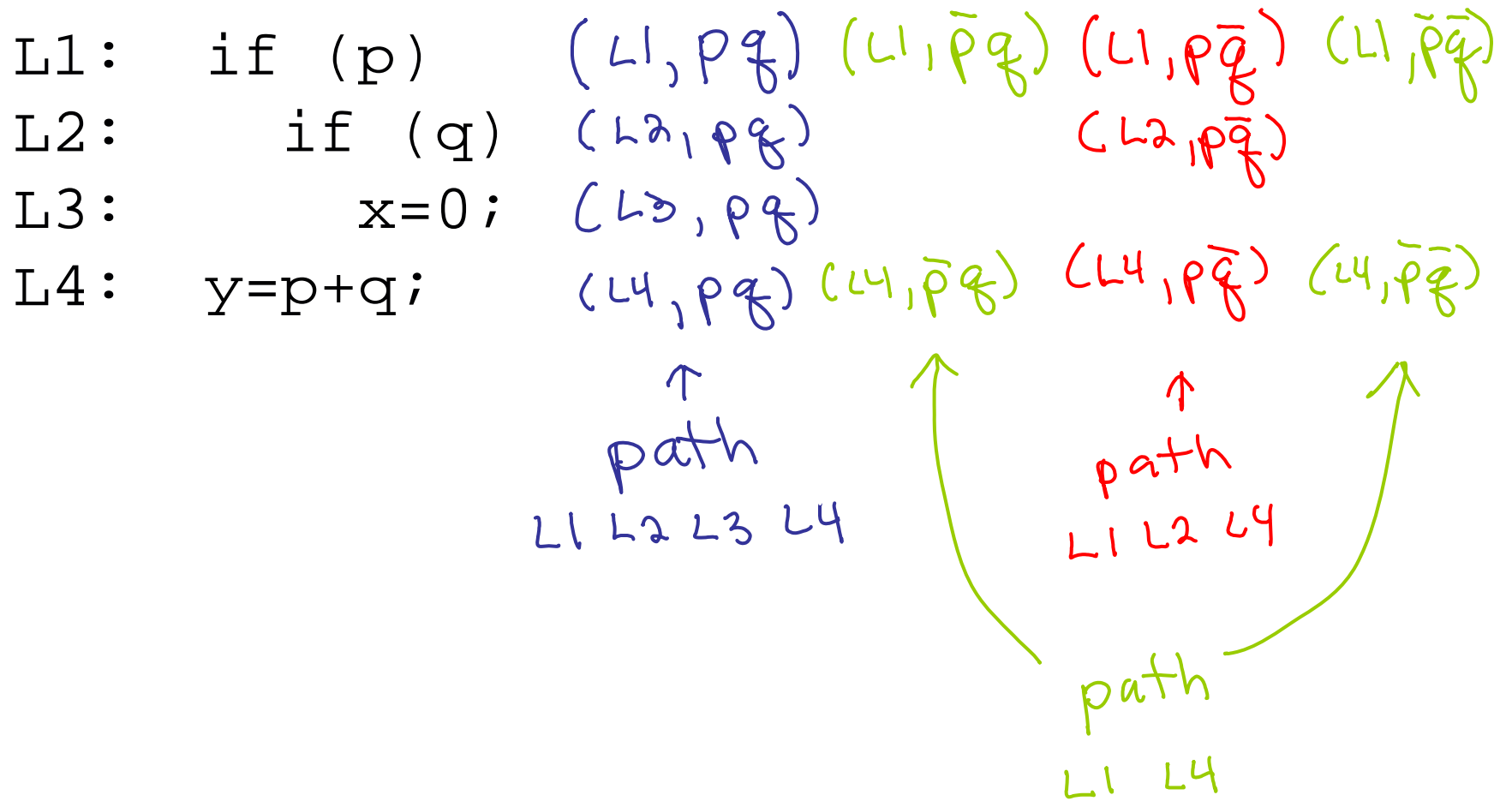
# Path Coverage does not imply PCT Coverage

L1:	if (p)	(L1, pq)	(L1, $\bar{p}q$ )	(L1, $p\bar{q}$ )	(L1, $\bar{p}\bar{q}$ )
L2:	if (q)	(L2, pq)		(L2, $p\bar{q}$ )	
L3:	x=0;	(L3, pq)			
L4:	y=p+q;	(L4, pq)	(L4, $\bar{p}q$ )	(L4, $p\bar{q}$ )	(L4, $\bar{p}\bar{q}$ )

		↑		↑	
		path		path	
		L1 L2 L3 L4		L1 L2 L4	

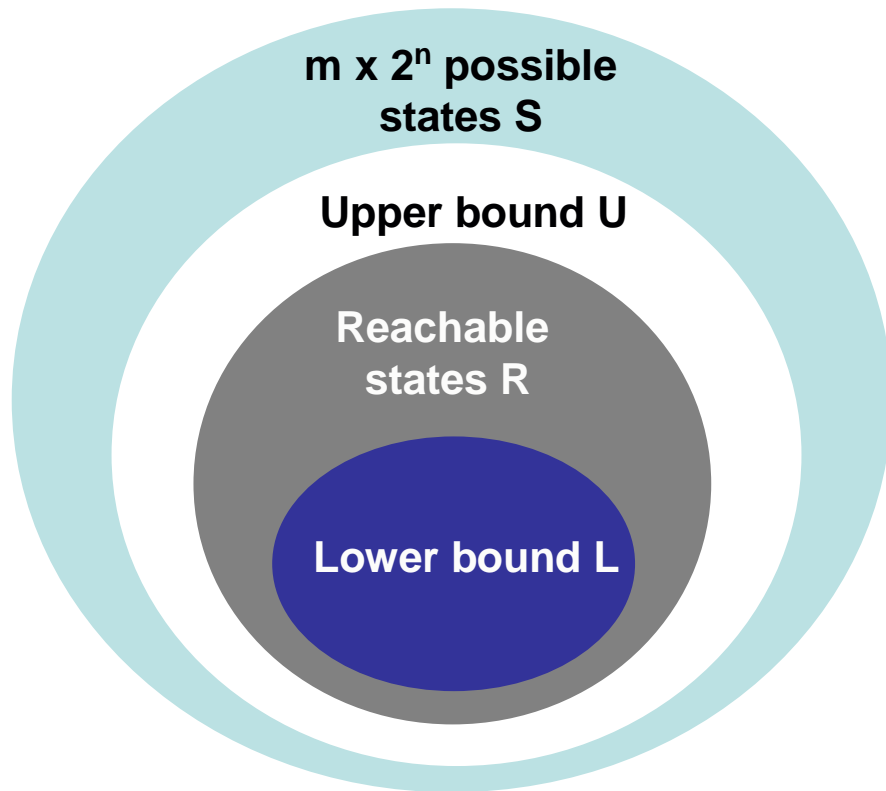
# Path Coverage does not imply PCT Coverage



# Denominator Problem

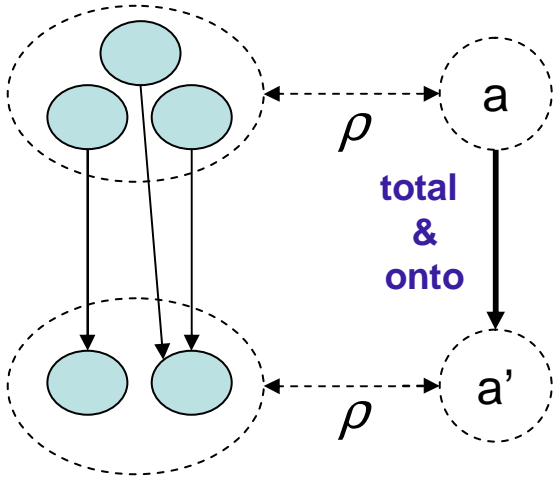
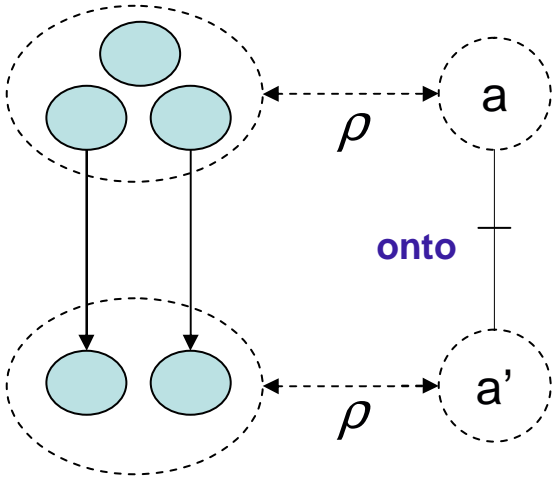
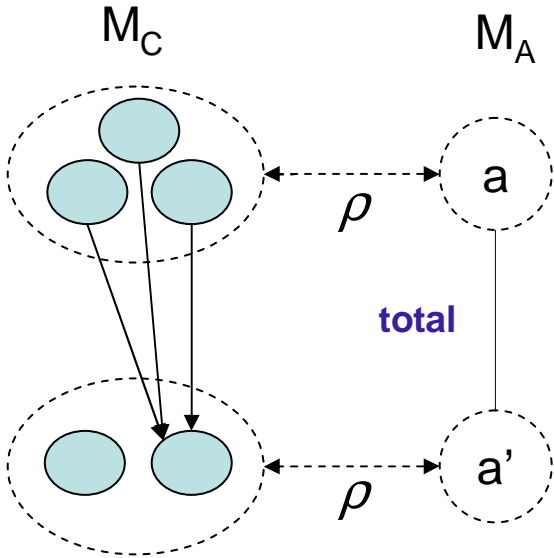
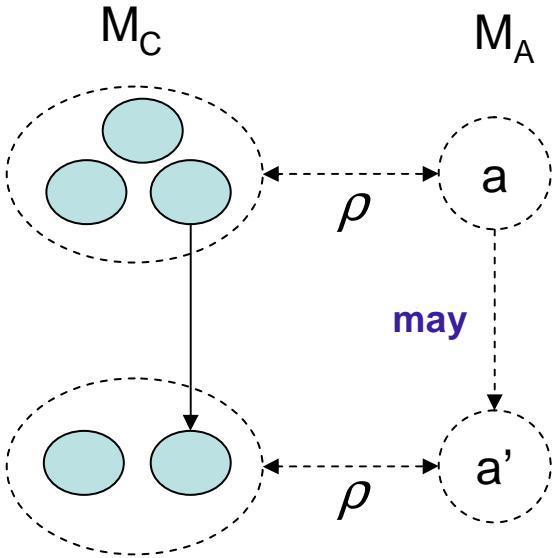
- Coverage metrics require a denominator
  - e.g. statements executed / total statements
- Easy to define for observable states
  - executed observable states / ( $m \times 2^n$ )
- But ( $m \times 2^n$ ) is not a very good denominator!
  - most observable states will not be reachable
  - $R \lll S$

# Upper and Lower Bounds

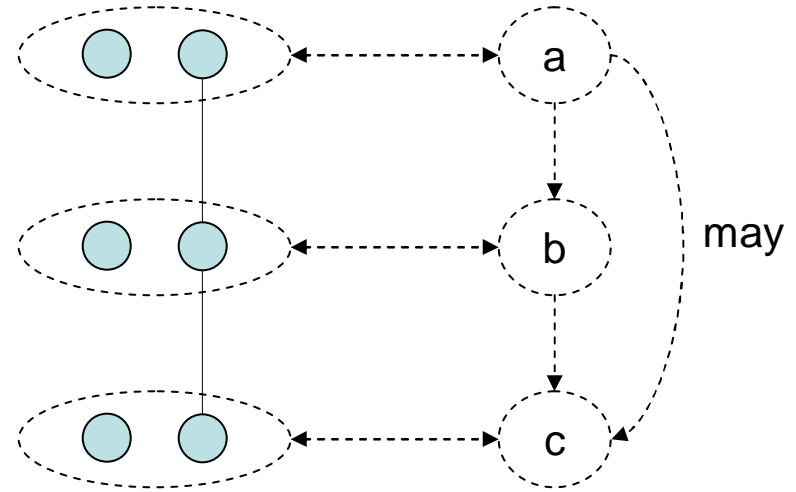
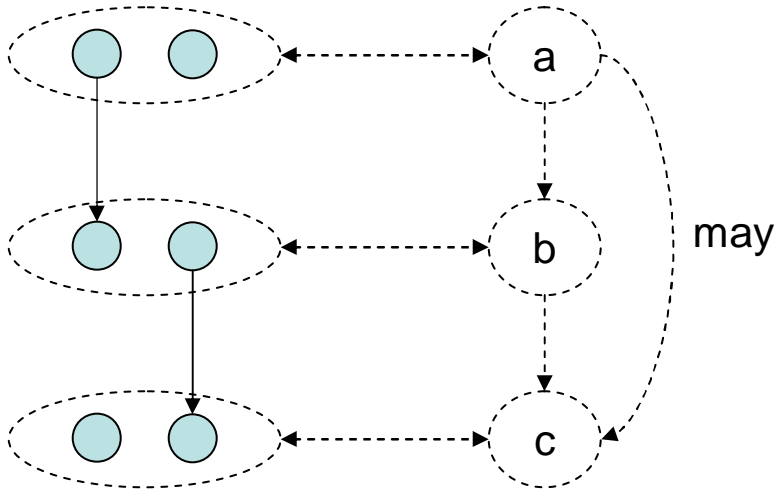


- Bound reachable observable states
  - modal transition systems and predicate abstraction
  - $|L| / |U|$  defines “goodness” of abstraction
- Test generation using lower bound L
- Refinement to increase  $|L| / |U|$  ratio

# Abstraction Construction

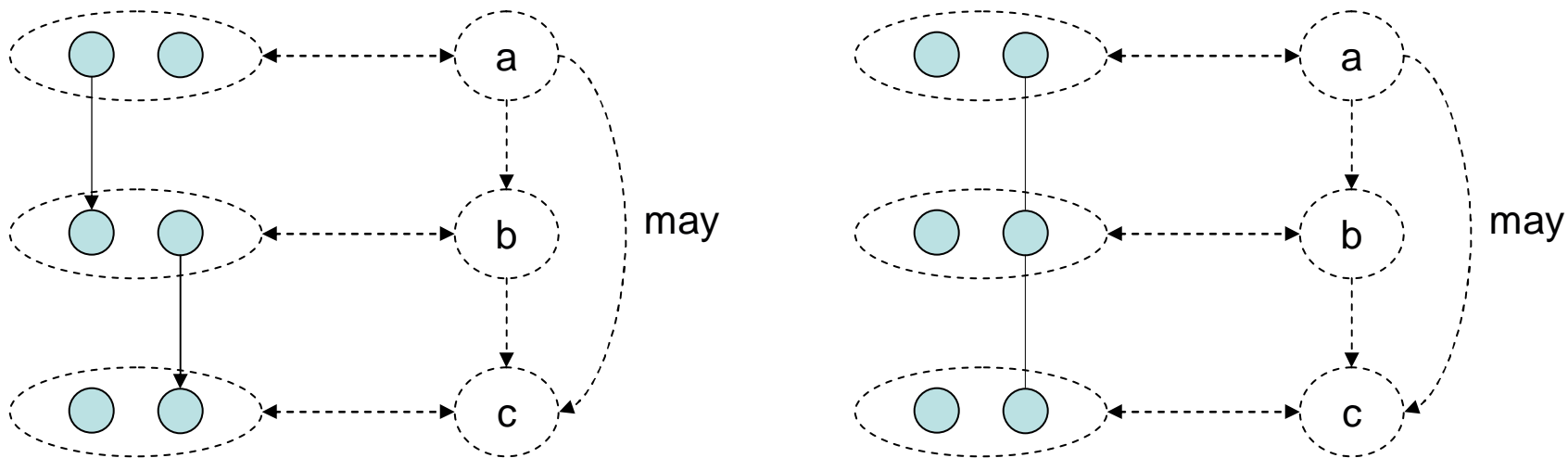


# Upper Bound: May-Reachability



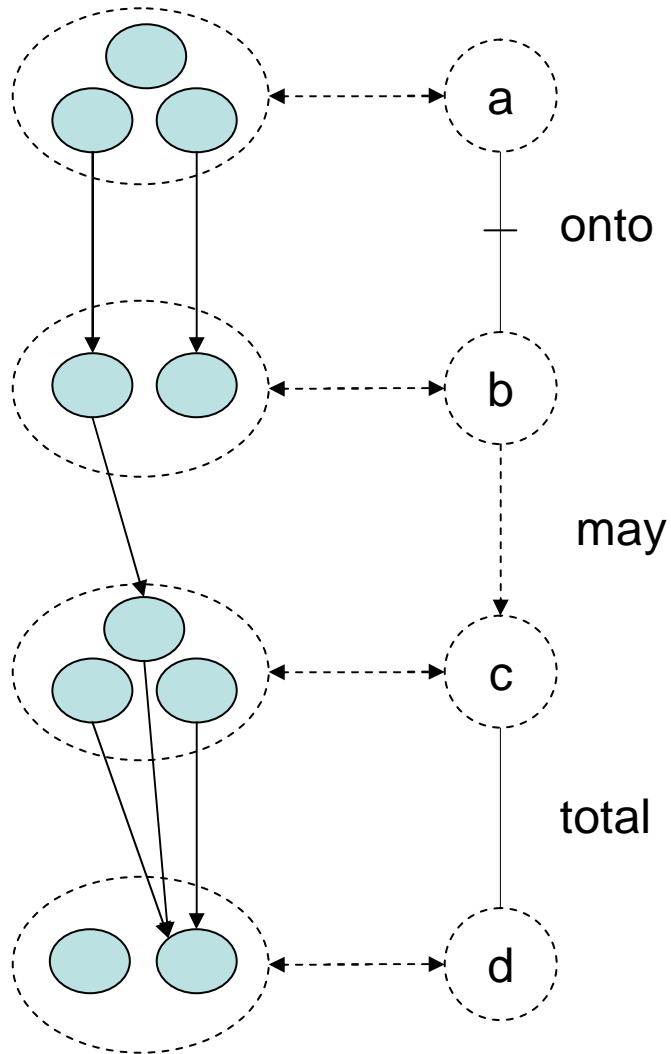


# Upper Bound: May-Reachability

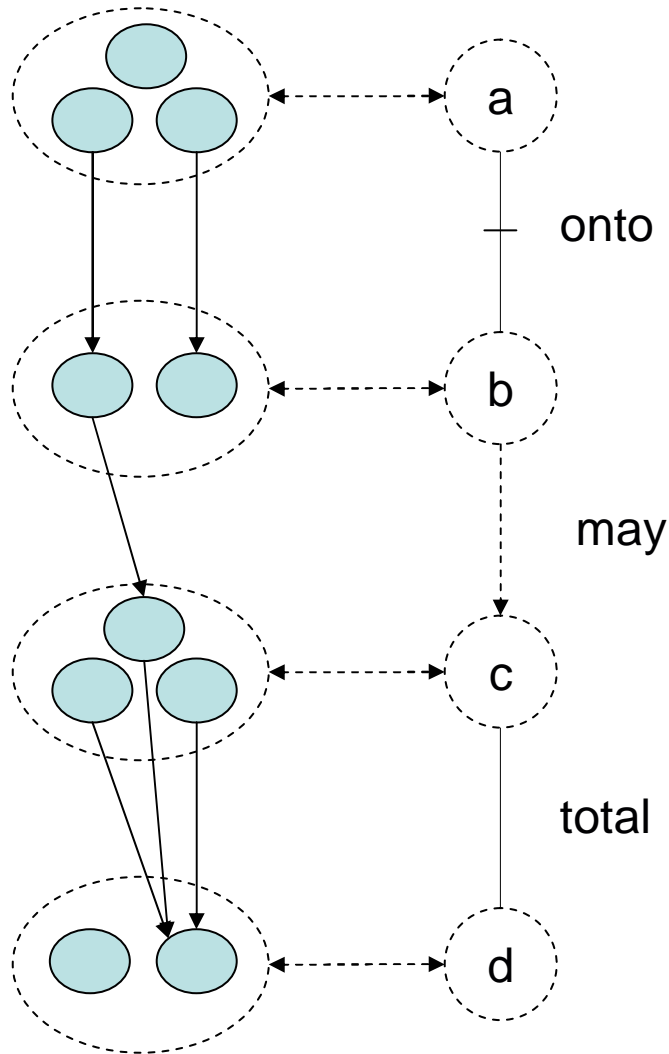


$$U = (\text{may})^*$$

# Pessimistic Lower Bound



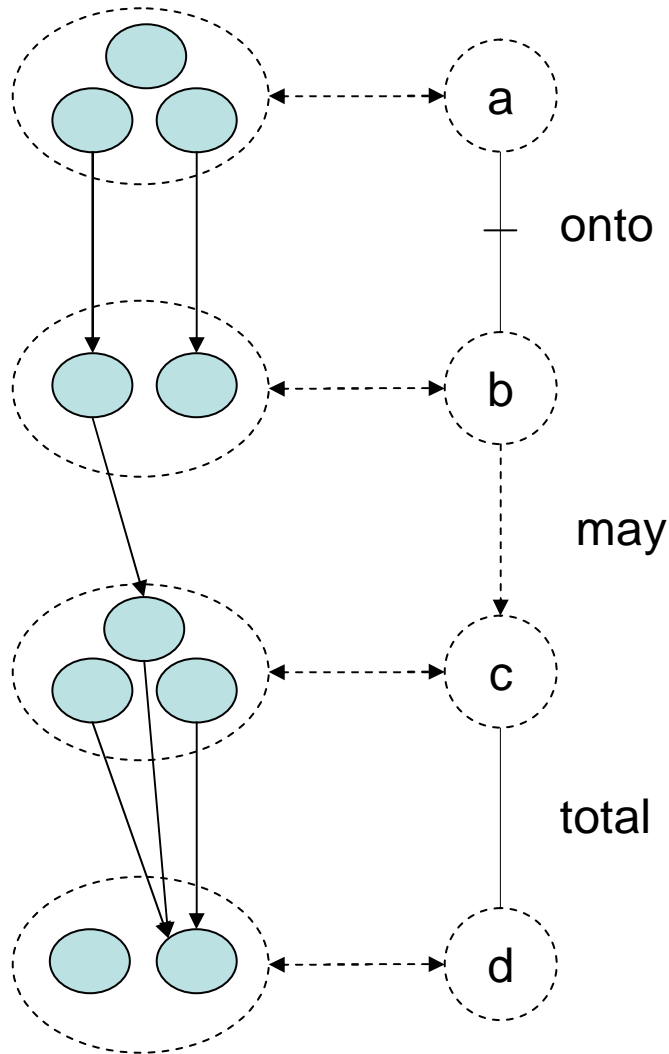
# Pessimistic Lower Bound



$(\text{onto})^*$  is onto

$(\text{total})^*$  is total

# Pessimistic Lower Bound



$(\text{onto})^*$  is onto

$(\text{total})^*$  is total

$$L_p = (\text{onto})^* \text{ may? } (\text{total})^*$$

# Example

```
void partition(int a[]) {  
    assume(a.length()>2);  
    int pivot = a[0];  
    int lo = 1;  
    int hi = a.length()-1;  
    while (lo<=hi) {  
        while (a[lo]<=pivot)  
            lo++;  
        while (a[hi]>pivot)  
            hi--;  
        if (lo<hi)  
            swap(a,lo,hi);  
    }  
}
```

# Observation Vector

[  $lo < hi$ ,  $lo \leq hi$ ,  $a[lo] \leq pivot$ ,  $a[hi] > pivot$  ]

- $lo < hi \Rightarrow lo \leq hi$
- $\neg lo < hi \wedge lo \leq hi \Rightarrow (a[lo] \leq pivot \wedge \neg a[hi] > pivot) \vee (\neg a[lo] \leq pivot \wedge a[hi] > pivot)$

Only 10/16 observations possible

```
void partition(int a[]) {
    assume(a.length()>2);
    int pivot = a[0];
    int lo = 1;
    int hi = a.length()-1;
```

13 labels x 10  
observations =  
**130** observable states

```
L0: while (lo<=hi) {
L1:     ;
L2:     while (a[lo]<=pivot) {
L3:         lo++;
L4:     ;}
L5:     while (a[hi]>pivot) {
L6:         hi--;
L7:     ;}
L8:     if (lo<hi) {
L9:         swap(a,lo,hi);
LA:     ;}
LB: ;}
LC: ;
}
```

But, program  
constrains **reachable**  
observable states  
greatly.

# Boolean Program

```
void partition() {
  decl lt, le, al, ah;
  enforce ( (lt=>le) &
            ((!lt&le)=>(al&!ah)|(!al&ah)) );
  lt,le,al,ah := T,T,*,*;
L0: while (le) {
L1:   ;
L2:   while (al) {
L3:     lt,le,al := (!lt ? F:*), lt, *;
L4:   ;}
L5:   while (ah) {
L6:     lt,le,ah := (!lt ? F:*), lt, *;
L7:   ;}
L8:   if (lt) {
L9:     al,ah := !ah,!al;
LA:  ;}
LB: ;}
LC: ;
}
```



# State Space of Boolean Program

[ lo < hi, lo <= hi, a[lo] <= pivot, a[hi] > pivot ]

	TTTT	TTTF	FTTF	FFTF	TTFT	FTFT	FFFT	TTF	FFFF	FFTT
L0	x	x			x			x	x	
L1	x	x			x			x		
L2	x	x	x	x	x	x		x	x	
L3	x	x	x	x						
L4	x	x	x	x	x	x		x	x	
L5					x	x	x	x	x	
L6					x	x	x			
L7					x	x	x	x	x	
L8								x	x	
L9								x		
LA	x									
LB	x									x
LC										x

Upper Bound = 49 states



# Test Generation

- DFS of  $L_p$  generates covering set of paths
- Symbolically execute paths to generate tests
- Run program on tests to find errors and compute coverage of observable states

# Generated Inputs

```
void partition(int a[]) {  
    assume(a.length()>2);  
    int pivot = a[0];  
    int lo = 1;  
    int hi = a.length()-1;  
  
    L0: while (lo<=hi) {  
        L1:     ;  
        L2:     while (a[lo]<=pivot) {  
            L3:         lo++;  
            L4:         ;}  
        L5:     while (a[hi]>pivot) {  
            L6:         hi--;  
            L7:         ;}  
        L8:     if (lo<hi) {  
            L9:         swap(a,lo,hi);  
            LA:        ;}  
        LB:    ;}  
        LC:    ;  
    }  
}
```

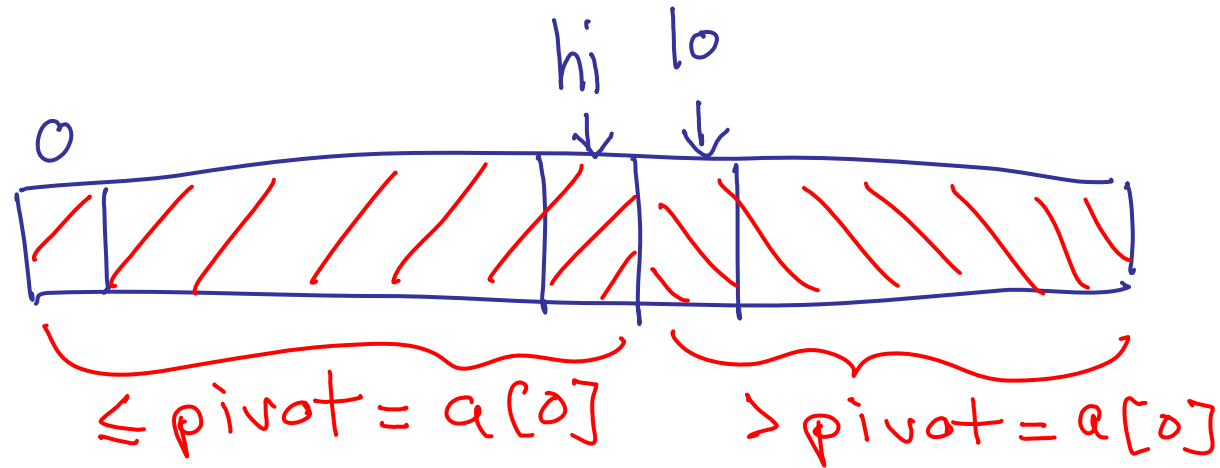
Array bounds violations

(L0:TTTT,L4:FTFT)	{ 0,-8,1 }
(L0:TTTT,L4:TTFT)	{ 0,-8,2,1 }
(L0:TTTT,L4:TTTT)	{ 0,-8,-8,1 }
(L0:TTTF,L4:TTFF)	{ 1,-7,3,0 }
(L0:TTTF,L4:FTTF)	{ 0,-7,-8 }
(L0:TTTF,L4:TTTF)	{ 1,-7,-7,0 }
(L0:TTFT,L7:TTFF)	{ 0,2,-8,1 }
(L0:TTFT,L7:FTFT)	{ 0,1,2 }
(L0:TTFT,L7:TTFT)	{ 0,3,1,2 }
(L0:TTFF,L0:TTTT)	{ 1,2,-1,0 }

# Results

- Buggy partition function
  - U=49, L=43, Tested=42
- Fixed partition function
  - U=56, L=37, Tested=43
- What about the remaining 13 states?

# Refinement



Existing predicates do not precisely track boundary condition

$$lo < hi, lo \leq hi, a[lo] \leq pivot, a[hi] > pivot$$

# New Observation Vector

[  $lo < hi$ ,  $lo \leq hi$ ,  $lo = hi + 1$ ,  
 $a[lo] \leq pivot$ ,  $a[hi] > pivot$ ,  
 $a[lo - 1] \leq pivot$ ,  $a[hi + 1] > pivot$   
]

Only 48/128 observations possible

***For this set of predicates,  $L_p = U$***

# Conclusions

- PCT coverage
  - new form of state-based coverage
  - similar to path coverage but finite
- Upper and lower bounds
  - computed using predicate abstraction and modal transitions
  - use lower bound to guide test generation
  - refine bounds